Bank fragility and the incentives to manage risk*

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Abstract

Shocks to banks' ability to raise liquidity at short notice can lead to depositor panics, as evidenced by recent bank failures. Why don't banks take a more active role in managing these risks? In a standard bank-run model, we show that risk management failures are most prevalent when exposures are more severe and managing risk would be particularly valuable. Bank capital and deposit insurance coverage act as substitutes for risk management on the intensive margin but as complements on its extensive margin, encouraging the adoption of risk management operations. We provide insights for the appropriate regulation of bank risk-management operations.

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1 Introduction

The recent collapse of a number of financial institutions, and the precarious positions of others, provides a stark reminder of how issues related to financial instability and panics are still of paramount concern, despite increases over the last decade on the dollar amount of deposits subject to deposit insurance coverage and the significant enhancements to the regulatory framework since the Global Financial Crisis (GFC). For instance, Silicon Valley Bank (SVB), a large regional lender in California, suffered unexpectedly large withdrawals in March 2023. This, coupled with reduced liquidation values for assets resulting from the increases in US interest rates, lead to massive withdrawals, triggering a liquidity crisis. While there appear to have been many reasons for SVB's difficulties (Fed Board, 2023), the ensuing "panics" occurred despite little evidence of long term insolvency for SVB. Similarly, large deposit outflows have been observed at other medium-sized banks (Caglio et al., 2024; Choi et al., 2024), creating a crisis of confidence at many of these institutions, with concerns of possible spillovers to the banking sector at large. For many of these financial institutions, the primary concern appeared to revolve around low values of assets subject to early liquidation when meeting depositor redemptions, rather than from accumulating loan losses or other asset impairments, even if these may have been lurking in the background.

While there is a broad literature on the possibility of depositor panics for financial institutions and the need to provide liquidity insurance to risk averse depositors, to our knowledge less attention has been paid on how reduced bank resources in the interim can trigger a crisis of confidence for depositors (notable exceptions include Allen and Gale, 2004; Vives, 2014; Liu, 2023, which we discuss in more detail below). Anticipating this possibility, why don't banks take a more active role in reducing their exposure to these risks and the consequent effects for the financial and non-financial sector? In other words, how do banks' incentives to manage risk, and in particular uncertainty related to the liquidation value of bank assets, depend on the degree of exposure to runs that banks face? Going further, how do capital structure considerations, which are often a tool for macroprudential policy, or deposit insurance, a common form of government guarantee, influence a bank's incentives to manage risk?

To study these issues, we present a two-period global-games model of bank runs, but where banks face uncertainty about the interim value of their assets if required to liquidate them early to meet withdrawals. This reflects that the bank, as well as investors, can only estimate what the interim value of the bank's assets may be when they first invest, and this value may be uncertain for a number of reasons, such as changes in interest rates, changes in the competitiveness of the secondary market for assets (such as in e.g., Shleifer and Vishny, 1986), or simply poor maintenance of assets purchased with the cash provided by the lender and which serve as loan collateral. In our model, banks collect deposits from risk-neutral consumers and invest them, along with their own capital, in a long term project with high, but uncertain, payoffs. The bank's objective is to maximize its profits, taking into account the need for consumers to be willing to deposit their funds with the bank. In order to meet any early withdrawals that take place before the maturity of investment, the bank must liquidate some fraction of its long-term assets and use the proceeds to satisfy redemptions. Following the literature using global games to model banking panics (e.g., Goldstein and Pauzner, 2005), each depositor receives a noisy private signal about the long-term fundamental value of the bank and uses that information to decide whether to withdraw early.

We first show, as is standard in similar settings, that there is a unique equilibrium in which all depositors choose to withdraw early if the signal about the fundamental is sufficiently bad. In other words, they *run* on the bank, with some of these runs being pure panics, driven by a coordination failure among depositors, with depositors withdrawing because other depositors are also withdrawing. The cutoff below which runs occur depends crucially on the realization of uncertainty—here constituting the shock to the value of the bank's assets at the interim date. We show that, consistent with other literature studying similar issues (see, e.g., Vives 2014; Liu 2023), the realization of a negative shock to the interim value of bank assets reduces financial stability and leads to a greater probability of depositor runs.

Given the greater run risk stemming from negative shocks to the interim value of assets, we ask whether banks have incentives to hedge this risk, and how such incentives depend on the magnitude of the risk they face. To study these issues, we incorporate a stylized model of contingent risk management where banks can take an action that increases the interim value of their assets whenever there is a negative shock to their value. This can be interpreted as the purchase of a derivative contract that pays in the event of a negative shock, but more generally can also represent any action that reduces the assets' exposure to other factors that may decrease their value, such as bank monitoring of assets or searching for more liquid resale markets.

While banks may have an incentive to manage much of their risk when the degree of exposure is relatively small, they find it optimal not to engage in risk management as their exposure becomes larger. More precisely, risk management failures are most prevalent when fragility is most extreme, so that managing such risk would be most beneficial. The intuition stems from recognizing that as the negative shock to the interim value of assets becomes large, financial fragility increases. At some point, the bank's benefit from surviving, which is inversely proportional to the probability of a run, becomes too small relative to the costs of establishing risk management capabilities. This cost is likely associated with the bank's size, with smaller banks finding it more costly to sustain upfront investments to engage in risk management. Thus, our result hints to a positive effect of size on risk management failures, with larger banks choosing to set up risk management operations even when faced with the possibility of more extreme negative shocks.

Our analysis highlights the importance of bank capital structure as a key determinant of the incentives to manage risk, as well as whether risk management failures occur. Likewise, the extent to which bank liabilities are subject to runs, which is determined by the fraction of deposits covered by government guarantees, plays an important role in driving a bank's risk management incentives. We first show that better capitalized banks, as well as those with a larger fraction of insured deposits, have, at the margin, a reduced need to manage risk, as well as lower incentives to do so—a reduction on the intensive margin. So for banks that engage in risk management practices, both capital and deposit insurance coverage act as substitutes for risk management, whereby both serve to reduce a bank's exposure to runs, but the marginal benefit to doing so is lower when the other component is higher. Second, however, both higher bank capital and having fewer deposits that are subject to runs act as a catalyst for risk management in the sense that, all else equal, banks that are better capitalized or that rely on a larger fraction of insured deposits choose to set up risk management operations in instances where less capitalized banks, as well as banks with a less stable funding base, would choose not to do so. In other words, bank capital (as well as deposit insurance coverage) and the establishment of a risk management desk are complementary.

These results highlight a differential effect of capital on the intensive and extensive margin of risk management. The substitutability between capital and risk management suggests that better-capitalized banks do less risk management on the intensive margin. On the other hand, their complementarity suggests better-capitalized banks face less risk management failures.

We consider several extensions and robustness checks. First, we endogenize the bank's capital structure and show that banks indeed find it optimal to raise capital, so that our argument that capital matters for risk management incentives is not undercut by a finding that banks never would want to issue capital anyway. Second, we show that bank risk-management decisions are inefficient. Due to its lack of commitment (a form of moral hazard), banks do less risk management than what would be chosen by a social planner with the power to commit to risk management. Moreover, banks choose to do no risk management at all for values of the interim asset for which the planner would find it optimal to engage in risk-management. Hence, the model exhibits true failures of risk management. Finally, we show that our result on the occurrence of risk-management failures is robust to alternative forms of risk management.

Our main contribution is to provide a fully rational framework where, even as their exposures to risks that can lead to depositor runs and disintermediation increases, banks may have reduced incentives to manage these risks. This then leads to an increased probability of bank failure, in a setting where such risk can be managed and where it would be socially efficient to do so. Banks may not find it optimal to hedge their exposure to this risk even though the distribution of uncertainty is known by everyone ex ante, including the depositors who are making withdrawal decisions. To the extent that there may also exist information asymmetries or behavioral factors that can in practice drive depositor behavior, we believe that those would magnify the effects we identify, but they are not necessary to generate an increased incidence of panics. Likewise, institutions' reduced incentives to manage risk emerge precisely as such risk increases arises as a natural consequence of the maximization problem banks face.

Literature. Our paper contributes to the long-standing literature on the determinants of bank fragility and policies to mitigate it. Panic-driven depositor runs, whereby crises are a self-fulfilling phenomenon hinging on strategic complementarities in depositors' withdrawal decisions, have received considerable attention following the seminal work by Diamond and Dybvig (1983). The application of global games methods (Carlsson and van Damme, 1993; Morris and Shin, 2003; Vives, 2005) establishes a link between depositors' withdrawal incentives and a bank's fundamentals (Rochet and Vives, 2004; Goldstein and Pauzner, 2005). This approach allows to endogenize the probability of a bank run and to study its dependence on bank characteristics and government policies. Examples include work on the implications for fragility of information disclosure (Bouvard et al., 2015), debt maturity structure (Eisenbach, 2017), the level of collateralization (Ahnert et al., 2019), government guarantees (Allen et al., 2018; Carletti et al., 2023a), bank capital structure and portfolio liquidity (Kashyap et al., 2023), and interventions in panic-based bank runs (Shen and Zou, 2024). Our paper also uses similar methods to analyze how risk management affects the probability of a bank run.

In our framework, the rationale for risk management arises from increased bank fragility upon a negative shock to the interim value of assets. A negative association between interim asset values and bank fragility has been previously established by Vives (2014) and Liu (2023), among others. Our paper takes this relationship as a starting point and analyzes the extent to which risk management can offset the detrimental impact of a lower interim asset value on fragility. More importantly, we also study how the severity of the shock, and the associated fragility, affects banks' incentives to manage risk in the first place, an aspect that to our knowledge has not previously been studied but which is important for the debate on the stability of the banking sector.

Our main result is that banks fail to engage in risk management when the shock to the interim asset value is severe and runs are likely. By focusing on the incentives to hedge against liquidity risk, our analysis complements the literature on failures in risk management, whose focus is, instead, on credit risk. Similarly to our paper, Bouvard and Lee (2020) develop a framework in which risk management failures emerge from a coordination problem among financial firms. However, they focus on the information acquisition process, with firms acquiring too little information related to the assessment of risks, while we analyze the decision to exert effort to mitigate the risks that have been identified. A similar focus is also present in (Rampini and Viswanathan, 2010, 2013). In the first paper, they show that risk management incentives depend on institutions' financial constraints in that those that are more financially constrained tend to hedge less and do not engage in any risk management. While the implication that smaller firms, which tend to be more financially constrained, are more exposed to risk management failures resembles our result on bank size, the key friction behind the result is very different. In our paper, depositors' coordination failure, rather than limited pledgeability, plays a crucial role and allows us to study specific policies that provide both contingent as well as non-contingent management of risk. Extending the analysis to a dynamic framework, Rampini and Viswanathan (2013) show that risk management failures occur when financial firms are hit by a series of adverse shocks. In this respect, despite the differences in the modelling approach, our paper shares with theirs the implication that banks fail to engage in risk management when it would be most needed.

The analysis in our paper is also related to the recent contributions analyzing the U.S. regional banking crisis of 2023. This turmoil in the US banking sector had a strong flavor of panic-induced runs, occurring with unprecedented speed in a digitized world and amplified by social media (Cookson et al., 2023). In view of the root causes, Jiang et al. (2023) document a substantial drop in asset values for SVB following the tightening in monetary policy. This, combined with its extreme reliance on uninsured deposits, emerges as the driver behind the solvency-driven run in their theoretical framework. In the model developed in Drechsler et al. (2023), runs stem from a pure coordination failure associated with a valuable deposit franchise (Drechsler et al., 2017). The deposit franchise contributes to bank value only if deposits are retained. Since the value of a bank's deposit franchise is very high in a high rate environment, deposit withdrawals have a large impact on bank value, making it optimal to withdraw if a depositor believes that others would do the same. Relative to these papers, we focus on the destabilizing effect of a reduced value of bank assets at the time of depositors' withdrawals and the reasons why banks fail to set up effective risk management strategies despite the fact that potential drops in interim asset values are anticipated. In this respect, our paper shares the view presented in Dursun-de Neef et al. (2023) that the SVB failure was driven by the losses incurred by the bank from the sale of their long-term assets.

Our result on the relationship between the use of contingent risk management tools and bank capital, which dampens the strategic complementarity in depositors' withdrawal decisions and so limits the occurrence of runs, speaks to the emerging discussion on the implications of the regional banking crisis for banking regulation (Acharya et al., 2023). In this respect, since bank incentives to put in place risk management operations increase with bank capital, our analysis highlights a novel complementarity between bank capital and contingent risk management tools that underscores the value of bank capital beyond its well-known stabilizing role.

Structure. The paper is organized as follows. Section 2 describes the model. Section 3 characterizes depositors' withdrawal decisions. Section 4 analyzes the incentives for risk management and its dependence on bank capital. Section 5 analyzes the role of deposit insurance in affecting bank risk management decisions. Section 6 discusses robustness and extensions, where we endogenize the deposit rate and bank capital, analyse an efficiency benchmark, and consider an alternative approach to modelling risk management. Finally, Section 7 concludes. All proofs are in the Appendix.

2 Model

The model builds on Goldstein and Pauzner (2005) and Carletti et al. (2023b). The economy extends over three dates $t = 0, 1, 2$ and is populated by a monopolistic bank and a unit continuum of consumers indexed by $i \in [0, 1]$. All agents are risk neutral and do not discount the future. Consumers are indifferent between consuming at either date. There is a single divisible good for consumption and investment. At $t = 0$, the bank is endowed with $k \in (0, 1)$ units (bank capital) and consumers are endowed with $1 - k$ units each.

The bank has access to a profitable but risky long-term investment technology, such as corporate loans, that requires an investment of one unit at $t = 0$. It returns $\ell \in (0, 1)$ if liquidated prematurely at $t = 1$ and $R\theta$ upon maturity at $t = 2$, where $\theta \sim U[0, 1]$ represents the fundamentals of the economy and R is a constant that reflects the return from financial intermediation, which is assumed to be high enough to ensure bank viability.[1]

To capture a role for risk management as a way to hedge against the realization of negative shocks, we assume that there is aggregate uncertainty about the interim value of the bank's assets (as in e.g., Allen and Gale, 2004, 2007; Eisenbach, 2017). This uncertainty is realized and observed at the beginning of $t = 1$:

$$\ell = \begin{cases} \ell_H & w.p. \;\; 1 - p_\ell \\ \ell_L & w.p. \;\; p_\ell, \end{cases} \tag{1}$$

for all $\theta \leq \overline{\theta}$, where $0 < \ell_L < \ell_H \leq 1$.[2] Following Goldstein and Pauzner (2005), we assume that there exists a value $\overline{\theta} \in (0, 1)$ such that, for $\theta \geq \overline{\theta}$, the bank always has sufficient resources to settle all date-1 claims. Specifically, we assume that $\ell = 1$ for $\theta > \overline{\theta}$, so that the asset's value is not impaired under liquidation. In what follows, we assume that $\overline{\theta} \to 1$.[3]

At $t = 0$, the bank can hedge against the shock of a low interim asset value $(\ell = \ell_L)$ by exerting risk management effort z. Hedging allows the bank to increase the interim asset value by an amount $z > 0$ to $\ell_L + z$ upon the negative shock. Risk management is costly and entails a non-pecuniary cost $c\frac{z^2}{2} + F$. We discuss in detail the interpretation for the risk management variable z and its cost in Section 4, as well as an alternative modelling approach to risk management in Section 6.4.

To finance investment, the bank collects external funds from consumers at $t = 0$ in exchange for a deposit contract that is demandable at par at $t = 1$.[4] At the beginning of $t = 0$, the bank chooses the repayment at $t = 2$, r_2, to maximize its expected profits subject to consumer participation. The opportunity cost of consumers is $\rho_D \geq 1$. Since debt is demandable, depositors can withdraw their funds at $t = 1$, before the bank's investment matures. At the beginning of $t = 1$, each depositor receives a noisy private signal about

[1] A necessary condition for this is $R > 2$.

[2] Allowing for a strictly positive probability that there is no shock to the interim asset value guarantees that intermediation is always feasible, even when the interim asset value in state L is very low. Moreover, the existence of two aggregate states also allows us to consider risk management tools that transfer resources from the good state (H) to the bad state (L), as we show in Section 6.4.

[3] This assumption is without loss of generality: $\overline{\theta}$ can be arbitrarily close to 1 without affecting any results.

[4] Bank debt is assumed to be demandable, which arises endogenously with liquidity needs (Diamond and Dybvig, 1983), as a commitment device to overcome agency conflicts (Calomiris and Kahn, 1991; Diamond and Rajan, 2001; Ahnert and Perotti, 2021), or as a way to maximize profits (Carletti et al., 2023a) by well-capitalized banks. Accordingly, uninsured deposits refer to any short-term or demandable debt instrument, including uninsured retail deposits and insured deposits when deposit insurance is not credible (Bonfim and Santos, 2020). In the U.S., three quarters of commercial bank funding are deposits, half of which are uninsured (Egan et al., 2017).

the fundamental:

$$s_i = \theta + \varepsilon_i, \tag{2}$$

with $\varepsilon_i \sim U[-\epsilon, +\epsilon]$. These signals are identically distributed and conditionally independent. A depositor's signal provides information about both the bank's profitability and the signals received by other depositors (and hence their withdrawal choices). Following much of the global-games literature, we assume vanishing noise to simplify the analysis, $\epsilon \rightarrow 0$.[5]

The bank satisfies early withdrawals n by liquidating the long-term investment. If withdrawal demand exceeds the liquidation value of the bank's assets, each depositor receives an equal share of the proceeds at $t = 1$. If the remaining proceeds from the project are not enough to satisfy depositors upon maturity at $t = 2$, the bank is bankrupt. In this case, the bank makes zero profits and depositors receive nothing due to the presence of bankruptcy costs.[6]

Table 1 summarizes the timing of the model. The model is solved backwards. We start with the analysis of the withdrawal game at $t = 1$ in Section 3. Then, we move on to the characterization of the bank's risk management choices in Section 4. In both sections, we take the deposit contract and bank capital structure as given, but we will endogenize these in later sections.

Date 0	**Date 1**	**Date 2**
1. Bank invests funds in a risky investment	3. The liquidation value ℓ is observed	6. Investment matures
2. Bank chooses its risk management effort	4. The fundamental θ is realized but unobserved; depositors receive a noisy private signal s_i and may withdraw	7. Residual depositors are paid
	5. Bank liquidates to meet withdrawals	8. Consumption

Table 1: Timeline of events.

[5]Vanishing private noise simplifies the analysis of the bank's ex-ante choices of the deposit contract and its risk management effort. Vives (2014) studies the properties of equilibria in a similar setup when this assumption is relaxed.

[6]For simplicity, we assume full bankruptcy costs. This is consistent with evidence that bankruptcy costs are large. E.g., James (1991) measures the losses associated with bank failure as the difference between the book value of assets and the recovery value less direct costs associated with failure. These losses amount to about 30% of failed banks' assets.

3 Depositors' withdrawal decisions and the probability of a run

In this section, we characterize depositors' interim withdrawal decisions for a given deposit contract $\{1, r_2\}$ and risk management choice z. We use ℓ to denote the interim asset value, which is either ℓ_H or $\ell_L + z$, depending on whether the negative shock has been realized and the extent of the bank's risk management effort. We assume that $z < \ell_H - \ell_L$, so that the bank's risk management does not fully compensate for the lower amount of interim resources available upon the negative shock.[7]

Depositors simultaneously decide whether to withdraw from the bank or to keep their funds deposited upon observing their private signal. As is standard in a global-games framework, when the fundamental of the economy is low, withdrawing early is a dominant action for depositors, even when all depositors do not withdraw. This is the case when $\theta < \underline{\theta}$, which solves:

$$R\theta - (1-k)r_2 = 0. \tag{3}$$

In this range, which we refer to as the *lower dominance region*, the bank is insolvent at date 2 even if no depositor has withdrawn early. As a result, each depositor expects to receive nothing at date 2 (due to costly bankruptcy) and so finds it optimal to withdraw early irrespective of what other depositors do. At the other extreme, when $\theta > \overline{\theta}$, waiting until date 2 is a dominant action. For such high values of the fundamental, the liquidation value fully covers the promised payment to all withdrawing depositors, so early withdrawals does not impose any loss on depositors who choose to wait until $t = 2$ to withdraw. Accordingly, these depositors prefer the payment of r_2 over the lower payment of $r_1 = 1$ at $t = 1$. Similarly, we refer to this range as the *upper dominance region*.

For the intermediate range $(\underline{\theta}, \overline{\theta})$, the withdrawal decision of a depositor depends on the withdrawal decisions of other depositors. The following proposition describes these withdrawal decisions and characterizes the bank failure threshold.

Proposition 1. *All depositors withdraw their funds at $t = 1$ and the bank fails if the fundamental θ falls below a threshold θ^*, which depends on the interim asset value ℓ and bank capital k:*

(i) For $\ell \geq 1 - k$, there are only fundamental runs and the threshold is the lower dominance bound:

$$\theta^* = \underline{\theta} \equiv \frac{(1-k)r_2}{R}. \tag{4}$$

[7] As shown in Section 4, $z^* < \ell_H - \ell_L$ is (privately) optimal when the variable cost parameter c is not too small.

(ii) For $1-k>\ell$, there are also panic runs and the threshold is:

$$\theta^* = \frac{(1-k)r_2\left(1-\frac{\alpha}{r_2}\right)}{R\left(1-\frac{1-k}{\ell}\frac{\alpha}{r_2}\right)} = \underline{\theta}\frac{r_2-\alpha}{r_2-\frac{\alpha(1-k)}{\ell}} > \underline{\theta}, \tag{5}$$

where $\alpha = \int_0^{\overline{n}} dn + \int_{\overline{n}}^1 \frac{\ell}{(1-k)n} dn$ is a depositor's expected payoff from withdrawing early and $\overline{n} = \ell/(1-k)$ is the maximal level of withdrawals that the bank can serve in full at the interim date.

Proof: See Appendix A.1.□

Proposition 1 establishes the existence of a unique equilibrium of the withdrawal subgame, where depositors use threshold strategies and optimally decide to withdraw upon unfavorable private signals, $s_i < s^*$. For vanishing private noise, this signal threshold converges to the threshold of the economic fundamental, $s^* \to \theta^*$. This implies that, in equilibrium, all depositors behave alike: they withdraw for $\theta < \theta^*$ and there are no withdrawals for $\theta \geq \theta^*$.

Depending on the interim asset value ℓ and bank capital k, pure panic runs arising from coordination failure may or may not be possible (see also Vives, 2014; Carletti et al., 2023b). When the interim value of the bank's asset ℓ is high enough to entirely cover repayment to all depositors, $1-k$, then there is no strategic complementarity among depositors' withdrawal decisions and runs only occur when the return of the bank's long-term investment is low. In other words, runs can only happen when the bank would be unable to meet its repayment obligations at the final date even if no depositors withdraw at the interim date, $t=1$. This is the case when $\theta < \underline{\theta}$. On the other hand, if the interim value ℓ is not enough to fully cover depositors' repayments at $t=1$, i.e., if $\ell < 1-k$, then panic runs are also possible since the bank's ability to fully repay depositors at the interim date depends on how many depositors are withdrawing. Thus, depositors fear that other depositors withdraw their funds and that the bank will not have enough to repay them as promised at the final date. In this case, $\theta^* > \underline{\theta}$ and panic runs occur when $\theta \in (\underline{\theta}, \theta^*)$.

Next, we examine the determinants of the failure threshold at $t=1$, for given risk management choice of the bank z and deposit rate r_2. Besides determining the nature of runs, i.e., whether they are fundamental-driven or also due to depositors' panic, the interim asset value ℓ and bank capital k also affect the probability of each type of run, as we show below.

Proposition 2. *The comparative statics with respect to bank capital and the interim asset value are:*

(i) For $\ell \geq 1-k$, the fundamental run threshold decreases in bank capital, $\partial \underline{\theta}/\partial k < 0$, while the interim asset value has no effect on the failure threshold: $\partial \underline{\theta}/\partial \ell = 0$.

(ii) For $1-k > \ell$, the run threshold decreases in bank capital and the interim asset value at a diminishing rate, $\partial \theta^*/\partial k < 0$, $\partial^2 \theta^*/\partial k^2 > 0$, and $\partial \theta^*/\partial \ell < 0$, $\partial^2 \theta^*/\partial \ell^2 > 0$. The effects of bank capital and the interim asset value on depositors' run risk are substitutes, $\partial^2 \theta^*/\partial \ell \partial k > 0$, so that the beneficial effect of a larger interim value for the bank's assets is lower when the bank has more capital. The range of fundamental realizations with panic runs decreases in bank capital and the interim asset value, $\partial(\theta^* - \underline{\theta})/\partial \ell < 0$ and $\partial(\theta^* - \underline{\theta})/\partial k < 0$.

Proof: See Appendix A.1. □

The failure threshold θ^* represents the ex-ante probability of a bank run, which is our measure of financial stability. Based on the results in Proposition 2, we have that $\theta_L^* \equiv \theta^*(\ell_L + z, k) > \theta_H^* \equiv \theta^*(\ell_H, k)$ given $\ell_L + z < \ell_H < 1$. This implies that, for values of the economy's fundamental θ such that $\theta \in (\theta_H^*, \theta_L^*)$, a run on the bank occurs when the interim asset value is low that would not have taken place otherwise. Furthermore, only the cutoff for panic runs θ^* depends on the realization of the interim asset value, so a negative shock to it not only increases the overall likelihood of a bank failure, but also the range in which runs are driven by the coordination failure among depositors.[8]

The lower interim value of the bank's assets exacerbates the coordination failure among depositors and induces them to want to withdraw their deposits for fear of others withdrawing as well, even though coordinating not to panic would be Pareto improving, benefiting both depositors and the bank. This problem becomes more severe the lower is ℓ, which is reflected in the increased sensitivity of the threshold. This effect makes risk management particularly valuable socially when the negative shock to the interim asset value is large. Importantly, the destabilizing effect of the negative shock to the interim asset value arises even though, by construction, there is no change in the long-term value of the project, $R\theta$, but rather only in the bank's resources at $t = 1$. Our focus therefore is purely on risk stemming from reduced interim values of assets, which in the absence of the possibility of depositor runs would have no implications for bank stability, but which exposes banks to possibly severe instability when depositors can decide whether

[8] These findings are consistent with and analogous to the result in Vives (2014), who considers comparative statics to the interim value of the bank's assets and shows that lower interim values lead to a greater risk of panics. Although the setting is slightly different, and does not consider the withdrawal incentives of depositors directly, the mechanism here is similar: the reduced interim asset value worsens the coordination failure among depositors (or fund managers in Rochet and Vives, 2004; Vives, 2014).

to withdraw early.[9]

In light of the increased fragility due to a negative shock to the interim asset value, we examine the implications for the bank's risk management choices when it rationally anticipates the possibility of such an event. Our main interest is in contingent risk management tools that are best suited to deal with severe negative shocks that occur with a potentially low, but positive probability.

4 Risk management

The probability of a run increases after a negative shock to the interim value of bank assets. Such a shock could arise, for example, because part of the bank's portfolio may represent fixed-income assets whose value at the interim date may decline substantially if interest rates rise, perhaps due to a rapid tightening of monetary policy. Alternatively, the shock may represent mismanagement of assets that further reduces their value in outside use, thus leading to depressed values at the interim date in the event they need to be sold to satisfy depositors' claims. It may also stem from sudden and unexpected drying up of secondary markets ("market freezes"), which cause the bank to suffer fire-sale losses when having to raise liquidity quickly. The effect of such a negative shock to the interim asset value can be quite large and result in highly fragile banks, which here is represented by having $\theta^*(\ell_L, k)$ approach $\overline{\theta}$, the bound for the upper dominance region.

Based on these considerations, we study the bank's incentives to hedge against such a negative shock to the interim value of its assets. Hedging against the shock results in the bank exerting effort z, so that the interim asset value after the negative shock becomes $\ell_L + z$. This formulation implies that ℓ_L and z are substitutes and that the effect of z on the failure threshold θ^* is the same as the effect of ℓ characterized in Proposition 2. By design, our notion here of risk management is narrow and is targeted to offset the exact risk faced by the bank, which is that it may be forced to liquidate assets whose value has dropped in the event of a run by depositors. We relax this assumption in Section 6.4 and study a setting in which risk management leads to a payout in the event of a negative shock, rather than only in the event of actual liquidation.

The amount z is chosen by the bank at $t = 0$, after the deposits are raised, and comes at non-pecuniary cost $c\frac{z^2}{2} + F$, borne by the bank. The variable part of the cost of hedging,

[9] Of course, shocks to the interim value of the bank's assets, ℓ, which represents a deterioration of the projects value, may also affect its long-term return, $R\theta$. This raises additional questions about the possibility of credit risk management in addition to the management of liquidity studied here. See the conclusion for a discussion of these aspects.

$c\frac{z^2}{2}$, may represent the search effort the bank must exert in finding an appropriate hedging instrument or partner, collateral that the seller might need to pledge against the derivative position, or counterparty risk. It may also reflect the cost of diligently monitoring the bank's assets or of identifying resale partners and opportunities, thus increasing the liquidity of the secondary market. The fixed part F, instead, can be interpreted as the cost of establishing a risk management desk, identifying the risk to be hedged, understanding the bank's exposure, etc. The assumption of a quadratic cost for hedging is clearly stylized, and is designed specifically to make at least some amount of risk management as easy as possible for the bank because the marginal cost of a minimal amount of effort z is zero.[10] In what follows, we assume that $\ell_L + z^* \leq \ell_H$ in any equilibrium, which effectively places a lower bound on the marginal cost of hedging, c. We also focus on the case where panic runs may arise, that is $\ell_H < 1 - k$, so the relevant failure thresholds are given by Equation (5) when evaluated at $\ell = \ell_L + z$ and $\ell = \ell_H$, respectively. In other words, we focus on cases where financial instability can clearly be inefficient, so that there is value to reducing the probability of a run.

As we will show in detail below, the choice of z depends crucially on the level of the interim asset value in the event of a shock ℓ_L. Since our primary focus is on identifying how the interim asset value affects depositor behavior and, in turn, bank incentives, in the analysis that follows in this section we hold all other variables fixed when we consider changes in ℓ_L. Doing so isolates the effect coming directly from changes in the interim asset value, leaving aside additional indirect effects stemming from the need to continue to satisfy depositors' participation constraint. It also has the additional benefit of allowing for a more tractable analysis, making the economics at work more transparent. Of course, changes in the distribution of the interim asset value, ℓ, also affect depositors' expected return, which may then require that the bank adjust r_2 in order to induce depositors to put their money in the bank. As we show in Section 6.1, our main result about risk management failures goes through also when r_2 is endogenized.

4.1 Risk management choice

The bank chooses whether and how much effort to exert with the objective of maximizing its expected profits. Since a run on the bank wipes out all resources, leaving nothing to

[10]Clearly, the costs of risk management in practice may differ from what we assume here and, in particular, even if convex these costs may not be quadratic. From this perspective, our framework is tilted against the possibility of extreme failures of risk management. As a result, we believe that our findings on risk management failures should be robust to the introduction of additional frictions and real-world considerations.

the bank (which occurs for $\theta < \theta_L^*$ and $\theta < \theta_H^*$), the bank only makes positive profits if no run occurs. In this case, the bank's investment project returns $R\theta$ and the bank keeps the remainder, $R\theta - (1-k)r_2$, after repaying depositors. Integrating over the possible realizations of the economic fundamentals, bank profits are:

$$\Pi(z) \equiv p_\ell \int_{\theta_L^*}^1 [R\theta - (1-k)r_2] d\theta + (1-p_\ell) \int_{\theta_H^*}^1 [R\theta - (1-k)r_2] d\theta - c\frac{z^2}{2} - F\mathbb{1}_{\{z>0\}}. \quad (6)$$

Differentiating (6) with respect to z gives us the first-order condition as follows:

$$\frac{d\Pi}{dz} = p_\ell [R\theta_L^* - (1-k)r_2] \left(-\frac{d\theta_L^*}{dz}\right) - cz = 0, \quad (7)$$

where $d\theta_L^*/dz = d\theta_L^*/d\ell$ because of the substitutability of ℓ_L and z and the latter derivative is given in Proposition 2. The first term in Equation (7) is the marginal benefit of risk management: higher z reduces the failure threshold θ_L^* in the bad aggregate state, which benefits the bank through the increase in residual profits around the failure threshold, $R\theta_L^* - (1-k)r_2 > 0$. The second term in (7) is the marginal cost of risk management.

Denote as $\widehat{z}$ the solution to Equation (7). The analysis above shows that, ignoring the fixed cost of setting up risk management operations, the bank always has an incentive to select a strictly positive risk management effort, $\widehat{z} > 0$. This follows directly from our specification designed to make risk management as easy as possible: the marginal cost of zero risk management is zero, while the marginal benefit is strictly positive. However, such incentives may be different depending on the the severity of the shock, as measured by ℓ_L. We have the following result.

Proposition 3. *The incentives to manage risk increase as the interim asset value in the event of a shock decreases: $d\widehat{z}/d\ell_L < 0$. Additionally, risk management incentives decrease as bank capital and the variable cost parameter c increase, while they increase as the probability of the negative shock increases: $d\widehat{z}/dk < 0$, $d\widehat{z}/dc < 0$, and $d\widehat{z}/dp_\ell > 0$.*

Proof: See Appendix A.2. □

A lower interim value of bank assets, as well as a lower level of capital and a more likely shock to the interim asset value all increase the banker's incentives to exert risk management effort. These effects are driven by the increased sensitivity of the probability of runs, which increases the marginal benefit from doing risk management and, in turn, the bank's incentives to exert effort.

Of course, whether the bank actually does any risk management depends on whether it is sufficiently profitable to do so. Whether bank profits are positive depends on the cost F of setting up risk management operations relative to the size of the shock, since the bank's expected profits will be small when the shock is large. In what follows, we show that, in line with the recent events in the US, the presence of these fixed costs—even if very small—may prevent the bank from engaging in any risk management. In particular, this occurs when the shock to the interim asset value is severe, so that the social value of mitigating the bank's exposure to risk would be particularly large.[11]

Before presenting our main result related to the bank's risk management choices, it is useful to first establish some notation that will be used in deriving the result. Denote by $\widehat{\ell}(F)$ the interim asset value ℓ_L such that bank profits in the event of a negative shock (i.e., if $\ell = \ell_L$) are equal to the cost of setting up risk management operations. In other words, $\widehat{\ell}(F)$ solves:

$$p_\ell \int_{\theta_L^*(\widehat{\ell}(F))}^{1} [R\theta - (1-k)r_2] d\theta \equiv F. \tag{8}$$

Note that, in defining $\widehat{\ell}(F)$, we are assuming the bank does no risk management, so that $z = 0$. Define by $\widehat{c}$ the value of the variable cost c such that, when $c = \widehat{c}$, the level of risk management chosen by the bank in the limiting case of $\ell \to 0$ is equal to $\widehat{\ell}(0)$. We establish the existence of such a $\widehat{c}$ in Appendix A.3. This assumption on c is primarily to guarantee that the bank cannot always fully offset negative shocks to the interim value of its assets, as previously discussed.

Proposition 4. *Let $c > \widehat{c}$. Then, for any $F > 0$, there exists an interim asset value $\widetilde{\ell}_L(F) \in (0, \widehat{\ell}(0))$ such that zero risk management is optimal, $z^* = 0$, when $\ell_L < \widetilde{\ell}_L(F)$.*

Proof: See Appendix A.3. □

The proposition shows that, for sufficiently large shocks, the bank prefers not to set up any risk management operations at all. Instead, the bank allows depositors to withdraw all their funds from the bank, leading to bank failures with near certainty upon a negative shock. This occurs even though the marginal value of hedging increases as ℓ_L decreases: $\partial\theta_L^*/\partial z$ increases as ℓ_L decreases. From the perspective of the bank, however, it only captures this value in the event a run does not occur, which becomes less likely as the interim liquidation value of assets decreases, even if the bank hedges optimally. Hence, for ℓ_L sufficiently low, the bank no longer finds it optimal to engage in risk management. Moreover, this is true even if the fixed cost F of setting up risk management operations is arbitrarily small, in the sense that for any cost F, there is always a value of the negative shock sufficiently large that ex ante the bank would prefer not to invest any resources in

[11]In Section 6.3 we show that the bank indeed does too little risk management relative to what would be socially optimal. Hence, the cessation of all hedging activities represents a failure in risk management.

creating a risk management desk.

Risk management choice

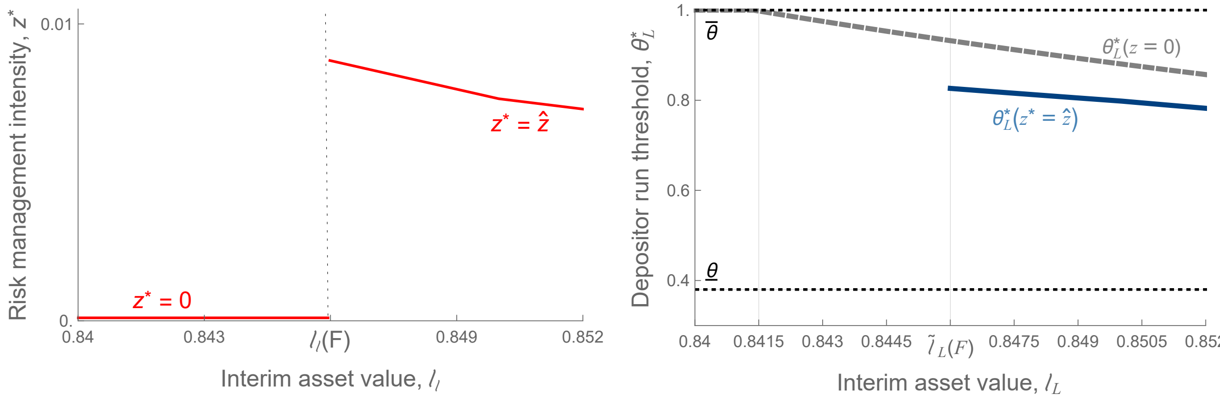

Figure 1: The left panel shows the risk management intensity z^* as a function of ℓ_L (red solid line) and the right panel shows the equilibrium run threshold θ_L^* as a function of ℓ_L for the case when $z = 0$ (gray dashed line) and for the case when $z^* = \widehat{z} > 0$ (blue solid line). This numerical illustration uses the parameters: $R = 3$, $p_\ell = k = 1/20$, $c = 100$, $F = 1/100$, and $r_2 = 6/5$.

The left panel of Figure 1 shows the optimal level of risk management effort z^* as a function of the interim asset value. For low interim asset values, $\ell_L < \widetilde{\ell}_L(F)$, the bank chooses not to set up a risk management desk and consequently $z^* = 0$ (Proposition 4). Conversely, the bank sets up a risk management desk when the negative shock to the interim asset is less severe, $\ell_L > \widetilde{\ell}_L(F)$. In this case, the marginal risk management incentive implies an optimal risk management effort $z^* = \widehat{z}$. The decrease in the marginal risk management incentive for higher levels of the interim asset value implies a negative relationship between $\widehat{z}$ and the interim asset value (Proposition 3). The right panel of Figure 1 shows the bank failure threshold as a function of the interim asset value, taking into account the optimal risk management choice. For $\ell_L > \widetilde{\ell}_L(F)$, the bank chooses to set up a risk management desk and the bank failure threshold is lowered from the gray dashed line to the blue solid line due to the the stabilizing effect of the risk management activity.

Through the cutoff $\widetilde{\ell}_L(F)$, Proposition 4 establishes that the realization of the shock to the interim asset value ℓ_L and the magnitude of the fixed cost F jointly matter for the bank's decision to engage in risk management. In other words, banks with different fixed cost F have a different tolerance to shocks and, in turn, ability to engage in risk management. Corollary 1 establishes an equivalent interpretation by considering that for any given shock ℓ_L, there must be a fixed cost F associated with setting up risk management operations such that banks with higher cost prefer not to do so. Denote such a cutoff value as $\widehat{F}$ and note that it is strictly positive for any $\ell_L > \widetilde{\ell}_L(0)$. We then have the following result for all cases where $\widehat{F} > 0$.

Corollary 1. *The cutoff* $\widehat{F}$ *increases in* ℓ_L: $\partial\widehat{F}/\partial\ell_L > 0$.

Proof: See Appendix A.4. □

This corollary emphasizes the relationship between the magnitude of the fixed cost F and the realized interim asset value ℓ_L. With a higher interim asset value, the bank can achieve a positive profit from risk management upon a negative shock even for a higher fixed cost of setting up risk management operations. Conversely, with a lower fixed cost, the bank can achieve a positive profit upon a negative shock even for lower interim asset values. This result suggests a link between bank size and their choice to set up risk management operations. To the extent that for larger banks the fixed cost F represents a smaller fraction of their total assets and revenue, we expect that larger banks find it optimal to set up risk management operations even when faced with the possibility of more extreme negative shocks to the interim value of their assets. By contrast, smaller bank engage in no risk management even for smaller possible shocks, recognizing that the benefit of managing this risk is insufficient to cover the costs in setting up such operations.[12]

4.2 Bank capital and risk management

In Section 4.1 we showed that a higher level of bank capital (e.g., due to higher regulatory bank capital requirements) is associated with a reduction in risk management effort if the solution to the risk management problem is interior (Proposition 3). However, k and z are not always substitutes. We show in this section that, all else equal, higher bank capital can be a catalyst for using contingent risk management, whereby it can facilitate the choice of a positive z^*. In other words, bank capital is complementary to the establishment of a risk management desk, even if it mutes the bank's marginal incentives to exert effort. To establish this complementarity, we analyze how $\widehat{F}$ is affected by bank capital.

Proposition 5. *The fixed cost threshold* $\widehat{F}$ *below which some risk management is optimal increases in bank capital,* $d\widehat{F}/dk > 0$.

Proof: See Appendix A.5. □

[12]It is straightforward to see that if all bank operations are scaled by a variable S, then the cutoff value of the fixed cost F above which banks do not engage in any risk management becomes $\widehat{F}/S$. Hence, for larger S, a given bank is more likely to have risk management operations in place, whereas smaller banks avoid such costs and instead risk collapse in the event of a sufficiently large negative shock.

The proposition establishes that the fixed cost threshold below which the bank finds it optimal to set up risk management operations and, hence, $z^* > 0$, increases in the bank's capital. This means that as bank capital increases, a larger set of banks should find it optimal to hedge their exposures. Therefore, a positive and *complementary* relationship between bank capital and setting up a risk management desk arises, which can be viewed as the effect of capital on the extensive margin of risk management, rather than the intensive margin, which would relate to the exact level of risk management whenever $z^* > 0$. Intuitively, more bank capital increases bank profits when it survives until the final date since it reduces the probability of runs as well as the repayment obligation to depositors, $(1-k)r_2$. This makes the bank more willing to bear the cost of setting up risk management operations, since it has more to gain by doing so.

Bank capital and risk management

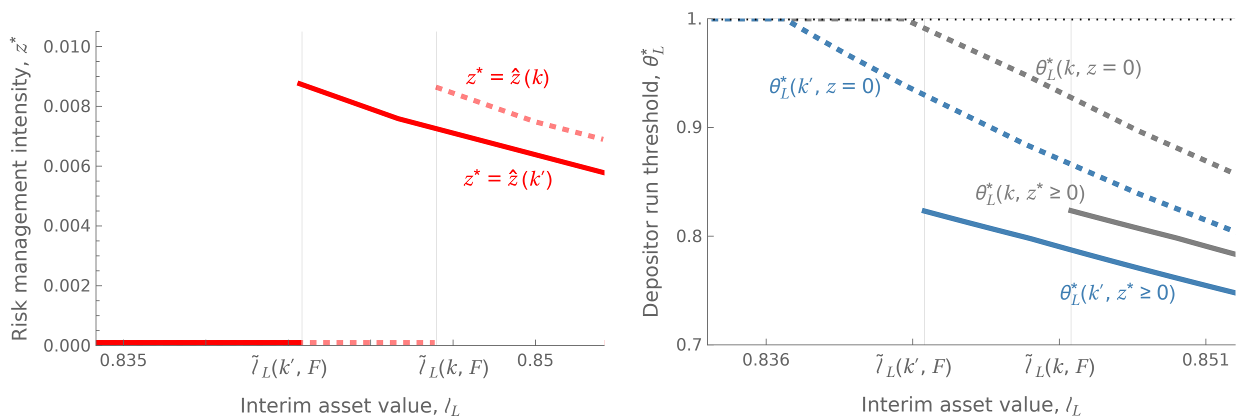

Figure 2: The left panel again shows z^* as a function of ℓ_L, but now for a higher level of bank capital $k' = 0.055$ (red solid line) and for the lower level of bank capital $k = 0.05$ that was previously shown in Figure 1 (red dotted line). The right panel shows depositors' run thresholds in equilibrium for both levels of bank capital, $\theta_L^*(k,z)$ and $\theta_L^*(k',z)$, as a function of ℓ_L. The gray and blue dashed (solid) lines depict the run thresholds for the case when $z = 0$ ($z^* = \hat{z} > 0$) for the lower and higher level of bank capital, respectively. All other parameters are the same as in Figure 1.

The left panel of Figure 2 shows the optimal level of risk management effort z^* and the right panel shows the bank failure threshold as a function of the interim asset value for two different levels of bank capital. We use the baseline level of bank capital $k = 0.05$ from Figure 1 and add a 10% higher level of bank capital $k' = 0.055$. The higher level of bank capital is associated with a lower risk management effort, i.e., $\hat{z}(k') < \hat{z}(k)$, conditional on setting up a risk management desk. That is, a bank with a more capital does less risk management at the intensive margin, implying that bank capital and risk management effort are *substitutes*, as shown in Proposition 3 and in the left panel of Figure 2. This result is due to the stabilizing role of bank capital, which is reflected in a lower run threshold for a higher levels of bank capital, i.e., $\theta_L^*(k', z=0) < \theta_L^*(k, z=0)$, as shown in Proposition 2 and represented by the dotted gray and blue lines. Also, $\theta_L^*(k', z^*) < \theta_L^*(k, z^*)$, as as shown by comparing the blue and gray solid lines.

However, the *complementary* relationship between bank capital and the establishment of a risk management desk can also be seen in Figure 2, and is illustrated by the interval $(\widetilde{\ell}_L(k', F), \widetilde{\ell}_L(k, F))$ of interim asset values where a higher level of bank capital is a catalyst for the establishment of a risk management desk. The figure thus illustrates how a bank with a higher amount of capital will engage in risk management activities even at lower levels of interim asset value, or equivalently, at higher levels of fixed costs, compared to a bank with less capital.

To show that banks indeed may want to raise capital as a source of financing even if, arguably, it represents a more expensive form of financing, we endogenize bank capital in Section 6.2. There we show that banks have incentives to use equity capital in equilibrium.

5 Deposit insurance

In the previous sections, we have assumed that all deposits are uninsured, as is common in the literature on financial fragility. In practice, however, many deposits are often insured, and this insurance is likely to affect depositor behavior and thus, by extension, how banks respond. In this section, we extend the analysis to consider the role that deposit insurance (DI) plays for the bank's risk management choice. To do so, we assume that a fraction $\sigma \in [0, 1]$ of depositors is insured. Insured depositors are certain to receive the promised repayment and thus have no incentive to withdraw early, regardless of the signal s_i.[13]

We obtain two main insights. First, we show that DI has a stabilizing effect on bank fragility. Second, we link this beneficial stabilizing effect of DI to the bank's risk management choice and find that it reduces the bank's marginal incentives to manage risk, much like the other non-contingent tool we studied earlier, namely capital. At the same time, however, DI acts as a catalyst for establishing a risk management desk. In other words, even though the degree of risk management decreases conditional on the bank being interested in managing such risk, the *extensive margin* effect of deposit insurance is to promote the creation of a risk management desk. In this sense, deposit insurance coverage again acts very similarly to bank capital, as analysed in Section 4.2.

[13] This specification resembles the sleepy (or inactive) depositor specification in Chen et al. (2010), although we assume that depositors' type is known and, as a consequence, the bank offers two deposit rates. Alternative approaches to modelling deposit insurance include Allen et al. (2018) and Dávila and Goldstein (2023).

Let r_2^I and r_2^U denote the promised time 2 repayment for insured and uninsured depositors, respectively. As before, we solve the model by working backwards, starting with withdrawal decisions of the fraction $1-\sigma$ of uninsured depositors, for a given deposit contract $\{1, r_2^I, r_2^U\}$, a fraction of insured depositors σ, capital structure k, and risk management choice z. To simplify the exposition, we introduce the subscript σ for all thresholds and define the average deposit rate as $\bar{r}_{2,\sigma} \equiv \sigma r_2^I + (1-\sigma) r_2^U$. As in the baseline model, we describe depositors' withdrawal decisions for a generic ℓ because its realization is known when depositors choose to withdraw.

In the baseline model, panic-driven runs exist only if there is strategic complementarity in depositors' withdrawal decisions, which occurs when the interim asset value ℓ is insufficient to cover the repayment of all depositors at time $t = 1$, $\ell < 1 - k$. Generalizing this bound by noting that a fraction σ of depositors is insured and never withdraws, strategic complementarity in depositors' withdrawal decisions arises only for interim asset values below a threshold $\check{\ell}_\sigma(k) = \left(1 - \sigma + \sigma / r_2^U\right)(1-k)$, which we derive in Appendix B.1. Note that $\check{\ell}$ equals $1-k$ for $\sigma = 0$, as in the baseline model, and $d\check{\ell}_\sigma / d\sigma < 0$. Proposition 6 extends our previous results in Propositions 1 and 2 on the existence of a unique equilibrium of the withdrawal game.

Proposition 6. *Uninsured depositors withdraw at time* 1 *only if the fundamental of the economy, θ, is below a critical cutoff. This threshold is given by $\underline{\theta}_\sigma$ whenever $\ell \geq \check{\ell}_\sigma$ and by θ^*_σ for $\ell < \check{\ell}_\sigma$, with $\theta^*_\sigma \geq \underline{\theta}_\sigma$. Both run thresholds, which are defined in the Appendix, decrease in σ: $\partial \underline{\theta}_\sigma / \partial \sigma < 0$ and $\partial \theta^*_\sigma / \partial \sigma < 0$. Furthermore, $\partial \theta^*_\sigma / \partial \ell < 0$, $\partial^2 \theta^*_\sigma / \partial \ell^2 > 0$, $\partial^2 \theta^*_\sigma / \partial \ell \partial \sigma > 0$, and $\partial \theta^*_\sigma / \partial r_2^I > 0$.*

Proof: See Supplemental Appendix B.1. □

As in the baseline model, a unique threshold equilibrium emerges in the characterization of depositors' withdrawal decisions. The exact threshold and the type of runs depend on the level of the interim asset value relative to the amount of bank capital and on the degree of DI coverage, summarized in the cutoff $\check{\ell}_\sigma(k)$. When the interim asset value, the fraction of insured depositors, and bank capital are high, so that $\ell \geq \check{\ell}_\sigma$, only fundamental runs occur, while panic runs emerge in equilibrium otherwise. Importantly, DI increases financial stability along two dimensions: it enlarges the range of parameters in which only fundamental runs occur, and it reduces the strategic complementarity in depositors' withdrawal decisions. To focus on situations in which financial fragility is of real concern, we again restrict attention to the case of $\ell < \check{\ell}_\sigma$ henceforth. This is equivalent to assuming that the degree of deposit insurance coverage, σ, is not so large that all fragility is eliminated.

Figure 3 shows the effect of deposit insurance on bank fragility, as established in Proposition 6. First, we can see that the equilibrium run threshold, for any given interim asset

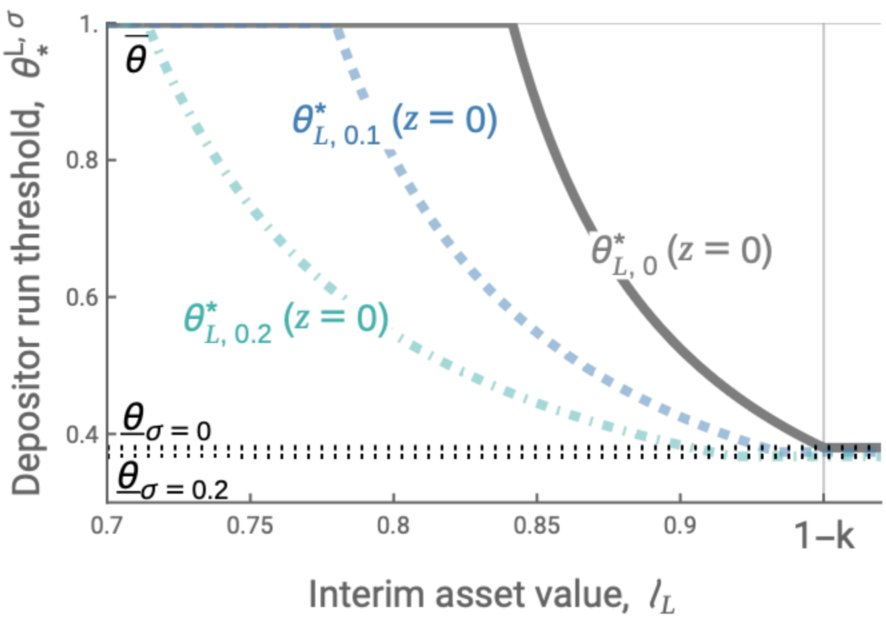

Figure 3: Bank failure thresholds, $\theta^*_{L,\sigma}$, for three different levels of deposit insurance σ as a function of ℓ_L in the absence of risk management $(z = 0)$. The gray solid line corresponds to the case without deposit insurance, $\sigma = 0$. The blue dashed and green dash-dotted lines correspond to deposit insurance coverage levels of $\sigma = 0.1$ and $\sigma = 0.2$, respectively. The figure uses the same parameters as in Figure 1; deposit rates are $r_2^I = 1$ and $r_2^U = 6/5$.

value, is higher when the deposit insurance coverage is lower, meaning that the probability of a run is higher. Second, the fundamental run threshold $\underline{\theta}_\sigma$ decreases in σ because insured depositors receive a lower deposit rate, i.e., $r_2^I < r_2^U$. Intuitively, the bank is more able to meet its payment obligation at time 2 if no uninsured depositor runs at time 1.

5.1 Deposit insurance and risk management incentives

Having characterized uninsured depositors' withdrawal decisions, we now move to the bank's choice of risk management. To do so, we first restate the expression for expected bank profits. Relative to the expression in (6), we need to replace θ^* with the one characterized in Proposition 6. We also account for the two types of depositors. Expected profits are then given by:

$$\Pi_\sigma(z) \equiv p_\ell \int_{\theta^*_{L,\sigma}}^{1} [R\theta - (1-k)\bar{r}_{2,\sigma}]d\theta + (1-p_\ell)\int_{\theta^*_{H,\sigma}}^{1} [R\theta - (1-k)\bar{r}_{2,\sigma}]d\theta - c\frac{z^2}{2} - F\mathbb{1}_{\{z>0\}},$$

as the bank only makes a positive profit if no run occurs. Next, we analyze the bank's risk management incentives, again ignoring the fixed cost, to find the level of risk management, $\widehat{z}_\sigma$, that maximizes profits. Proposition 7 complements Proposition 3 by analyzing how the bank's incentives are affected by DI and by the deposit rates r_2^I and r_2^U.

Proposition 7. *Let $\ell_L < \check{\ell}_\sigma$. The incentives to manage risk increase as the interim asset value in the event of a shock decreases, $d\widehat{z}_\sigma / d\ell_L < 0$, and the incentives decrease as the fraction of insured depositors increases, $d\widehat{z}_\sigma / d\sigma < 0$.*

Proof: See Supplemental Appendix B.3. □

The first result is identical to that in Proposition 3, suggesting that the key mechanics with deposit insurance are similar to the baseline model. The second result about the effect of DI coverage is intuitive: more insured depositors have a stabilizing effect on the bank, reducing its incentives and need to engage in risk management.

5.2 Deposit insurance and risk management failures

Having analyzed the bank's risk management incentives in the presence of deposit insurance, we next revisit the existence of risk management failures (as characterized in Proposition 4) and show that our previous finding—the bank chooses not to set up any risk management operations for low enough interim asset values—continues to hold. What is more, DI can be a catalyst for setting up a risk management desk, similar to bank capital.

Proposition 8. *Let $\ell_L < \check{\ell}_\sigma$ and $c > \widehat{c}_\sigma$, for some $\widehat{c}_\sigma > 0$. Then, for any $F > 0$, there exists an interim asset value $\widetilde{\ell}_L(F) \in (0, \widehat{\ell}_\sigma(0))$ such that the optimal amount of risk management is zero, $z_\sigma^* = 0$, when $\ell_L < \widetilde{\ell}_L(F)$. The fixed cost threshold $\widehat{F}_\sigma$ below which some risk management is optimal increases with the level of DI coverage, $d\widehat{F}_\sigma / d\sigma > 0$.*

Proof: See Supplemental Appendix B.4. □

Intuitively, the *complementary* relationship between deposit insurance and setting up a risk management desk arises because of the stabilizing effect of DI, which increases the expected profits of the bank from risk management by reducing the probability of a run.

6 Discussion of robustness and extensions

We discuss the robustness of our finding of risk management failures to the endogenization of deposit rates in Section 6.1. We then extend our model by endogenizing bank capital structure in 6.2. Section 6.3 solves a constrained planner problem and shows that there is a wedge between the private and social incentives for risk management, confirming our argument that there are failures in risk management. Finally, we consider an alternative modelling approach to risk management in Section 6.4 and show that our main results are robust.

6.1 Endogenous deposit rates

In order to clearly illustrate the main forces at work, in the analysis above we held all variables constant, including the promised repayment to depositors, when varying the interim asset value. However, the equilibrium repayment to depositors, r_2, clearly should depend on the interim asset value since changes in the distribution of ℓ affect the expected value of the deposit claim and hence shift depositors' participation constraint. Here, we demonstrate that our main result on risk management failures continues to hold even when the indirect effects arising from the need to satisfy depositors' participation constraint are taken into account. As a preliminary first step, the following lemma establishes the effect of a change in the interim asset value ℓ_L on r_2 for the baseline model with $\sigma = 0$ and no risk management, $z = 0$.

Lemma 1. *A lower interim asset value $\ell = \{\ell_L, \ell_H\}$ is associated with a higher deposit repayment r_2 in equilibrium, i.e., $dr_2^*/d\ell_L < 0$ and $dr_2^*/d\ell_H < 0$.*

Proof. See Supplemental Appendix B.5. □

The lemma above shows that the anticipation of a larger negative shock leads the bank to offer a higher repayment r_2 to depositors. In the next lemma we study the implication for the overall effect of a change in the level of the interim asset value ℓ_L on the run threshold.

Lemma 2. *For $\ell_H < 1 - k$, the run threshold decreases in the interim asset value upon a negative shock, i.e., $d\theta_L^*/d\ell_L < 0$.*

Proof. See Supplemental Appendix B.6. □

Lemma 2 establishes that similar comparative statics to those obtained in Proposition 2 (for the case when r_2 is held fixed) also hold when r_2 is endogenous and pinned down by the binding participation constraint of investors.[14] This occurs even though, as shown above in Lemma 1, r_2 must increase whenever ℓ_L decreases in order to compensate depositors for the potentially larger drop in the interim liquidation value of the bank's assets. The increase in r_2, however, reduces depositors' incentives to run, thus lowering θ^*, and at least partially offset the effect of the drop in ℓ_L. Nevertheless, Lemma 2 establishes that the overall equilibrium effect of a change in ℓ_L is unchanged, since the indirect effect coming from the change in r_2 does not fully offset the direct effect. The implication is that, as before, a larger negative shock to the interim value of bank assets is destabilizing. Having characterized how the probability of a run changes with ℓ, the following proposition restates our main result in the context where $r_2(\ell_L)$ endogenously responds to the size of the negative shock to the interim asset value of the bank.

Proposition 9. *There is a value $\check{c}$ (defined in the appendix) such that, if $c > \check{c}$, then for any $F > 0$ there exists a sufficiently low but positive interim asset value ℓ_L such that the optimal amount of risk management effort is zero, $z^* = 0$.*

Proof. See Supplemental Appendix B.7. □

Proposition 9 shows that our main result on risk management failures continues to hold when r_2 is endogenized. As before, the constraint that c not be too small is there only to guarantee that risk management is not so cheap that the bank can simply fully offset any shock, no matter how negative. Hence, the bank's decision not to manage risk does not arise from the assumption that deposit interest rates were fixed and did not appropriately reflect economic conditions, but rather stems from how bank incentives are altered as shocks become sufficiently large.

6.2 Endogenous bank capital

So far, we have treated bank capital as exogenous, so that the comparative statics with respect to k could be interpreted as the response to a change in binding capital requirements, for instance. In this section, we endogenize the bank's capital structure. We show that the bank finds it optimal to raise a strictly positive amount of capital as long as the cost of bank capital is not too high relative to the cost of deposit funding. Intuitively, the

[14]The lemma focuses on the deposit rate corresponding to the solution to the binding participation constraint. In principle, the bank may find it optimal to increase r_2 further in this setting, leaving some slack in the depositors' participation constraint. Such strategy could be optimal since a higher deposit rate reduces the run threshold and so may have a beneficial effect on the bank's expected profits. As shown in Ahnert et al. (2023), parametric restrictions on the investment profitability R ensure that the participation constraint binds in equilibrium.

bank values the role of bank capital as a non-contingent risk management tool that can reduce the probability of bank runs in all states.

To study this issue, suppose that the banker raises capital and deposits at the beginning of $t = 0$, so that k and r_2 are jointly determined. Assume also that the outside options of depositors and bank equity holders are ρ_D and $\rho_E \geq \rho_D$, respectively.[15]

To ease the exposition, we focus on a parameter space where depositors' participation constraint binds. Thus, the banker's problem is:

$$\max_{k,r_2} \Pi = p_\ell \int_{\theta_L^*}^{1} [R\theta - (1-k)r_2] d\theta + (1-p_\ell) \int_{\theta_H^*}^{1} [R\theta - (1-k)r_2] d\theta - \rho_E k - c\frac{z^2}{2} - F\mathbb{1}_{\{z>0\}}, \quad (9)$$

where the term $\rho_E k$ represents the cost of equity for the bank, or equivalently the bank shareholders. This maximization problem is subject to the participation constraints of the banker and depositors:

$$\Pi \geq 0 \quad (10)$$

$$V \equiv p_\ell \left[\int_0^{\theta_L^*} \frac{\ell_L + z}{1-k} d\theta + \int_{\theta_L^*}^{1} r_2 \, d\theta \right] + (1-p_\ell) \left[\int_0^{\theta_H^*} \frac{\ell_H}{1-k} d\theta + \int_{\theta_H^*}^{1} r_2 \, d\theta \right] = \rho_D, \quad (11)$$

where we used the fact that depositors receive an equal share of liquidation proceeds at time 1. Note that risk management effort is determined at the end of time 0, so $z^* = z^*(k, r_2)$.

We can now state the following result, which characterizes the bank's optimal capital structure.

Proposition 10. *The bank is funded with equity and debt, $k^* \in (0,1)$, if the cost of equity, ρ_E, is not too high relative to the cost of debt, ρ_D.*

Proof: See Supplemental Appendix B.8. □

This proposition states that a positive but not too high equity premium results in a positive and interior choice of bank capital, $k^* \in (0,1)$.[16] The intuition stems from recognizing that panic runs, which are the result of coordination failures among depositors, are

[15] While it is widely accepted that the cost of bank equity is higher than the cost of debt, due to factors such as taxes (Modigliani and Miller 1958, 1963), bankruptcy costs (Myers 1977), asymmetric information (Myers and Majluf 1984), as well as a cost of contingencies and limited financial market participation (Barberis et al. 2006; Guiso et al. 2008; Guiso and Sodini 2013), the magnitude of the equity premium is debated (Admati et al. 2013). See Allen et al. (2015) for a general equilibrium framework in which the cost of equity endogenously emerges as higher than the cost of deposits.

[16] The first-order condition for bank capital may have multiple solutions, so there may be several local maxima. This arises from the interaction between the bank capital choice and its risk management effort, captured by the term $\frac{dz^*}{dk}$. In any case, the bank will select the global maximum, which is interior.

welfare destroying. As a result, the use of instruments or tools that can reduce the probability of a run will, all else equal, lead to a Pareto improvement as both depositors and the bank benefit from this reduction. Therefore, as long as the cost of equity capital, ρ_E, is not too much higher than the cost of debt, ρ_D, raising at least some amount of equity will be optimal for the bank. Ceteris paribus, depositors also benefit from capital because (i) it reduces the run threshold θ^*, thus making it more likely that they receive the promised payment at date 2; and (ii) it increases the repayment upon a run. In equilibrium, this then allows the bank to reduce the date 2 repayment, r_2^*, and increases its profits.

6.3 Constrained inefficiency

In this section, we consider a planner's problem and compare its solution to the one obtained privately by a bank. This analysis aims at establishing that the private incentives for risk management effort are insufficient and that our main result is indeed one of a failure in risk management. The key friction that the planner can resolve is that it can commit, when raising deposit funding, to a risk management choice. By contrast, the bank has no such commitment since risk management actions are not contractible and any decision it may make about managing risk only occurs *after* it has raised funds and, as a consequence, promised repayments to depositors have been set.

We consider a constrained planner who maximizes utilitarian welfare and takes the incomplete information structure of the economy as given and, in particular, takes depositors' withdrawal decisions as given. Thus, the planner P takes the failure threshold θ^* specified in Proposition 1 as given, as does the bank. We focus again on the case where bank leverage is relatively high, $1-k > \ell_H$, so there are panic runs. We also assume that the participation constraint of investors binds, which pins down the equilibrium deposit rate. We have the following results.

Proposition 11. *There are risk management failures along two dimensions:*

(i) When engaging in risk management, the planner chooses a higher level than the bank, $\widehat{z}_P > \widehat{z}$.

(ii) The planner engages in risk management for a larger range of parameters, $\widehat{F}^P > \widehat{F}$.

Proof: See Supplemental Appendix B.9. □

The intuition for these results is as follows. The commitment power of the planner implies that the deposit rates are responsive to future risk management choices, reducing the funding cost of the bank. As a result, the planner chooses more risk management on the intensive margin. On the extensive margin, it also sets up a risk management desk for a larger range of fixed costs. In this sense, the privately optimal risk management failures are *excessive*. A direct implication of these results is that the bank is excessively fragile (because it engages in too little risk management), which is socially costly due to panic runs.

6.4 Alternative modelling of risk-management: An interest rate swap

In the baseline model, we considered the benefit of risk management to be narrowly targeted to address the specific risk to which the bank is exposed, namely the possibility that the interim value of assets turns out to be low. As a result, our notion of risk management is geared towards raising the liquidation value from ℓ_L to $\ell_L + z$ in the state with the negative shock. In this section, we broaden the analysis by considering an alternative modelling approach to risk management that resembles an interest rate swap. This serves two purposes. First, it demonstrates that our result on risk-management failures is robust to allowing for more general, and perhaps also more realistic, tools for managing risk. Second, it provides a microfoundation for the assumption of a convex cost of risk management used in the previous sections by showing that a similar cost structure can arise endogenously with other risk management tools.

Specifically, an interest rate swap is a derivative contract in which two counterparties exchange cash payments based on the realization of the underlying risk-factor, i.e., changes in the interest rate. Interpreting state L and the associated low interim asset value ℓ_L as the result of an interest rate hike, entering into an interest rate swap at date 0 allows the bank to obtain extra resources z in state L in exchange for a cash payment in state H. Importantly, the transfer of resources is independent of whether the bank winds up liquidating any assets (i.e., independent of whether a run occurs or not) and does not require funding at $t = 0$ beyond setting up the risk management desk at fixed cost F, consistent with Bretscher et al. (2018).

For simplicity, we assume that the swap contract is free from counterparty risk and senior to other claims on the bank (see Dasgupta 2004 for a similar assumption). This could be because of collateral set aside for the transaction, which is then exempt from the resources of the bank depositors have access to when running (see, e.g., Ahnert et al. 2019). Also for simplicity, we focus on the symmetric case $p_\ell = 1/2$, which makes the design of an actuarially fair swap particularly simple: the bank receives z in state L and pays z in state H, for which it has to partially liquidate some investment. The rest of the model is unchanged and we introduce the subscript S, for "Swaps," for all thresholds and for bank profits.

The analysis proceeds following the same steps as in the baseline model. First, we characterize depositors' withdrawal decisions. Then, we move to analyze the bank's incentives to manage risk and demonstrate that when the shock to the interim asset value is severe, the bank has no incentives to enter into an interest rate swap, i.e., a risk-management failure emerges.

Proposition 12. *Depositors withdraw at the interim date in state L and H if the fundamental of the economy θ falls below the cutoffs $\theta^*_{L,S}$ and $\theta^*_{H,S}$, respectively. The threshold $\theta^*_{L,S}$ decreases in z, while $\theta^*_{H,S}$ increases.*

Proof: See Supplemental Appendix B.10. □

Having characterized depositors' withdrawal decisions, expected bank profits are as follows:

$$\Pi_s(z) \equiv p_\ell \int_{\theta^*_{L,S}}^1 [R\theta + z - (1-k)r_2] d\theta + (1-p_\ell) \int_{\theta^*_{H,S}}^1 [R\theta \frac{\ell_H - z}{\ell_H} - (1-k)r_2] d\theta - F\mathbb{1}_{\{z>0\}}, \quad (12)$$

where the first term captures the profits the bank accrues in state L, $\Pi_{S,L}$, while the second one represents the profits accrued in state H, $\Pi_{S,H}$. The effect of the risk management is twofold. First, it affects the bank's exposure to runs (via changes in the failure threshold). Second, it affects the amount of profits the bank accrues conditional on surviving a run.

The left panel of Figure 4 illustrates the result of Proposition 12: a higher z is associated with a lower failure threshold in the low state, θ^*_L, and a higher failure threshold in the high state, θ^*_H. Moreover, the effect of risk management is different in each state: when z is high, an increase in z has a larger impact on bank fragility in the high state, θ^*_H, than in the low state, θ^*_L. This differential effect on the failure thresholds contributes to explaining the different shapes of the bank's profits in the right panel of Figure 4. Entering into the swap contract benefits the bank in state L, but entails a cost in state H. When z is high, an increase in z has a stronger impact on the profits in state H line than in state L, so that the endogenous cost of additional risk management at that point exceeds its benefits.

Figure 4 also shows that the endogenous cost of risk management, which here is the reduction in the bank's profits in state H, is convex, much like the reduced-form assumption on risk management used in the baseline model. Denoting such cost as C, the following proposition provides a formal characterization of this result.

Proposition 13. *If $\ell_H \leq (1-k)/2$, then the endogenous cost of risk management is convex, $d^2C/dz^2 > 0$.*

Equilibrium run thresholds and bank profits

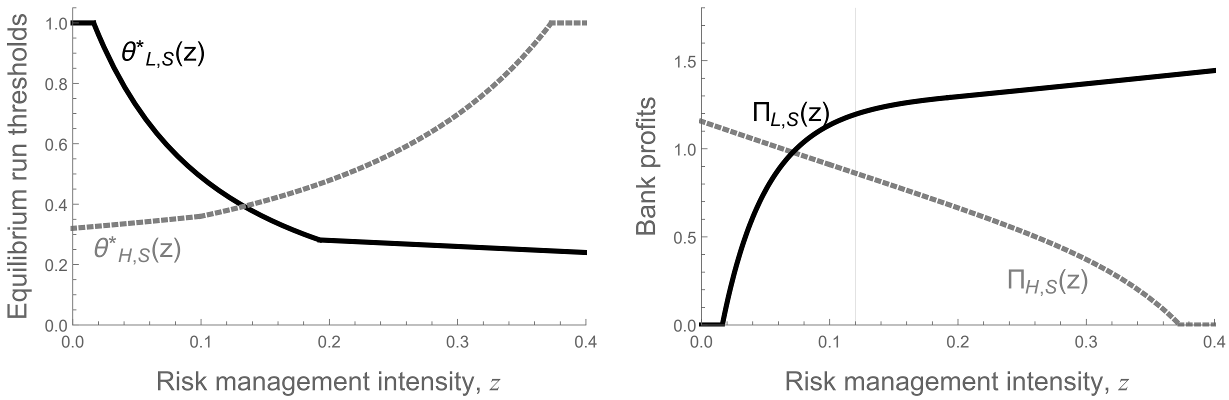

Figure 4: The left panel shows the run thresholds $\theta^*_{L,S}$ (solid line) and $\theta^*_{H,S}$ (dotted line) as a function of the risk management intensity z. The right panel shows bank profits $\Pi_{L,S}$ (solid line) and $\Pi_{H,S}$ (dotted line) as a function of z. Parameters: $R = 5$, $k = 1/5$, $\ell_L = 2/5$, $\ell_H = 9/10$, and $r_2 = 2$.

Proof: See Supplemental Appendix B.11. □

The proposition shows that in the context of an interest rate swap, an endogenous convex cost of risk management emerges, thus providing a micro-foundation for the convex cost used in the main model. The convexity of the risk-management cost hinges on the convexity of the failure threshold $\theta^*_{H,S}$ with respect to z. To prove this analytically, Proposition 13 makes use of a sufficient condition related to the degree of leverage of the bank. However, the result holds much more generally, as various numerical examples, including the one illustrated in Figure 4, show.

The endogenous costs of risk management makes it difficult to characterize the bank's risk-management decision analytically. For this reason, we perform the remainder of the analysis in this section numerically. The left panel of Figure 5 shows that, consistent with the baseline model, the choice of a positive level of risk management, which we denote as $z^* \equiv \widehat{z}_S$, decreases in the interim asset value ℓ_L and drops to zero when the shock to the interim asset value is severe. As in the baseline model, for a positive fixed cost F, the bank will find it optimal not to do any risk management, i.e., not to enter into the swap, if the interim asset value in state L is very low. The right panel in Figure 5 illustrates the underlying mechanism for this result: as ℓ_L decreases, the bank's expected profits (across both states) decrease due to the bank's choice of a progressively higher z. As illustrated in Figure 4, for sufficiently high values of z the losses in state H become progressively larger and overcome the gains in state L, so it is optimal for the bank to do no risk management.

Risk management choice and expected bank profits

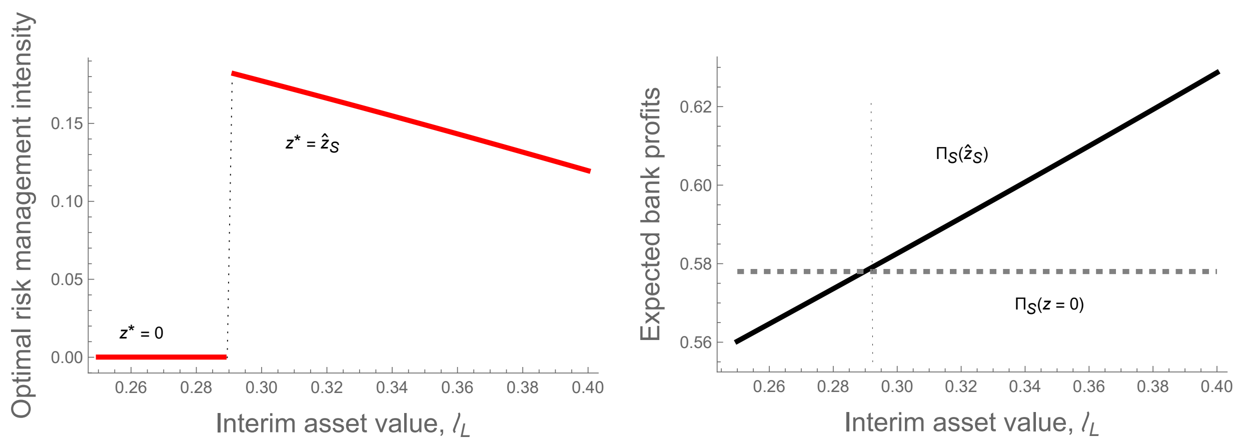

Figure 5: The left panel shows the equilibrium level of risk management z^* as a function of the realized interim asset value ℓ_L. The right panel shows the expected profits when the bank chooses a positive risk management intensity $\widehat{z}_S$ (solid line) and those when the bank does not engage in any risk management (dotted gray line). The parameters are the same as in Figure 4 and $F = 2/5$.

7 Conclusion

Runs on banks have been pervasive historically, and recent episodes demonstrate that these continue to be a concern for bank shareholders, depositors, and regulators. Moreover, the failure of a number of financial institutions in 2023 proved once again that the conditions at which banks can obtain funds at short notice to meet withdrawals play a crucial role for their stability. Specifically, negative shocks to the interim value of bank assets, even if fully anticipated, can lead to greater instability and, ultimately, increase the risk of a run. Despite the apparent availability of appropriate tools to manage liquidity risk, these channels of instability and their implications for risk management have been largely ignored up to now.

Incorporating such uncertainty into a canonical global-games model of bank runs, we consider the question of what incentives a bank may have to hedge these risks and engage in activities geared toward reducing exposure to shocks to the interim value of their assets, hence reducing the likelihood of a panic among depositors. We show that even if a bank's marginal incentives to manage risk may increase as the severity of the possible negative shock increases, the bank's profitability nevertheless declines. For sufficiently large negative shocks, therefore, the bank does not find it profitable to set up risk management operations if it faces any kind of fixed, upfront cost to doing so. As a result, risk management operations are abandoned (or never entertained in the first place) precisely when managing those risk would have the largest impact in terms of improving financial stability. Our framework is useful for understanding how private sector incentives to manage risk may differ from social incentives, and what role the costs of setting up such operations play. It also helps understand the benefit of different tools that might be available either to the banker or to a regulator, such as the use of bank capital as an alternative, non-contingent tool for managing risk, or of deposit insurance coverage.

One aspect that we have ignored, however, is the possibility that economic shocks that reduce the interim value of the bank's assets may similarly affect their long term value, thus also reducing the bank's revenue even if the project is held until maturity. In the context of the model, this would be like assuming that ℓ_L and R are positively correlated. While a full analysis of this case is beyond the scope of this paper, it is clear that similar issues related to failures in risk management are likely to arise, and may even be compounded: as the negative shock reduces R, depositors are more inclined to run, so that financial fragility increases. This effect complements the increased fragility stemming from reductions in the interim asset value, and further increases the probability of a run, making it more likely that the bank will abandon all efforts to manage risk since they will not be able to get sufficient compensation for doing so. A similar (and related) result likely obtains if one considers only the issue of credit risk management, and its impact on depositor run risk. Alternative: The insights here are then useful for the policy debate on how best to encourage bank risk management more broadly, and how to reduce systemic risk in the financial system.

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A Appendix

A.1 Proofs of Proposition 1 and 2

A.1.1 Low liquidation value relative to depositor base

Consider $\ell < 1-k$. The proof builds closely on Carletti et al. (2023b), which adapt the proof in Goldstein and Pauzner (2005) to prove the existence and uniqueness of a monotone equilibrium in a setting with profit-maximizing banks.

In the intermediate region $\theta \in (\underline{\theta}, \overline{\theta})$, a depositor's withdrawal decision depends on the actions taken by the other depositors as these determine whether the bank is illiquid or insolvent. Recall that $n \in [0,1]$ is the fraction of consumers who withdraw at $t=1$. When the liquidation proceeds at $t=1$ are insufficient to meet withdrawals, $n > \overline{n} \equiv \ell/(1-k) < 1$, the bank is illiquid. Otherwise, it continues to operate until $t=2$. If the bank cannot meet the remaining withdrawals, $n > \widehat{n}$, it is bankrupt due to insolvency at $t=2$, where $\widehat{n}$ can be derived from the following solvency condition:

$$R\theta\left(1-\frac{(1-k)n}{\ell}\right)-(1-k)(1-n)r_2=0, \tag{13}$$

which gives

$$\widehat{n} \equiv \frac{R\theta-(1-k)r_2}{\frac{R\theta(1-k)}{\ell}-(1-k)r_2} < 1. \tag{14}$$

The first term of Equation (13) is the return on the part of the project that was not liquidated at $t=1$ and the second term represents remaining withdrawals at $t=2$. Table 2 shows the payoffs of depositors associated with the two withdrawal actions. Depositors' payoffs depend on the realized economic fundamental θ and the proportion n of depositors withdrawing:

aggregate action / *individual action*	$n \leq \hat{n}$ bank is solvent	$\hat{n} < n \leq \overline{n}$ bank is insolvent but liquid at $t=1$	$n > \overline{n}$ bank is insolvent and illiquid at $t=1$
Withdraw	1	1	$\begin{cases} 1 & w.p.\ \frac{\ell}{(1-k)n} \\ 0 & w.p.\ 1-\frac{\ell}{(1-k)n} \end{cases}$
Don't withdraw	r_2	0	0

Table 2: Payoffs in the withdrawal game for intermediate realized fundamentals, $\theta \in (\underline{\theta}, \overline{\theta})$.

For vanishing signal noise, the Laplacian property holds and we can write the indifference condition of the marginal depositor as:

$$\int_0^{\hat{n}} r_2 dn = \int_0^{\overline{n}} dn + \int_{\overline{n}}^1 \frac{\ell}{(1-k)n} dn \equiv \alpha = \ell/(1-k)\left[1-\ln\left(\ell/(1-k)\right)\right]. \tag{15}$$

Using the marginal depositor's expected payoff from withdrawing at $t = 1$, α, we can express:

$$\theta^* = \frac{(1-k)r_2\left(1-\frac{\alpha}{r_2}\right)}{R\left(1-\frac{1-k}{\ell}\frac{\alpha}{r_2}\right)} = \underline{\theta}\,\frac{r_2-\alpha}{r_2-\frac{\alpha(1-k)}{\ell}} > \underline{\theta}, \tag{16}$$

which we refer to as the panic-run threshold. Throughout we focus on $r_2 > \alpha(1-k)/\ell$. Otherwise, a marginal depositor would not exist, resulting in certain bank runs. This cannot be optimal for the bank, which would make zero expected profits in this case. Hence, the bank chooses to offer a contract that satisfies the above inequality. That is, the long-term deposit rate is sufficiently high to compensate for the risk of (panic) runs at $t = 1$. Formally, when the marginal depositor does not exist, it is a dominant strategy for depositors to withdraw at time 1, so $\theta^* = \overline{\theta}$. This arises if Equation (15) does not hold, which is the case if

$$r_2 \quad < \quad \widehat{r}_2 \equiv \frac{\left(\frac{\overline{\theta}}{\underline{\theta}}-\frac{\ell}{1-k}\right)\left(1-\ln\left(\frac{\ell}{1-k}\right)\right)}{\frac{\overline{\theta}}{\underline{\theta}}-1}, \tag{17}$$

where $\widehat{r}_2 > \frac{\alpha(1-k)}{\ell}$.

A.1.2 High liquidation value relative to depositor base

Consider $\ell \geq 1-k$. For $\ell = 1-k$, we have $\overline{n} = 1$ and $\alpha = 1$. Furthermore, the bank solvency condition, characterized in (13) can be rearranged as follows:

$$\left[R\theta-(1-k)r_2\right](1-n) = 0,$$

which is positive for $\theta > \underline{\theta}$ and negative for $\theta < \underline{\theta}$. It follows that a depositor's expected payoff from withdrawing at $t = 1$ and $t = 2$ is independent of n, and so the relevant bank failure threshold is $\underline{\theta}$ because $r_2 > 1$. Intuitively, the withdrawal of deposits does not impose a loss on other depositors when $\ell = 1-k$. The fundamental-run threshold in (4) decreases in bank capital:

$$\frac{\partial\underline{\theta}}{\partial k} = -\frac{r_2}{R} < 0.$$

Hence, the solvency condition in (13) becomes less binding for any n when ℓ exceeds $1-k$. Intuitively, a high liquidation value relative to the deposit base means that withdrawals by other depositors increases a given depositor's incentive *not* to withdraw. It follows that, for any $\ell > 1-k$, the relevant threshold is still $\underline{\theta}$. Similarly, we can derive $\partial\underline{\theta}/\partial R < 0$.

Finally, we have $\partial^2\underline{\theta}/\partial k^2 = 0$ and that the lower dominance bound is independent of the liquidation value, $\partial\underline{\theta}/\partial\ell = 0 = \partial^2\underline{\theta}/\partial\ell^2 = \partial^2\underline{\theta}/\partial k\partial\ell$.

A.1.3 Comparative statics of the failure threshold with panics

Next, we analyze how the bank failure threshold with panics, θ^*, changes with bank capital, the interim liquidation value of the bank asset, and both of them. Since ℓ is used to generically denote both $\ell_L + z$ and ℓ_H, and since z and ℓ_L are perfect substitutes, it follows that the comparative statics with respect to ℓ below applies equally to changes in ℓ_L and ℓ_H, as well as changes in z.

Note that the interim liquidation value and bank capital only enter into the failure threshold in Equation (5) as their ratio, $\ell/(1-k) \equiv x$. Thus, $dx/d\ell = (1-k)^{-1} > 0$, $dx/dk = \ell(1-k)^{-2} > 0$, and $d^2x/dkd\ell = (1-k)^{-2} > 0$. Using this definition, we can express the marginal depositor's expected utility from withdrawing as

$$\alpha = x(1-\ln(x)), \tag{18}$$

with $d\alpha/dx = -\ln(x) > 0$ because $x < 1$.

Inserting the definitions of α and x into the failure threshold in Equation (5) yields:

$$\theta^* = \underline{\theta}\frac{r_2 - x(1-\ln(x))}{r_2 - (1-\ln(x))} > \underline{\theta}. \tag{19}$$

Define β as $\theta^* \equiv \underline{\theta}\beta$, so $\beta > 1$ for panic runs. We can write the difference as:

$$\theta^* - \underline{\theta} = \underline{\theta}\frac{(1-x)(1-\ln(x))}{r_2 - (1-\ln(x))} = \underline{\theta}\frac{1-x}{\frac{r_2}{1-\ln(x)} - 1}, \tag{20}$$

which decreases in x because both effects of x reduce the left-hand side: $d(\theta^* - \underline{\theta})/dx < 0$. This implies that $d(\theta^* - \underline{\theta})/d\ell < 0$ and $d(\theta^* - \underline{\theta})/dk < 0$, so both higher bank capital and a higher liquidation value reduce coordination failure among depositors.

Note that $d\theta^*/d\ell = (d\theta^*/dx)(dx/d\ell)$ because $d\underline{\theta}/d\ell = 0$. Moreover, $d\theta^*/dk = \beta d\underline{\theta}/dk + (d\theta^*/dx)(dx/dk)$. Thus, it is critical to determine the derivative with respect to x:

$$\frac{d\theta^*}{dx} = \underline{\theta}\frac{(1-\ln(x))^2 - r_2\,(1/x - \ln(x))}{[r_2 - (1-\ln(x))]^2} < 0, \tag{21}$$

because $x < 1$ and $r_2 > \alpha/x = 1-\ln(x)$ in any equilibrium. (Recall that this is the condition to ensure that there is not a certain run on the bank, as described above.) The numerator is negative even at $r_2 = \alpha/x = 1-\ln(x)$, and the numerator decreases in r_2. As a result of $d\theta^*/dx < 0$, we obtain $d\theta^*/dk < 0$ and $d\theta^*/d\ell < 0$.

Turning to second derivatives, note that $d^2\theta^*/d\ell^2 = (d^2\theta^*/dx^2)(dx/d\ell)$ because $d^2x/d\ell^2 = 0$. Thus, $d^2\theta^*/d\ell^2 > 0$ if and only if $d^2\theta^*/dx^2 > 0$. Moreover, $d^2\theta^*/d\ell dk = (d\theta^*/dx)(d^2x/dkd\ell) + (d^2\theta^*/dx^2)(dx/d\ell)$. Hence, a sufficient condition for $d^2\theta^*/d\ell dk > 0$ is also $d^2\theta^*/dx^2 > 0$. We next determine the second derivative with respect to x:

$$\underline{\theta}\frac{[-\frac{1}{x}2(1-ln(x))-r_2(-\frac{1}{x^2}-\frac{1}{x})][r_2-(1-ln(x))]-\frac{2}{x}\left[(1-ln(x))^2-r_2(\frac{1}{x}-ln(x))\right]}{[r_2-(1-ln(x))]^3},$$

which after simple manipulations gives:

$$\frac{d^2\theta^*}{dx^2}=\frac{\underline{\theta}\,r_2}{x^2\left[r_2-(1-\ln(x))\right]^3}\left\{2(1-x)+(1+x)\left[r_2-(1-\ln(x))\right]\right\}>0. \tag{22}$$

As a result, $d^2\theta^*/d\ell^2>0$ and $d^2\theta^*/d\ell dk>0$. To obtain the sign of $d^2\theta^*/dk^2$, we directly differentiate θ^* to obtain:

$$\frac{d\theta^*}{dk} = \frac{r_2^2\left(x-r_2-\ln(x)\right)}{R\left(r_2-(1-\ln(x))\right)^2}<0 \tag{23}$$

$$\frac{d^2\theta^*}{dk^2} = \frac{r_2^2}{R(1-k)}\frac{2\left(r_2-(x-\ln(x))\right)+(x-1)\left(r_2-(1-\ln(x))\right)}{\left(r_2-(1-\ln(x))\right)^3}>0. \tag{24}$$

A.2 Proof of Proposition 3

The comparative statics of $\widehat{z}$ with respect to ℓ_L, k, and p_ℓ can be obtained using the implicit function theorem. The second-order condition is:

$$\frac{d^2\Pi}{dz^2}=-p_\ell R\left(\frac{d\theta_L^*}{dz}\right)^2-p_\ell[R\theta_L^*-(1-k)r_2]\frac{d^2\theta_L^*}{dz^2}-c<0. \tag{25}$$

Together with the sign of the cross-partial with respect to ℓ_L:

$$\frac{d^2\Pi}{dzd\ell_L}=-p_\ell R\frac{d\theta_L^*}{dz}\frac{d\theta_L^*}{d\ell_L}-p_\ell[R\theta_L^*-(1-k)r_2]\frac{d^2\theta_L^*}{dzd\ell_L}<0, \tag{26}$$

we arrive at $d\widehat{z}/d\ell_L<0$. Similarly, we analyze the sign of the cross-partial with respect to k:

$$\frac{d^2\Pi}{dzdk}=-p_\ell\left[R\frac{d\theta_L^*}{dk}+r_2\right]\frac{d\theta_L^*}{dz}-p_\ell[R\theta_L^*-(1-k)r_2]\frac{d^2\theta_L^*}{dzdk}<0, \tag{27}$$

because $\frac{d^2\theta_L^*}{dzdk}=\frac{d^2\theta_L^*}{d\ell dk}>0$ as shown in the proof of Proposition 1 and $Rd\theta_L^*/dk+r_2<0$. To see this, we substitute the expression for $d\theta_L^*/dk$ in (23). After a simple manipulation, we can rewrite:

$$R\frac{d\theta_L^*}{dk}+r_2=-r_2\left[\frac{r_2}{r_2-(1-ln(x))}\frac{r_2-(x-ln(x))}{r_2-(1-ln(x))}-1\right]<0,$$

since $\frac{r_2}{r_2-(1-ln(x))} > 1$ and $x = \frac{\ell}{1-k} < 1$ so that also $\frac{r_2-(x-ln(x))}{r_2-(1-ln(x))} > 1$. Therefore, $d\widehat{z}/dk < 0$. Finally:

$$\frac{d^2\Pi}{dzdp_\ell} = [R\theta_L^* - (1-k)r_2]\left(-\frac{d\theta_L^*}{dz}\right) > 0, \tag{28}$$

so $d\widehat{z}/dp_\ell > 0$. Hence, the results in Proposition 3 follow.

A.3 Proof of Proposition 4

We start the proof by establishing some preliminary results in the following three Lemmas. Denote $\widehat{z}(\ell)$ by the optimal level of risk management (ignoring F) for a given interim liquidation value.

Lemma 3. *For all ℓ_L such that $\theta_L^*(\widehat{z}(\ell_L),\ell_L) < \overline{\theta}$, the equilibrium run threshold $\theta_L^*(\widehat{z}(\ell_L),\ell_L)$ decreases in ℓ_L, since the change in risk management does not fully offset changes in the interim liquidation value.*

Proof. We prove Lemma 3 by contradiction. Recall that $d\widehat{z}/d\ell_L < 0$ from Proposition 3 and that $\partial\theta_L^*/\partial z \equiv \partial\theta_L^*/\partial\ell_L < 0$. Consider two different values for ℓ_L, ℓ_L' and ℓ_L'', with $\ell_L' < \ell_L''$, and suppose that, contrary to the claim, $\theta_L^*(z^*(\ell_L),\ell_L)$ actually *increases* in ℓ_L, so that $\theta_L^*(z'(\ell_L'),\ell_L') < \theta_L^*(z''(\ell_L''),\ell_L'')$, where z' and z'' represent the bank's optimal choices of z in each case. Since we know $\theta_L^*(\widehat{z}(\ell_L),\ell_L)$ decreases in ℓ_L for a given z, we can only have $\theta_L^*(z'(\ell_L'),\ell_L') < \theta_L^*(z''(\ell_L''),\ell_L'')$ if $z' >> z''$. Moreover, it must be that $\ell_L' + z' > \ell_L'' + z''$.

Consider the FOC for the case of $\ell_L = \ell_L''$. To be explicit, it is worth rewriting it with ℓ_L'' and z'':

$$-p_\ell\frac{\partial\theta_L^*(z''(\ell_L''),\ell_L'')}{\partial z}\left(R\theta_L^*(z''(\ell_L''),\ell_L'') - (1-k)r_2\right) - cz'' = 0.$$

Suppose the solution to the risk management problem is interior and denote by $\check{z}$ the value of z that would give us the same run threshold if instead $\ell_L = \ell_L'$: $\theta_L^*(\check{z}(\ell_L'),\ell_L') = \theta_L^*(z''(\ell_L''),\ell_L'')$, and note that $\check{z} \in (z'',z')$. Now consider the derivative of the bank's profit expression for the case where $\ell_L = \ell_L'$, evaluated at $z = \check{z}$:

$$-p_\ell\frac{\partial\theta_L^*(\check{z}(\ell_L'),\ell_L')}{\partial z}\left(R\theta_L^*(\check{z}(\ell_L'),\ell_L') - (1-k)r_2\right) - c\check{z}.$$

Since $\theta_L^*(\check{z}(\ell_L'),\ell_L') = \theta_L^*(z''(\ell_L''),\ell_L'')$ and $\ell_L' + \check{z} = \ell_L'' + z''$, this can be rewritten as:

$$-p_\ell\frac{\partial\theta_L^*(z''(\ell_L''),\ell_L'')}{\partial z}\left(R\theta_L^*(z''(\ell_L''),\ell_L'') - (1-k)r_2\right) - c\check{z}.$$

The expression above is the same as the FOC for the case where $\ell_L = \ell_L''$, except for the last term, $c\check{z}$. However, we know that at equilibrium the derivative of bank profits equals zero when evaluated at $z = z''$. Hence, this derivative must be negative for $z = \check{z}$. Hence, contrary to the supposition, it cannot be optimal for the bank to choose a value of z such that $\ell'_L + z' > \ell''_L + z''$, which then implies that the inequality $\theta^*_L(z'(\ell'_L), \ell'_L) < \theta^*_L(z''(\ell''_L), \ell''_L)$ cannot hold. The result of Lemma 3 follows. □

Lemma 4. *For all ℓ_L such that $\theta^*_L(\widehat{z}(\ell_L), \ell_L) < \overline{\theta}$, the bank's expected profits $\Pi(\widehat{z}; \ell_L, \ell_H)$ increase in ℓ_L.*

Proof. We prove Lemma 4 using the Envelope theorem. Denote as $\Pi(\widehat{z}(\ell_L); \ell_L, \ell_H)$ the bank's expected profits evaluated at the $\widehat{z}$ obtained from the bank's first order condition with respect to z:

$$\begin{aligned} \Pi(\widehat{z}(\ell_L); \ell_L, \ell_H) &= (1 - p_\ell) \int_{\theta^*_H}^1 (R\theta - (1-k)r_2)\, d\theta + p_\ell \int_{\theta^*_L(\widehat{z}(\ell_L), \ell_L)}^1 (R\theta - (1-k)r_2)\, d\theta \\ &\quad - \frac{c}{2} [\widehat{z}(\ell_L)]^2 - F\mathbb{1}_{\{z>0\}}. \end{aligned}$$

Hence, we can compute:

$$\frac{d\Pi(\widehat{z}(\ell_L), \ell_L)}{d\ell_L} = \underbrace{\frac{\partial \Pi(\cdot)}{\partial z} \frac{d\widehat{z}(\ell_L)}{d\ell_L}}_{=0} + \frac{\partial \Pi(\cdot)}{\partial \ell_L},$$

where we use that $\partial \Pi(\cdot) / \partial z = 0$ from the FOC with respect to z for an interior solution. It follows that the overall sign is just equal to the sign of $\partial \Pi(\cdot) / \partial \ell_L$, which is equal to:

$$\frac{\partial \Pi(\cdot)}{\partial \ell_L} = -p_\ell \frac{\partial \theta^*_L(\widehat{z})}{\partial \ell_L} (R\theta^*_L(\widehat{z}) - (1-k)r_2) > 0. \tag{29}$$

It follows that profits increase in ℓ_L, as desired. □

Lemma 5. *There exists a positive and finite value of the variable cost parameter, $\widehat{c}$, such that $\widehat{z}(0) = \widehat{\ell}(0)$.*

Proof. The proof of Lemma 5 builds on the previous lemmas. From Proposition 3, $\widehat{z}$ decreases in ℓ_L and in c. Moreover, $\widehat{z}(\ell_L, c) \to 0$ as $c \to \infty$ for any ℓ_L. Thus, there exists a positive and finite value of the variable cost parameter, $\widehat{c}$, such that $\widehat{z}(0) = \widehat{\ell}(0)$, where $\widehat{\ell}$ is defined in Equation (8). □

Based on the Lemmas 3-5, we can prove the result in the proposition. For the bank, engaging in risk management is optimal, as long as its expected profits in the event of the negative shock (i.e., when $\ell = \ell_L$ and $z > 0$, net of the fixed cost F, are higher than the expected profits in the absence of risk management, i.e., if $\Pi(\widehat{z}(\ell_L)|\ell_L) - \Pi(0|\ell_L) \geq 0$. Since the part of the expected profits without the negative shock to the interim asset value is not affected by z in Equation (6), the above comparison boils down to:

$$p_\ell \int_{\theta^*_L(z)}^{\theta^*_L(0)} [R\theta - (1-k)r_2] d\theta - \frac{cz^2}{2} \geq F. \tag{30}$$

We continue by deriving a sufficient condition for Inequality (30) to be violated. Recall that $\theta_L^*(0) = \overline{\theta} \approx 1$ for $\ell_L < \widehat{\ell}(0)$, so the upper bound of the integral in Inequality (30) can be replaced by 1 for all $\ell_L < \widehat{\ell}(0)$.

Take $c > \widehat{c}$ and $\ell_L = 0$, then $\widehat{z}(0) < \widehat{\ell}(0)$ from Lemma 5 and Inequality (30) is violated for any $F \geq 0$. From Lemma 3, we know that $\theta^*(\widehat{z}(\ell_L), \ell_L)$ decreases in ℓ_L. From Lemma 4, we know that bank profits are increasing in ℓ_L. Thus, for $c > \widehat{c}$ there exists a strictly positive cutoff value $\widetilde{\ell}_L(F) \in (0, \widehat{\ell}(0))$ with associated optimal amount of risk management effort $\widehat{z}(\widetilde{\ell}_L(F))$ that solves (30) with equality. The result in Proposition 4 follows.

A.4 Proof of Corollary 1

The proof is straightforward. From the condition $\ell = \ell_L > \widetilde{\ell}_L(F)$, we can obtain a cutoff $\widehat{F}$ as simply the inverse function of $\widetilde{\ell}_L(F)$. Since the LHS in (30) increases with ℓ_L, it follows immediately that $\widehat{F}$ increases in ℓ_L. This completes the proof. □

A.5 Proof of Proposition 5

First, we define the threshold level of the fixed cost $\widehat{F}$ as:

$$\widehat{F} \equiv p_\ell \int_{\theta_L^*(z^*)}^1 [R\theta - (1-k)r_2] d\theta - c\frac{(z^*)^2}{2},$$

where $z^*(\ell_L, c; k)$ solves Equation (7). Taking the derivative with respect to bank capital gives:

$$\frac{d\widehat{F}}{dk} = -p_\ell \left(\underbrace{\frac{\partial \theta_L^*}{\partial k}}_{<0} + \underbrace{\frac{\partial \theta_L^*}{\partial z}}_{<0} \underbrace{\frac{dz^*}{dk}}_{<0} \right) [R\theta_L^* - (1-k)r_2] + p_\ell \int_{\theta_L^*}^1 r_2 d\theta - cz \underbrace{\frac{dz^*}{dk}}_{<0}.$$

The expression above can be rearranged as follows:

$$\frac{d\widehat{F}}{dk} = -p_\ell \frac{\partial \theta_L^*}{\partial k} [R\theta_L^* - (1-k)r_2] + \frac{dz^*}{dk} \left(\underbrace{-p_\ell \frac{\partial \theta_L^*}{\partial z} [R\theta_L^* - (1-k)r_2] - cz}_{FOC_z} \right) + p_\ell \int_{\theta_L^*}^1 r_2 d\theta.$$

The first term is positive because $\partial \theta_L^* / \partial k < 0$, as is the third term. The second term is instead zero because the term in brackets is the same as the first-order condition for z as given in (7). It follows that $d\widehat{F}(\ell_L, c; k)/dk > 0$ as desired, and the proposition follows.

Online Appendix for
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B Supplemental Appendix

This supplemental appendix contains the derivations for four extensions: the introduction of deposit insurance in Section 5, the endogenization of deposit rates in Section 6.1, the endogenization of the bank's capital structure in Section 6.2, and the alternative modelling approach to risk management in Section 6.4, as well as the analysis of the planner problem in Section 6.3.

B.1 Proof of Proposition 6

The existence proof is analogous to that of Proposition 1. We derive the thresholds for a generic ℓ, knowing that it takes one of two values depending on the realization of the shock. The lower dominance region is structurally the same as in the baseline model, as it is computed under the assumption that no one runs. However, we now have to average over deposit rates:

$$\underline{\theta}_\sigma \equiv \frac{1-k}{R}[\sigma r_2^I + (1-\sigma) r_2^U] = \frac{1-k}{R}\bar{r}_{2,\sigma}.$$

For any $\theta > \underline{\theta}_\sigma$, it is useful to distinguish between different levels of deposit insurance coverage σ as follows:

Case *(i)*: High level of DI coverage, $1 \geq \sigma \geq r_2^U \frac{\frac{1-k}{\ell}-1}{(r_2^U-1)\frac{1-k}{\ell}} \Leftrightarrow \ell \geq \check{\ell}_\sigma(k)$

Case *(ii)*: Intermediate level of DI coverage,
$1-\ell/(1-k) \leq \sigma < r_2^U \frac{\frac{1-k}{\ell}-1}{(r_2^U-1)\frac{1-k}{\ell}} \Leftrightarrow (1-\sigma)(1-k) \leq \ell < \check{\ell}_\sigma(k)$

Case *(iii)*: Low level of DI coverage, $0 \leq \sigma < 1-\ell/(1-k) \Leftrightarrow \ell \leq (1-\sigma)(1-k)$.

The equilibrium run threshold takes a different functional form for each case, and we consider the three cases in turn. We also formally derive the threshold $\check{\ell}_\sigma$.

For low levels of deposit insurance coverage, i.e. if $0 \leq \sigma < 1-\ell/(1-k)$, the bank may become illiquid at time 1 if a large fraction of uninsured depositors decide to withdraw, leading to *rationing* at the interim date as in our baseline model. Given a proportion n of uninsured depositors running, the bank becomes insolvent whenever $n > \widehat{n}_\sigma(\theta)$, which corresponds to the solution to the following equation:

$$R\theta\left(1-\frac{n\left(1-k\right)\left(1-\sigma\right)}{\ell}\right)-\left(1-n\right)\left(1-k\right)\left(1-\sigma\right)r_2^U-\left(1-k\right)\sigma r_2^I=0, \tag{31}$$

which gives:

$$\widehat{n}_\sigma\left(\theta\right)=\frac{R\theta-\left(1-k\right)\left(1-\sigma\right)r_2^U-\sigma(1-k)r_2^I}{R\theta\frac{(1-k)(1-\sigma)}{\ell}-\left(1-k\right)\left(1-\sigma\right)r_2^U}.$$

It is easy to see that (31) decreases with n:

$$\begin{aligned} & -R\theta\frac{\left(1-k\right)\left(1-\sigma\right)}{\ell}+\left(1-k\right)\left(1-\sigma\right)r_2^U<0 \\ \iff & \frac{\left(1-k\right)\left(1-\sigma\right)}{\ell}\left[-R\theta+\ell r_2^U\right]<0, \end{aligned}$$

since $\ell<(1-k)$ and $\theta>\underline{\theta}_\sigma$, as we are now considering the intermediate region where $\theta\in\left(\underline{\theta}_\sigma,\overline{\theta}\right)$.

We can derive the indifference condition of uninsured depositors as follows:

$$\int_0^{\widehat{n}_\sigma(\theta)}r_2^U dn=\int_0^{\overline{n}_\sigma}1dn+\int_{\overline{n}_\sigma}^1\frac{\ell}{\left(1-k\right)\left(1-\sigma\right)n}dn,$$

where the liquidation needs are insufficient to meet withdrawals by uninsured depositors if $n>\overline{n}_\sigma\equiv\ell/((1-k)(1-\sigma))>\overline{n}$. Note that $\overline{n}_\sigma\leq 1$ if and only if $0\leq\sigma\leq 1-\ell/(1-k)$, which describes the parameter condition for Case *(iii)*, where the level of deposit insurance coverage is low.

Defining $\alpha_{\sigma,r}\equiv\int_0^{\overline{n}}1dn+\int_{\overline{n}}^1\frac{\ell}{(1-k)(1-\sigma)n}dn$ and following the same steps as in the baseline model, we obtain:

$$r_2^U\widehat{n}_\sigma\left(\theta\right)=\alpha_{\sigma,r}\Leftrightarrow\theta^*_{\sigma,r}\equiv\underline{\theta}_\sigma\frac{r_2^U-\frac{r_2^U(1-\sigma)}{\overline{r}_{2,\sigma}}\alpha_{\sigma,r}}{r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_{\sigma,r}}. \tag{32}$$

Recognising the the first term is the fundamental run threshold, we can immediately see that $\theta^*_{\sigma,r}$ is above it since:

$$\frac{r_2^U-\frac{r_2^U(1-\sigma)}{\overline{r}_{2,\sigma}}\alpha_{\sigma,r}}{r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_{\sigma,r}}>1,$$

given that $r_2^U/\overline{r}_{2,\sigma}<(1-k)/\ell$ for $\sigma\leq 1-\ell/(1-k)$, meaning that Case *(iii)* features panic runs as in our baseline model.

Next, we move to Case *(ii)*, where the level of deposit insurance coverage is in the intermediate range. The lower bound of the intermediate range follows from $\overline{n}_\sigma\geq 1$, which implies that the bank is never illiquid at time 1. In other words, there is *no rationing* at the interim date. We continue by deriving the equilibrium run threshold for Case *(ii)* and, thereafter, the upper bound of the intermediate range, which demarks the point when there are only fundamental and no panic runs.

For Case *(ii)* the indifference condition of uninsured depositors is:

$$\int_0^{\widehat{n}_\sigma(\theta)} r_2^U d\theta = 1,$$

since $\overline{n}_\sigma > 1$. Defining $\alpha_{\sigma,nr} \equiv 1$ and following the same steps as before, we obtain:

$$r_2^U \widehat{n}_\sigma(\theta) = \alpha_{\sigma,nr} \Leftrightarrow \theta^*_{\sigma,nr} \equiv \underline{\theta}_\sigma \frac{r_2^U - \frac{r_2^U(1-\sigma)}{\overline{r}_{2,\sigma}} \alpha_{\sigma,nr}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell} \alpha_{\sigma,nr}}.$$

Also Case (ii) features panic runs despite the absence of rationing at time 1, since:

$$\frac{r_2^U - \frac{r_2^U(1-\sigma)}{\overline{r}_{2,\sigma}}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}} > 1,$$

which holds if and only if the level of deposit insurance coverage is below the upper bound of the intermediate range, or equivalenly, if the interim asset value falls below $\check{\ell}$:

$$\sigma < r_2^U \frac{\frac{1-k}{\ell} - 1}{\left(r_2^U - r_2^I\right) \frac{1-k}{\ell}} \Leftrightarrow \ell < \check{\ell}_\sigma(k) \equiv \left((1-\sigma) + \sigma \frac{r_2^I}{r_2^U}\right)(1-k) = \left(1 - \sigma + \frac{\sigma}{r_2^U}\right)(1-k), \tag{33}$$

where we used the result that $r_2^I = 1$. This is the case because the run threshold increases with r_2^I and bank profits if no run occurs decreases in it as well. Therefore, it is optimal for the bank is to choose the lowest possible level of r_2^I. This combined with the fact that depositors receive 1 at time 1 when a run occurs implies that, with $\rho_D = 1$, the lowest possible level is $r_2^I = 1$. An aspect that we will revisit in Section 6.1.

Finally, we move on to Case *(i)*, where the level of deposit insurance coverage is in the the upper range such that Inequality (33) holds. Importantly, by rearranging $\sigma \geq \left(\frac{1-k}{\ell} - 1\right) / \left(\left(r_2^U - 1\right)\frac{1-k}{\ell}\right)$ and expressing it as a function of ℓ, we obtain the cutoff $\check{\ell}$ defined in the proposition, which completes the characterization of the run thresholds. In this range, the indifference condition does not apply for uninsured depositors, since $r_2^U \geq 1$, and they optimally decide to run if and only if $\theta < \underline{\theta}_\sigma$. To see this, observe that:

$$\widehat{n}_\sigma(\theta)\,|_{\ell=\check{\ell}} = \frac{R\theta - (1-k)(1-\sigma)r_2^U - \sigma(1-k)r_2^I}{R\theta r_2^U \frac{1-\sigma}{\overline{r}_{2,\sigma}} - (1-k)(1-\sigma) r_2^U} = \frac{\overline{r}_{2,\sigma}}{(1-\sigma) r_2^U}$$

and:

$$\int_0^{\widehat{n}_\sigma|_{\ell=\check{\ell}}(\theta)} r_2^U d\theta = \frac{\overline{r}_{2,\sigma}}{1-\sigma} > 1.$$

To complete the proof we compute $\partial\theta^*_\sigma/\partial\sigma$, $\partial\theta^*_\sigma/\partial\ell$, $\partial^2\theta^*_\sigma/\partial\ell^2$, $\partial^2\theta^*_\sigma/\partial\ell\partial\sigma$, $\partial\theta^*_\sigma/\partial r_2^I$ and $\partial\theta^*_\sigma/\partial r_2^U$. Differentiating equation (32) with respect to σ, we obtain:

$$\begin{aligned}\frac{\partial\theta^*_\sigma}{\partial\sigma} &= \frac{(1-k)\left(r_2^I-r_2^U\right)}{R}\frac{r_2^U-\frac{r_2^U(1-\sigma)\alpha_\sigma}{\overline{r}_{2,\sigma}}}{r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} \\ &+\frac{(1-k)\overline{r}_{2,\sigma}}{R}\frac{-\frac{\partial\alpha_\sigma}{\partial\sigma}\frac{(1-\sigma)r_2^U}{\overline{r}_{2,\sigma}}-\alpha_\sigma\frac{-r_2^U\overline{r}_{2,\sigma}-r_2^U(1-\sigma)\left(r_2^I-r_2^U\right)}{\overline{r}^2_{2,\sigma}}}{r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} \\ &-\frac{(1-k)\overline{r}_{2,\sigma}}{R}\frac{\left(r_2^U-\frac{r_2^U(1-\sigma)\alpha_\sigma}{\overline{r}_{2,\sigma}}\right)\left(-\frac{\partial\alpha_\sigma}{\partial\sigma}\frac{(1-k)(1-\sigma)}{\ell}+\alpha_\sigma\frac{1-k}{\ell}\right)}{\left(r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2}<0,\end{aligned}$$

where $\alpha_\sigma=1$ and $\frac{\partial\alpha_\sigma}{\partial\sigma}=0$ in Case *(ii)* and $\frac{\partial\alpha_\sigma}{\partial\sigma}=\int_{\overline{n}}^1\frac{\ell}{(1-k)(1-\sigma)^2n}dn>0$ and $\alpha_\sigma-\frac{\partial\alpha_\sigma}{\partial\sigma}(1-\sigma)=\int_0^{\overline{n}_\sigma}dn+\int_{\overline{n}_\sigma}^1\frac{\ell}{(1-k)(1-\sigma)n}dn-\int_{\overline{n}_\sigma}^1\frac{\ell}{(1-k)(1-\sigma)n}dn=\int_0^{\overline{n}_\sigma}dn>0$ in Case *(ii)*.

Next, taking the derivative with respect to ℓ we obtain:

$$\frac{\partial\theta^*_\sigma}{\partial\ell}=\frac{(1-k)r_2^U}{R}\frac{-\frac{\partial\alpha_\sigma}{\partial\ell}(1-\sigma)}{r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma}-\theta^*_\sigma\frac{\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell}+\frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma}{r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma}<0,$$

where $\alpha_\sigma=1$ and $\frac{\partial\alpha_\sigma}{\partial\ell}=0$ in Case *(ii)* and $\frac{\partial\alpha_\sigma}{\partial\ell}=\int_{\overline{n}_\sigma}^1\frac{1}{(1-k)(1-\sigma)n}dn>0$ and $-\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell}+\alpha_\sigma\frac{(1-k)(1-\sigma)}{\ell^2}=\frac{(1-k)(1-\sigma)}{\ell^2}\left(-\ell\frac{\partial\alpha_\sigma}{\partial\ell}+\alpha_\sigma\right)=\frac{(1-k)(1-\sigma)}{\ell}\int_0^{\overline{n}_\sigma}dn>0$ in Case *(ii)*.

The second derivative with respect to ℓ can be derived as:

$$\begin{aligned}\frac{\partial^2\theta^*_\sigma}{\partial\ell^2} &= \frac{(1-k)r_2^U}{R}\frac{(1-\sigma)}{\left(r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2}\cdot \\ &\cdot\left\{-\frac{\partial^2\alpha_\sigma}{\partial\ell^2}\left(r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)-\left(\frac{\partial\alpha_\sigma}{\partial\ell}\right)^2\frac{(1-k)(1-\sigma)}{\ell}+\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma\right\} \\ &-\frac{\partial\theta^*_\sigma}{\partial\ell}\frac{\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell}+\frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma}{r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} \\ &-\theta^*_\sigma\frac{\frac{\partial^2\alpha_\sigma}{\partial\ell^2}\frac{(1-k)(1-\sigma)}{\ell}-\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell^2}-2\alpha_\sigma\frac{(1-k)(1-\sigma)}{\ell^3}+\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell^2}}{r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} \\ &+\theta^*_\sigma\frac{\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell}+\alpha_\sigma\frac{(1-k)(1-\sigma)}{\ell^2}}{\left(r_2^U-\frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2}\frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma>0.\end{aligned}$$

Note that $\frac{\partial^2\alpha_\sigma}{\partial\ell^2}=0$ in Case *(ii)* and $\frac{\partial^2\alpha_\sigma}{\partial\ell^2}=-\frac{\partial\overline{n}_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{(1-k)(1-\sigma)\ell}=-\frac{1}{\ell}\frac{1}{(1-k)(1-\sigma)}<0$ in Case *(ii)*. The terms in the second line (curly bracket) sum up to a positive for Case *(iii)* and they are zero for Case *(ii)*. The third, forth and fifth lines are positive.

Next, we derive the cross-partial as:

$$
\begin{aligned}
\frac{\partial \theta_{\sigma}^{*}}{\partial \ell \partial \sigma} = & \frac{(1-k)\, r_2^U}{R} \frac{-\frac{\partial^2 \alpha_\sigma}{\partial \ell \partial \sigma}(1-\sigma) + \frac{\partial \alpha_\sigma}{\partial \ell}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} + \frac{(1-k)\, r_2^U}{R} \frac{-\frac{\partial \alpha_\sigma}{\partial \ell}(1-\sigma)\left(\frac{\partial \alpha_\sigma}{\partial \ell}\frac{(1-k)(1-\sigma)}{\ell} - \alpha_\sigma \frac{(1-k)}{\ell}\right)}{\left(r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2} \\
& - \frac{\partial \theta_\sigma^*}{\partial \ell} \frac{\frac{\partial \alpha_\sigma}{\partial \ell}\frac{(1-k)(1-\sigma)}{\ell} + \frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} - \theta_\sigma^* \frac{\frac{\partial^2 \alpha_\sigma}{\partial \ell \partial \sigma}\frac{(1-k)(1-\sigma)}{\ell} - \frac{\partial \alpha_\sigma}{\partial \ell}\frac{1-k}{\ell} - \frac{1-k}{\ell^2}\alpha_\sigma + \frac{(1-k)(1-\sigma)}{\ell^2}\frac{\partial \alpha_\sigma}{\partial \sigma}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} \\
& + \theta_\sigma^* \frac{\left(+\frac{\partial \alpha_\sigma}{\partial \ell}\frac{(1-k)(1-\sigma)}{\ell} + \frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma\right)^2}{\left(r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2} > 0,
\end{aligned}
$$

where again $\frac{\partial^2 \alpha_\sigma}{\partial \ell \partial \sigma} = 0$ in Case *(ii)* and $\frac{\partial^2 \alpha_\sigma}{\partial \ell \partial \sigma} = -\frac{\partial \bar{n}_\sigma}{\partial \ell} / (1-\sigma) + \int_{\bar{n}_\sigma}^{1} \frac{1}{(1-k)(1-\sigma)^2 n} dn = -\frac{1}{(1-k)(1-\sigma)^2} + \int_{\bar{n}_\sigma}^{1} \frac{1}{(1-k)(1-\sigma)^2 n} dn < 0$ in Case *(iii)*. Recall that $\alpha_\sigma - \frac{\partial \alpha_\sigma}{\partial \sigma}(1-\sigma) = \int_0^{\bar{n}_\sigma} dn > 0$ and also $\frac{(1-k)(1-\sigma)}{\ell^2}\left(-\ell \frac{\partial \alpha_\sigma}{\partial \ell} + \alpha_\sigma\right) = \frac{(1-k)(1-\sigma)}{\ell} \int_0^{\bar{n}_\sigma} dn > 0$, as well as $\ell \frac{\partial^2 \alpha_\sigma}{\partial \ell \partial \sigma} + \frac{\partial \alpha_\sigma}{\partial \sigma} < 0$. Therefore, all terms are positive in Case *(iii)*, while the second, third and forth terms are positive in Case *(iii)*, and the other terms are zero.

Finally, taking the derivative with respect to r_2^I and r_2^U, we obtain:

$$
\begin{aligned}
\frac{\partial \theta_\sigma^*}{\partial r_2^I} &= \frac{\partial \underline{\theta}_\sigma}{\partial r_2^I} \frac{r_2^U - \frac{r_2^U (1-\sigma)\alpha_\sigma}{\bar{r}_{2,\sigma}}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} + \underline{\theta}_\sigma \frac{\frac{r_2^U (1-\sigma)\alpha_\sigma}{\bar{r}_{2,\sigma}^2}\sigma}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} > 0 \\
\frac{\partial \theta_\sigma^*}{\partial r_2^U} &= \frac{\partial \underline{\theta}_\sigma}{\partial r_2^U} \frac{r_2^U - \frac{r_2^U (1-\sigma)\alpha_\sigma}{\bar{r}_{2,\sigma}}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} + \underline{\theta}_\sigma \frac{1 - \frac{1-\sigma}{\bar{r}_{2,\sigma}}\alpha_\sigma + \frac{r_2^U (1-\sigma)^2 \alpha_\sigma}{\bar{r}_{2,\sigma}^2}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} - \underline{\theta}_\sigma \frac{r_2^U - \frac{r_2^U (1-\sigma)\alpha_\sigma}{\bar{r}_{2,\sigma}}}{\left(r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2},
\end{aligned}
$$

where the last derivative has an ambiguous sign as in the baseline model. This completes the proof of Proposition 6.

B.2 Proof of Lemma 6

Lemma 6. ***Positive profits.*** *The bank only makes a positive profit if no run occurs.*

Proof: Suppose there is a panic run, i.e. $\theta < \theta^*$, then the bank obtains a positive profit for all $\theta > \widetilde{\theta}_\sigma$, where $\widetilde{\theta}_\sigma$ follows from solvency for $n = 1$, i.e. in the event of a panic run:

$$
R\theta\left(1 - \frac{(1-k)(1-\sigma)}{\ell}\right) - (1-k)\,\sigma r_2^I = 0 \Leftrightarrow \widetilde{\theta}_\sigma \equiv \frac{1-k}{R} \frac{\sigma r_2^I}{1 - \frac{(1-k)(1-\sigma)}{\ell}}. \tag{34}
$$

We can show that $\theta^* < \widetilde{\theta}_\sigma, \forall \ell < 1-k$ and $r_2^U \geq r_2^I$. Moreover, $\lim_{\ell \to 1-k} \theta^* = \lim_{\ell \to 1-k} \widetilde{\theta}_\sigma = \underline{\theta}_\sigma$ if $r_2^U = r_2^I$, with $\lim_{\ell \to 1-k} \widetilde{\theta}_\sigma / \theta^* > 1$, which leads to a contradiction since it is not optimal to run if $\theta > \widetilde{\theta}_\sigma$. This concludes the proof. □

B.3 Proof of Proposition 7

To analyze risk management incentives, we differentiate $\Pi_\sigma(z;\ell_L,\ell_H)$ with respect to z, again ignoring the fixed cost, to compute the level of risk management that maximizes bank profits. The first-order condition is:

$$\frac{d\Pi_\sigma(z;\ell_L,\ell_H)}{dz} = p_\ell[R\theta_L^* - (1-k)(\sigma r_2^I + (1-\sigma)r_2^U)]\left(-\frac{d\theta_L^*}{dz}\right) - cz = 0. \tag{35}$$

As in equation (7), the first term is the marginal benefit of risk management and the second term is the marginal cost. However, the solution to (35) is now a function of σ and we denote it with $\hat{z}_\sigma$.

In Proposition 7 we focus on the model with panic runs, i.e. $\ell < \check{\ell}_\sigma$. Differentiating the FOC with respect to ℓ_L and σ gives:

$$\begin{aligned}
\frac{d^2\Pi_\sigma}{dzd\ell_L} &= -p_\ell\left(\frac{d^2\theta_L^*}{dzd\ell_L}\right)[R\theta_L^* - (1-k)(\sigma r_2^I + (1-\sigma)r_2^U)] - p_\ell\frac{d\theta_L^*}{dz}\frac{d\theta_L^*}{d\ell_L}R < 0 \\
\frac{d^2\Pi_\sigma}{dzd\sigma} &= -p_\ell\left(\frac{d^2\theta_L^*}{dzd\sigma}\right)[R\theta_L^* - (1-k)(\sigma r_2^I + (1-\sigma)r_2^U)] - p_\ell\frac{d\theta_L^*}{dz}[R\frac{d\theta_L^*}{d\sigma} - (1-k)(r_2^I - r_2^U)] < 0,
\end{aligned}$$

where $d\theta_L^*/d\ell = d\theta_L^*/dz < 0$, $d^2\theta_L^*/d\ell dz > 0$, $d\theta_L^*/d\sigma < 0$, $d^2\theta_L^*/d\ell d\sigma > 0$ and:

$$R\frac{\partial\theta_L^*}{\partial\sigma} - (1-k)\left(r_2^I - r_2^U\right) \quad < 0,$$

which together with $d^2\Pi_\sigma/dz^2 < 0$ leads to the results in Proposition 7. This concludes the proof.

B.4 Proof of Proposition 8

The first part of this proof discusses the existence of risk management failures, while the second part analyzes the role of DI coverage.

We follow the argument of the proof of Proposition 4. Lemmas 3 and 4 continue to hold with minor modifications. Lemma 5 needs to be adjusted as follows.

Denote by $\widehat{\ell}_\sigma(F)$ the interim asset value ℓ_L such that bank profits in the event of a negative shock are equal to the cost of setting up risk management operations, i.e., $\widehat{\ell}_\sigma(F)$ solves:

$$p_\ell\int_{\theta^*_{L,\sigma}(\widehat{\ell}_\sigma(F))}^{1}[R\theta - (1-k)r_2]d\theta = F. \tag{36}$$

Recall that in defining $\widehat{\ell}_\sigma(F)$ we are assuming the bank does no risk management, so that $z = 0$. We also assume that the variable cost parameter c, is above a threshold $\widehat{c}_\sigma$, which we define below such that $\widehat{z}_\sigma(0,\widehat{c}_\sigma)=\widehat{\ell}_\sigma$, where $\widehat{\ell}_\sigma$ is defined in Equation (36).

Lemma 7. *There exists a positive and finite value of the variable cost parameter, $\widehat{c}_\sigma$, such that $\widehat{z}_\sigma(0,\widehat{c})=\widehat{\ell}_\sigma(0)$.*

Proof. The proof of Lemma 7 replicates the proof of Lemma 5. □

Note that $\theta^*_{L,\sigma}(0)=\overline{\theta}\approx 1$ for $\ell_L<\widehat{\ell}_\sigma(0)$, so the upper bound in the integral of the inequality describing the differential payoff from setting up risk management operations,

$$p_\ell\int_{\theta^*_{L,\sigma}(z)}^{\theta^*_{L,\sigma}(0)}[R\theta-(1-k)\overline{r}_{2,\sigma}]d\theta-\frac{cz^2}{2}\geq F, \tag{37}$$

can be replaced by 1 for all $\ell_L<\widehat{\ell}_\sigma(0)$, as in the proof of Proposition 4.

Take $c>\widehat{c}_\sigma$ and $\ell_L=0$, then $\widehat{z}_\sigma(0,c)<\widehat{\ell}_\sigma(0)$ from Lemma 5 and Inequality (30) is violated for any $F\geq 0$. Since $\theta^*_\sigma(\widehat{z}_\sigma(\ell_L),\ell_L)$ decreases with ℓ_L, bank profits are increasing in ℓ_L. Thus, for $c>\widehat{c}_\sigma$ there exists a strictly positive cutoff value $\widetilde{\ell}_L(F)\in(0,\widehat{\ell}_\sigma(0))$ with associated optimal amount of risk management effort $\widehat{z}_\sigma(\widetilde{\ell}_L(F),c)$ that solves (37) with equality. The first result in Proposition 8 follows.

Next, we study the role of DI coverage. Denote $\widehat{F}_\sigma$ as in the proof of Proposition 5, as the solution to:

$$F=p_\ell\int_{\theta^*_{L,\sigma}(z^*_\sigma)}^{1}[R\theta-(1-k)\overline{r}_2]\,d\theta-c\frac{(z^*_\sigma)^2}{2}, \tag{38}$$

where z^*_σ solves the $FOC_z=0$, as defined in Equation (35). Taking the derivative of the RHS of Equation (38) with respect to σ, we obtain:

$$\begin{aligned}\frac{\partial\widehat{F}_\sigma}{\partial\sigma} &= -p_\ell\left(\underbrace{\frac{\partial\theta^*_{L,\sigma}(z^*_\sigma)}{\partial\sigma}}_{<0}+\underbrace{\frac{\partial\theta^*_{L,\sigma}(z^*_\sigma)}{\partial z}}_{<0}\underbrace{\frac{d\widehat{z}_\sigma}{d\sigma}}_{<0}\right)[R\theta^*-(1-k)\overline{r}_2]\\ &\quad -p_\ell\int_{\theta^*_{L,\sigma}(z^*_\sigma)}^{1}(1-k)\underbrace{\left(r^I_2-r^U_2\right)}_{<0}d\theta-cz^*_\sigma\underbrace{\frac{d\widehat{z}_\sigma}{d\sigma}}_{<0},\end{aligned}$$

which can be rearranged as follows:

$$\begin{aligned}\frac{\partial\widehat{F}_\sigma}{\partial\sigma} &= -p_\ell\frac{\partial\theta^*_{L,\sigma}(z^*_\sigma)}{\partial\sigma}[R\theta^*-(1-k)\overline{r}_2]+p_\ell\int_{\theta^*_{L,\sigma}(z^*_\sigma)}^{1}(1-k)\underbrace{\left(r^I_2-r^U_2\right)}_{<0}d\theta\\ &\quad +p_\ell\frac{d\widehat{z}_\sigma}{d\sigma}\left(\underbrace{-\frac{\partial\theta^*_{L,\sigma}(z^*_\sigma)}{\partial z}[R\theta^*-(1-k)\overline{r}_2]-cz^*_\sigma}_{FOC_z=0}\right)>0,\end{aligned}$$

since $\partial\theta^*_{L,\sigma}(z^*_\sigma)/\partial\sigma < 0$ and the second term is zero as it is just the FOC with respect to z. Therefore, $d\widehat{F}_\sigma/d\sigma > 0$ follows. This concludes the proof of Proposition 8.

B.5 Proof of Lemma 1

The bank chooses r_2 so as to maximize its expected profits as given in (4) subject to the participation constraint in Condition (11). Differentiating (4) with respect to r_2, we obtain the following FOC:

$$\begin{aligned} FOC &= -p_\ell \int_{\theta^*_L}^1 (1-k)\,d\theta - p_\ell \frac{\partial\theta^*_L}{\partial r_2}\left[R\theta^*_L - (1-k)\,r_2\right] \\ &\quad -(1-p_\ell)\int_{\theta^*_H}^1 (1-k)\,d\theta + (1-p_\ell)\frac{\partial\theta^*_H}{\partial r_2}\left[R\theta^*_H - (1-k)\,r_2\right] = 0. \end{aligned} \tag{39}$$

The equilibrium is $r_2^* = \max\left\{r_2^\Pi, r_2^V\right\}$, where r_2^Π is the solution to (39), while r_2^V is the solution to (11) holding with equality.

We start with r_2^Π and compute $SOC \equiv \partial FOC/\partial r_2$ by differentiating (39) with respect to r_2. Hence, we obtain:

$$\begin{aligned} SOC &= +p_\ell(1-k)\frac{\partial\theta^*_L}{\partial r_2} - p_\ell\frac{\partial^2\theta^*_L}{\partial r_2^2}\left[R\theta^*_L - (1-k)\,r_2\right] - p_\ell R\left(\frac{\partial\theta^*_L}{\partial r_2}\right)^2 + p_\ell(1-k)\frac{\partial\theta^*_L}{\partial r_2} + (1-p_\ell)(1-k)\frac{\partial\theta^*_H}{\partial r_2} \\ &\quad -(1-p_\ell)\frac{\partial^2\theta^*_H}{\partial r_2^2}\left[R\theta^*_H - (1-k)\,r_2\right] - (1-p_\ell)R\left(\frac{\partial\theta^*_H}{\partial r_2}\right)^2 + (1-p_\ell)(1-k)\frac{\partial\theta^*_H}{\partial r_2} < 0, \end{aligned}$$

because $\partial\theta^*_i/\partial r_2 < 0$ and $\partial^2\theta^*_i/\partial r_2^2 > 0$. The former derivative was derived in Proposition 2 and can be rearranged as $\frac{1-k}{R}\frac{r_2-\alpha}{r2-\alpha\frac{(1-k)}{L}} + \underline{\theta}\frac{1}{r2-\alpha\frac{(1-k)}{L}} - \theta^*_i\frac{1}{r2-\alpha\frac{(1-k)}{L}}$. This must be negative as otherwise the FOC will be negative and so r_2^Π cannot be a solution. The latter derivative is equal to:

$$\frac{\partial^2\theta^*_i}{\partial r_2^2} = \frac{1-k}{R}\frac{-\alpha+\alpha\frac{(1-k)}{L}}{\left(r2-\alpha\frac{(1-k)}{L}\right)^2} + \frac{(1-k)}{R}\frac{1}{r2-\alpha\frac{(1-k)}{L}} - \underline{\theta}\frac{1}{r2-\alpha\frac{(1-k)}{L}} - \frac{\partial\theta^*_i}{\partial r_2}\frac{1}{r2-\alpha\frac{(1-k)}{L}} + \theta^*_i\frac{1}{r2-\alpha\frac{(1-k)}{L}} > 0.$$

for $i = \{L, H\}$. Given $SOC < 0$, using the implicit function theorem, the sign of $dr_2^\Pi/d\ell_L$ is equal to the sign of $\partial FOC/\partial\ell_L$, which equals:

$$p_\ell\frac{\partial\theta^*_L}{\partial\ell_L}(1-k) - p_\ell\frac{\partial^2\theta^*_L}{\partial r_2\partial\ell_L}\left[R\theta^*_L - (1-k)\,r_2\right] - p_\ell\frac{\partial\theta^*_L}{\partial r_2}R\frac{\partial\theta^*_L}{\partial\ell_L} < 0,$$

since $\partial\theta^*_L/\partial\ell_L < 0$, $\partial^2\theta^*_L/\partial r_2\partial\ell_L > 0$ and $\partial\theta^*_L/\partial r_2 < 0$, which is a necessary condition for $r_2^* = r_2^\Pi$. Thus, $dr_2^*/d\ell_L < 0$ if $r_2 = r_2^\Pi$.

Next, consider r_2^V. The effect of a change in ℓ_L on r_2^V can be computed using the IFT:

$$\frac{dr_2^V}{d\ell_L} = -\frac{\partial V/\partial \ell_L}{\partial V/\partial r_2}.$$

The numerator is always positive; differentiating (11) with respect to ℓ_L gives:

$$p_\ell \int_0^{\theta_L^*} \frac{1}{1-k} d\theta - \frac{\partial \theta^*}{\partial \ell_L}\left(r_2 - \frac{\ell_L + z}{1-k}\right) > 0.$$

To sign the denominator, we consider separately the case in which $\partial\theta^*/\partial r_2 < 0$ and when $\partial\theta^*/\partial r_2 > 0$. In the former case, we can immediatelly see that if r_2 increases V increases due to both an increase in the repayment if no run occurs and because the run thresholds decreases with r_2. When $\partial\theta^*/\partial r_2 > 0$, it is less straightforward as the increase in the run thresholds decreases the expected payoffs of depositors, which constitutes an opposing effect.

We, next, prove that $\partial\theta^*/\partial r_2 > 0$ leads to a contradiction and can, thus, be excluded. Note first that $\partial\theta^*/\partial r_2 > 0$ is incompatible with a slack depositor participation constraint. To see this, observe that if the constraint were slack, then the bank would want to reduce r_2 in order to increase its profits, and would do so until the participation constraint becomes binding, i.e., $V = \rho_D$. Next, along the binding participation constraint, if r_2 is chosen such that $\partial\theta^*/\partial r_2 > 0$, then it must be that depositor expected payoffs are increasing in r_2: $dV/dr_2 > 0$. This is the case because otherwise the bank would instead prefer to reduce r_2 instead, since that would increase its profits, and would make depositors better off, which would be inconsistent with saying that r_2 has been chosen optimally. We arrive at a contradiction, because the depositor participation constraint cannot be slack. A similar argument can be made for changes in ℓ_H. The result in Lemma 1 follows.

B.6 Proof of Lemma 2

To study the role of the indirect effect via adjustments in the repayment of depositors, we study the bank's problem. The bank's profits are given by:

$$\max_{r_2} p_\ell \int_{\theta_L^*(r_2)}^1 (R\theta - (1-k)r_2)\,d\theta + (1-p_\ell)\int_{\theta_H^*(r_2)}^1 (R\theta - (1-k)r_2)\,d\theta - k\rho_E \quad s.t.\, V(r_2, \ell_L, \ell_H) \geq \rho_D,$$

where we introduce a modified notation to highlight the dependence of the run thresholds on the repayment to depositors, i.e., $\theta_{L,H}^*(r_2)$. Next, assuming that the depositor participation constraint binds, we can rewrite the bank's problem as:

$$\max_{r_2} p_\ell \int_0^{\theta_L^*(r_2)} (\ell_L + z)d\theta + p_\ell \int_{\theta_L^*(r_2)}^1 R\theta d\theta + (1-p_\ell)\int_0^{\theta_H^*(r_2)} \ell_H d\theta + (1-p_\ell)\int_{\theta_H^*(r_2)}^1 R\theta d\theta - (1-k)\rho_D - k\rho_E.$$

Observe that the bank has one policy variable, r_2, which affects the equilibrium run thresholds in both states in a deterministic way. From the above, it is clear that the objective of the bank is to select r_2 so as to minimize the overall run probability.

For a given ℓ_L, we have an optimal solution r_2^* which pins down θ_L^* and θ_H^*. Now suppose ℓ_L decreases to some $\ell_L' < \ell_L$. From Lemma 3 we know that $\ell_L' + (z^*)' < \ell_L + z^*$ and $\theta_L^*(\ell_L' + (z^*)') > \theta_L^*(\ell_L + z^*)$ for a given r_2. Now the depositors' participation constraint would no longer be satisfied, both because ℓ_L went down, but also because all things equal the run risk would increase as well. So r_2 needs to increase in order to satisfy depositors' participation constraint.

Next, we argue that the fall in ℓ_L cannot lead to both θ_L^* and θ_H^* going down in equilibrium, after the adjustment in r_2. First, observe that the bank's profits are unambiguously decreasing in the level of fragility. Second, let $r_2(\ell_L')$ be the level of deposit repayment chosen when ℓ_L falls to ℓ_L'. If both run thresholds were to go down when $\ell_L = \ell_L'$ and $r_2 = r_2(\ell_L')$, then it would be the case that profits evaluated at the original ℓ_L and $r_2 = r_2(\ell_L')$ would be even higher. This means that it would have been optimal for the bank to choose this level of deposit repayment even before the drop in ℓ_L since depositors' participation constraint was for sure satisfied with $r_2 = r_2(\ell_L')$. Since the bank had chosen a lower one, it cannot be that $r_2(\ell_L')$ is associated with a lower level of fragility in both states. Given that θ_H^* does not depend on ℓ_L and $r_2(\ell_L') > r_2(\ell_L)$ from Lemma 1, it must be the case that $\theta_H^*(r_2(\ell_L')) < \theta_H^*(r_2(\ell_L))$ from Proposition 2. Therefore, we must also have that $\theta_L^*(\ell_L') > \theta_L^*(\ell_L)$, as stated in the lemma. Note that this argument holds taking into account the risk management choice. This completes the proof.

B.7 Proof of Proposition 9

As in the proof of Proposition 4, it is optimal for the bank to engage in risk management as long as it obtains a net gain in terms of expected profits from it in the event of a negative shock (i.e., when $\ell = \ell_L$). This is given by Inequality (30) in the proof of Proposition 4, which we restate here for convenience:

$$p_\ell \int_{\theta_L^*(z)}^{\theta_L^*(0)} \left[R\theta - (1-k)\, r_2\left(\ell_L\right)\right] d\theta - c\frac{z^2}{2} \geq F. \tag{40}$$

The two extremes of the integral only differ because of z. Let's focus on the upperbound.

The run threshold $\theta_L^*(0)$ converges to 1 when ℓ_L falls below some positive threshold $\check{\ell}_L$. Formally, we show that $\exists\, \check{\ell}_L \in (0, 1-k)$ such that $\theta_L^* \to \overline{\theta} \approx 1$ for $\ell_L \to \check{\ell}_L > 0$. To see this, it is useful to rewrite the expression for $\theta_L^*(0)$ as follows:

$$\theta_L^*(0) = \frac{(1-k)\, r_2}{R} \frac{r_2\left(\ell_L\right) - \frac{\ell_L}{1-k}\left(1 - \ln\left(\frac{\ell_L}{1-k}\right)\right)}{r_2\left(\ell_L\right) - \left(1 - \ln\left(\frac{\ell_L}{1-k}\right)\right)}.$$

First, notice that for θ_L^* to exist the following two conditions are necessary:

$$r_2(\ell_L) - \ell_L/(1-k)\,(1-\ln(\ell_L/(1-k))) \quad > \quad r_2(\ell_L) - (1-\ln(\ell_L/(1-k))) \tag{41}$$
$$r_2(\ell_L) - (1-\ln(\ell_L/(1-k))) \quad > \quad 0, \tag{42}$$

where the second inequality follows from depositor indifference. Both the numerator and the denominator of θ_L^* are monotonically decreasing in ℓ_L. Furthermore, the bank operates only with non-negative profits, which imposes an upper bound on $r_2\,(\ell_L)$, i.e. $r_2\,(\ell_L) < R/(1-k)$ independent on whether it is determined by (11) or (39).

As $\ell_L \to 0$, the left-hand-side of Inequality (41) goes to $r_2 < R/(1-k)$, while the right-hand side is strictly negative for all $\ell_L < \check{\check{\ell}}_L$, where $\check{\check{\ell}}_L > 0$ solves:

$$R/(1-k) = 1 - ln\left(\check{\check{\ell}}_L/(1-k)\right).$$

Conversely, for $\ell_L \to 1-k$, both sides of Inequality (41) go to r_2. Thus, by continuity and monotonicity, $\exists\ \check{\ell}_L(0) > \check{\check{\ell}}_L$ such that $\theta_L^*(0) = 1$ for all $\ell_L \in [0, \check{\ell}_L(0)]$.

When $z > 0$, the same argument can be used to show that the lower extreme of the integral in condition (40), $\theta_L^*\,(z)$, goes to the upperbound when ℓ_L is small. However, as θ_L^* is decreasing in z, so that $\theta_L^*\,(z) < \theta_L^*\,(0)$, the value of ℓ_L for which $\theta_L^*\,(z) \to 1$ is lower than $\check{\ell}\,(0)$ and we denote it as $\check{\ell}\,(z)$. Therefore, if $c \to \infty$, then $\ell_L + \hat{z}(\ell_L) < \check{\ell}_L(0)$ for all $\ell_L \in [0, \check{\ell}_L(0))$ and $z^* = 0$. Analogous to the proof of Proposition 4, we can define a $\check{c}$ such that $\hat{z}(0, \check{c}) = \check{\ell}_L(0)$. Then, for any $c > \check{c}$ and $F > 0$, there exists a sufficiently large negative shock to the interim asset value such that zero risk management is optimal. This completes the proof.

B.8 Proof of Proposition 10

We assume that R is high enough to ensure that financial intermediation is profitable and the participation constraint of the banker is always slack, $\Pi \geq 0$. Since the participation constraint of depositors binds, it pins down r_2^* for any level of bank capital k. That is, $V(r_2^*, k) = \rho_D$ for all $k < 1-\ell$. Multiplying the binding participation constraint by deposits $(1-k)$ and inserting into the bank's expected profits yields the following reduced problem:

$$\begin{aligned}\max_k \Pi \quad = \quad & p_\ell\left(\int_0^{\theta_L^*}(\ell_L+z)d\theta + \int_{\theta_L^*}^1 R\theta d\theta\right) + (1-p_\ell)\left(\int_0^{\theta_H^*}\ell_H d\theta + \int_{\theta_H^*}^1 R\theta d\theta\right) \\ & -(1-k)\rho_D - k\rho_E - c\frac{z^2}{2} - F\mathbb{1}_{\{z>0\}}.\end{aligned}$$

Focusing on the interior solution $\widehat{z} > 0$ and invoking the envelope theorem and $d\Pi/dz = 0$, we have the following first-order condition for bank capital:

$$\frac{d\Pi}{dk} = p_\ell \left[R\theta_L^* - (\ell_L + z)\right]\left(-\frac{d\theta_L^*}{dk}\right) + (1-p_\ell)\left[R\theta_H^* - \ell_H\right]\left(-\frac{d\theta_H^*}{dk}\right) - (\rho_E - \rho_D) = 0. \tag{43}$$

Equation (43) shows the trade-off associated with more bank capital. The first two terms are the endogenous expected marginal benefit of capital in terms of improving bank stability in both states of the world. The last term is the marginal cost of capital because capital is assumed to be a more expensive form of bank funding. Each of the first two terms are strictly positive, while the last term converges to zero as $\rho_E \to \rho_D$. Therefore, it is optimal for the bank to raise a strictly positive amount of bank capital as long as ρ_E is not too much larger than ρ_D. This completes the proof.

B.9 Proof of Proposition 11

To see the effect of commitment on deposit rates, consider the participation constraint of investors, given in Condition (11). Using the implicit function theorem, higher risk management reduces the deposit rate, i.e. $\frac{dr_2^*}{dz} < 0$. To see this, note that a greater amount of risk management and a higher deposit rate increase the value of the deposit claim V:

$$\begin{aligned}
\frac{\partial V}{dz} &= p_\ell \left\{\int_0^{\theta_L^*} \frac{1}{1-k} d\theta + \left[r_2 - \frac{\ell_L + z}{1-k}\right]\left(-\frac{d\theta_L^*}{dz}\right)\right\} > 0 \\
\frac{\partial V}{dr_2} &= p_\ell \int_{\theta_L^*}^1 d\theta + (1-p_\ell)\int_{\theta_H^*}^1 d\theta + \left[r_2 - \frac{\ell_L + z}{1-k}\right]\left(-\frac{d\theta_L^*}{dr_2}\right) + \left[r_2 - \frac{\ell_H}{1-k}\right]\left(-\frac{d\theta_H^*}{dr_2}\right),
\end{aligned}$$

because risk management reduces bank fragility upon a shock, $d\theta_L^*/dz < 0$. A higher deposit rate directly increases the value of the deposit claim and has an indirect effect via bank fragility. A sufficient condition for the value of the claim to increase in deposit rates, $\partial V/dr_2 > 0$, is that a higher deposit rate reduces fragility, which arises for a low equilibrium deposit rate r_2^* (Proposition 2). For example, a high enough return on investment R suffices for this to arise.

Similarly, a higher deposit rate directly reduces bank profits and has an indirect effect via fragility:

$$\begin{aligned}
\frac{\partial \Pi}{\partial r_2} = &-(1-k)\left[p_\ell(1-\theta_L^*) + (1-p_\ell)(1-\theta_H^*)\right] \\
&-p_\ell\left[R\theta_L^* - (1-k)r_2\right]\frac{\partial \theta_L^*}{\partial r_2} - (1-p_\ell)\left[R\theta_H^* - (1-k)r_2\right]\frac{\partial \theta_H^*}{\partial r_2}.
\end{aligned}$$

If the value of the deposit claim increases in the deposit rate, then $\partial\Pi/\partial r_2 < 0$ must hold in equilibrium. The intuition is as follows. If the participation constraint were to bind for a deposit rate at which marginal profits still increase in the deposit rate, the bank would voluntarily pay higher deposit rates (in order to benefit from the beneficial effect via lower fragility). Then, the participation constraint would be slack and the equilibrium deposit rate pinned down by zero marginal profits. Since we focus on parameters for which the participation constraint of investors binds in equilibrium, it must be that higher deposit rates reduce expected bank profits, as was to be shown.

Equipped with these results, we can turn to the planner's choice of risk management effort, where $\widehat{z}_P$ is the equivalent of $\widehat{z}$ for the planner, and similarly for r_2^P. The first-order condition that pins down $\widehat{z}_P$ is:

$$\frac{d\Pi}{dz} \equiv \frac{\partial \Pi}{\partial z} + \frac{\partial \Pi}{\partial r_2}\frac{dr_2}{dz},$$

where $\frac{dr_2}{dz}$ comes from the binding participation constraint and $\frac{\partial \Pi}{\partial z} = 0$ pins down $\widehat{z}$ for the bank. Since $\frac{\partial \Pi}{\partial r_2}\frac{dr_2}{dz} > 0$, we must have $\frac{\partial \Pi}{\partial z}|_{z=\widehat{z}_P} < 0$. By the concavity of Π in z, it immediately follows that $\widehat{z}_P > \widehat{z}$. In words, when abstracting from the fixed cost,the planner chooses a higher risk management than the bank, because the planner internalizes its benefit for reducing deposit rates.

Turning to risk management failures, we see that the planner's commitment to future risk management also has implications for whether the bank engages in any risk management in the first place. The result is straightforward from the following considerations. The planner maximizes welfare $SW(z)$, while the bank maximizes $\Pi(z)$. Therefore, it is immediate that $SW(\widehat{z}_p) \geq SW(\widehat{z})$, with the inequality strict whenever $\widehat{z}_p \neq \widehat{z}$. Now, $\Pi(z) = SW(z) - V(z)$. Since the participation constraint binds throughout, we have that $V(\widehat{z}_p) = V(\widehat{z})$, which then implies that $\Pi(\widehat{z}_p) \geq \Pi(\widehat{z})$, with the inequality again strict whenever $\widehat{z}_p \neq \widehat{z}$. This then implies that for any given ℓ_L, if the differential in bank profits between hedging and not is exactly equal to zero when $F = \widehat{F}$ and the bank chooses $z = \widehat{z}$, they must be strictly positive when the planner chooses $z = \widehat{z}_p$. Hence, $\widehat{F}^P > \widehat{F}$. This completes the proof.

B.10 Proof of Proposition 12

We begin by analysing the alternative model of risk management and deriving the equilibrium depositor run thresholds to obtain some preliminary results, focusing on the aspects that differ from the main specification. First, consider the impact of risk management z on bank fragility. We start from the bad state, $\ell = \ell_L$. There are now three cases for interim withdrawals. We consider them in turn.

Case 1: $n \leq z/(1-k) \equiv \underline{n}$**.** When withdrawals are below $\underline{n}$, no liquidation is needed to serve withdrawals because the cash received from the risk management contract suffices. Thus, the bank is liquid at $t = 1$ and stores the remainder, $z - n(1-k)$, until time 2. The bank is solvent at $t = 2$

whenever:

$$R\theta + z - n(1-k) \geq (1-k)(1-n)r_2.$$

Note that the lower dominance bound is also different relative to the baseline model and now solves $R\theta + z = (1-k)r_2$, so the equilibrium fundamental threshold changes to:

$$\underline{\theta}_{L,S} \equiv \frac{(1-k)r_2 - z}{R},$$

where A stands for the alternative modelling of risk management. We have the following ranking: $\underline{\theta}_A < \underline{\theta}$. Since $\theta \geq \underline{\theta}_A$ holds when establishing the bank failure threshold, the bank is always solvent at $t = 2$. To see this, $R\underline{\theta} + z - n(1-k) \geq (1-k)(1-n)r_2 \Leftrightarrow r_2 \geq 1$, which always holds.

Case 2: $z/(1-k) < n \leq \overline{n}_{L,S} \equiv (\ell_L + z)/(1-k)$**.** For intermediate levels of withdrawals, the bank is liquid at $t = 1$ and can meet all withdrawals, so depositors who withdraw early are repaid in full. To ensure this, some liquidation is required, namely the fraction $(1-k)(n - \underline{n})/\ell_L$ of investment. Thus, the bank is solvent at $t = 2$ if:

$$R\theta \left(1 - \frac{(1-k)n - z}{\ell_L}\right) \geq (1-k)(1-n)r_2.$$

The insolvency threshold of withdrawals solves this condition with equality, so:

$$\widehat{n}_{L,S} \equiv \frac{R\theta\left(1 + \frac{z}{\ell_L}\right) - (1-k)r_2}{R\theta\left(\frac{1-k}{\ell_L}\right) - (1-k)r_2}.$$

We focus on parameters such that z^* is low enough in order to ensure that $\widehat{n}_{L,S} \leq 1$.

Case 3: $\overline{n}_A < n$**.** Full liquidation occurs at $t = 1$ and withdrawing depositors receive a pro-rata share of liquidation proceeds. Depositors who wait until $t = 2$ receive nothing.

For vanishing private noise, the usual Laplacian property holds, $n \sim U[0,1]$. Thus, a depositor's expected payoff from withdrawing early is:

$$\alpha_{L,S} = \int_0^{\overline{n}_{L,S}} dn + \int_{\overline{n}_{L<S}}^1 \frac{\ell_L + z}{(1-k)n} dn.$$

Thus, $\alpha_{L,S}$ has the exact same functional form as α; the argument ℓ_L is just replaced by $\ell + z$.

As in the baseline model, risk management effort z increases depositors' expected payoff from withdrawing. The effect is twofold. First, it increases the pro-rata share in the event the bank fails at $t = 1$. Second, it makes bank failure at $t = 1$ less likely. It follows that $\partial \alpha_{L,S}/\partial z > 0$.

The equilibrium failure threshold again solves $\int_0^{\widehat{n}_A} r_2 dn = \alpha_A$, so

$$\theta^*_{L,A} = \frac{(1-k)r_2}{R} \frac{r_2 - \alpha_A}{\left(1+\frac{z}{\ell_L}\right) r_2 - \frac{\alpha_A(1-k)}{\ell_L}}.$$

The characterization of the run threshold in the good state, i.e., $\ell = \ell_H$ is simpler and follows the same steps as in the baseline model, which we omit for brevity. The bank is illiquid at $t = 1$ if withdrawals and swap payment exhaust the liquidation value of investment: $(1-k)n + z = \ell_H$, so

$$\overline{n}_{H,S} = \frac{\ell_H - z}{1-k} < 1,$$

because $\ell_H < 1-k$. Intuitively, a higher interest rate swap payment reduces the liquidity available to depositors at the interim date, $\frac{d\overline{n}_{H,S}}{dz} = -\frac{1}{1-k} < 0$.

For $n \leq \overline{n}_{H,S}$, the bank continues until date 2 and fails due to insolvency if

$$n > \widehat{n}_{H,S} \equiv \frac{R\theta\left(1-\frac{z}{\ell_H}\right) - (1-k)r_2}{\frac{R\theta(1-k)}{\ell_H} - (1-k)r_2},$$

where the insolvency bound $\widehat{n}_{H,S}$ solves the insolvency condition:

$$R\theta\left[1 - \frac{z + (1-k)n}{\ell_H}\right] - (1-k)(1-n)r_2 = 0.$$

Intuitively, a higher swap payment z at the interim triggers more partial liquidation of the asset, so the bank is insolvent at the final date for a smaller proportion of withdrawals, $\frac{d\widehat{n}_{H,S}}{dz} < 0$.

Using the insolvency condition, the lower dominance bound ($n = 0$) is

$$\underline{\theta}_{H,S} = \frac{(1-k)r_2}{R\left(1-\frac{z}{\ell_H}\right)} = \frac{r_2}{R}\frac{\ell_H}{\overline{n}_{H,S}}.$$

Using these bounds, we can define the following terms: $\alpha_{H,S} \equiv \frac{\ell_H - z}{1-k}\left[1 - \ln\left(\frac{\ell_H - z}{1-k}\right)\right] = \overline{n}_{H,S}\left[1 - \ln\left(\overline{n}_{H,S}\right)\right]$, so $\frac{d\alpha_{H,S}}{d\overline{n}_{H,S}} = -\ln(\overline{n}_{H,S}) > 0$. For future reference, we have $\frac{d\alpha_{H,S}}{dz} = \frac{\partial \alpha_{H,S}}{\partial z} = \frac{1}{1-k}\ln\left(\frac{\ell_H - z}{1-k}\right) = \frac{1}{1-k}\ln\left(\overline{n}_{H,S}\right) < 0$ from the chain rule, where the first inequality arises from $\frac{\partial \alpha_{H,S}}{\partial \overline{n}_S} = 0$. In words, a higher swap payment reduces the expected payoff from withdrawing, which tends to stabilize the bank.

The indifference condition $r_2 \widehat{n}_{H,S} \equiv \alpha_{H,S}$. Rewriting yields the bank failure threshold:

$$\theta^*_{H,S} \equiv \frac{r_2(1-k)}{R} \frac{r_2 - \alpha_{H,S}}{r_2\left(1-\frac{z}{\ell_H}\right) - (1-k)\frac{\alpha_{H,S}}{\ell_H}} = \underline{\theta}_{H,S} \frac{r_2 - \alpha_{H,S}}{r_2 - \alpha_{H,S}\frac{1-k}{\ell_H - z}} > \underline{\theta}_{H,S}, \tag{44}$$

where the existence of panic runs arises from $1-k > \ell_H - z$.

As it would be useful later for the derivatives, it is algebraically convenient to re-express the failure threshold in terms of $\bar{n}_S$ and α_S only:

$$\theta^*_{H,S} \equiv \frac{r_2}{R}\,\ell_H\,\frac{r_2-\alpha_{H,S}}{r_2\bar{n}_{H,S}-\alpha_{H,S}}.$$

Next, we proceed to sign the effect of higher risk management on bank fragility. Let $x_{L,S} \equiv \frac{\ell_L+z}{1-k}$. Then, we can again express the expected payoff from withdrawing early compactly as $\alpha_{L,S} \equiv x_{L,S}(1-\ln(x_{L,S}))$. Using this expression for $\alpha_{L,S}$, we can express the failure threshold as:

$$\theta^*_{L,S} = \underline{\theta}\,\frac{\ell_L}{\ell_L+z}\,\frac{r_2-x_{L,S}(1-\ln(x_{L,S}))}{r_2-(1-\ln(x_{L,S}))}.$$

It is immediate that $d\theta^*_{L,S}/dz<0$. The first factor is independent of risk management, the second factor decreases in risk management, and the third factor has the same mathematical structure as before, so $d\theta^*_{L,S}/dx_{L,S}<0$ and $dx_{L,S}/dz>0$ (see also Appendix A.1.3).

Consider now state H. Differentiating (44) with respect to z, we obtain:

$$\begin{aligned}\frac{d\theta^*_{z,H}}{dz} \;=\;& \frac{(1-k)\,r_2}{R\left(1-\frac{z}{\ell_H}\right)^2}\frac{1}{\ell_H}\frac{r_2-\alpha_{z,H}}{r_2-\alpha_{z,H}\frac{1-k}{\ell_H-z}}\\ &+\frac{(1-k)\,r_2}{R\left(1-\frac{z}{\ell_H}\right)}\frac{-\frac{d\alpha_{z,H}}{dz}}{\left(r_2-\alpha_{z,H}\frac{1-k}{\ell_H-z}\right)}-\frac{(1-k)\,r_2}{R\left(1-\frac{z}{\ell_H}\right)}\frac{(r_2-\alpha_{z,H})\left[-\frac{d\alpha_{z,H}}{dz}\frac{1-k}{\ell_H-z}-\alpha_{z,H}\frac{1-k}{(\ell_H-z)^2}\right]}{\left(r_2-\alpha_{z,H}\frac{1-k}{\ell_H-z}\right)^2},\end{aligned}$$

which can be rearranged as follows:

$$\frac{d\theta^*_{z,H}}{dz}=\frac{\theta^*_{z,H}}{\ell_H-z}-\frac{\theta^*_{z,H}\ln\left(\frac{\ell_H-z}{1-k}\right)}{(1-k)(r_2-\alpha_{z,H})}+\frac{\theta^*_{z,H}}{\ell_H-z}\frac{1}{r_2-\alpha_{z,H}\frac{1-k}{\ell_H-z}}>0,$$

since

$$\frac{d\alpha_{z,H}}{dz}\frac{1-k}{\ell_H-z}+\alpha_{z,H}\frac{1-k}{(\ell_H-z)^2}=\frac{1}{\ell_H-z}.$$

This completes the proof.

B.11 Proof of Proposition 13

Differentiating the second integral in Equation (12) with respect to z, we obtain the marginal costs of risk management can be expressed as follows:

$$C'(z)\equiv(1-p_\ell)\left[R\theta^*_{H,S}\left(1-\frac{z}{\ell_H}\right)-(1-k)r_2\right]\frac{d\theta^*_{H,S}}{dz}+\frac{1-p_\ell}{\ell_H}\int_{\theta^*_{H,S}}^1 R\theta d\theta>0,$$

because the marginal profit at the failure threshold is positive (due to panic runs).

It follows from the expression above that the cost of risk management is convex if:

$$C''(z) \equiv (1-p_\ell)R\left(\frac{d\theta^*_{H,S}}{dz}\right)^2\left(1-\frac{z}{\ell_H}\right)+(1-p_\ell)[R\theta^*_{H,S}(z)-(1-k)r_2]\frac{d^2\theta^*_{H,S}}{dz^2}$$
$$-2\frac{1-p_\ell}{\ell_H}R\theta^*_{H,S}\frac{d\theta^*_{H,S}}{dz}>0.$$

Note that $\frac{d^2\theta^*_{H,S}}{dz^2}\geq 0$ and $\frac{\ell_H-z}{2}\frac{d\theta^*_{H,S}}{dz}\geq\theta^*_{H,S}$ are sufficient for convexity (where the second sufficient condition arises from combining the first and third term of the second derivative).

We can then express the derivative of $\theta^*_{H,S}$ obtained in the proof of Proposition 12 as follows:

$$\frac{d\theta^*_{z,H}}{dz}=\theta^*_{z,H}\left[\frac{1}{\ell_H-z}-\frac{\ln\left(\frac{\ell_H-z}{1-k}\right)}{(1-k)(r_2-\alpha_{z,H})}+\frac{1}{\ell_H-z}\frac{1}{r_2-\alpha_{z,H}\frac{1-k}{\ell_H-z}}\right]$$

Denote the expression in the square bracket as Φ. Then, we can write the second derivative as follows:

$$\frac{d^2\theta^*_{z,H}}{dz^2}=\frac{d\theta^*_{z,H}}{dz}\Phi+\theta^*_{z,H}\frac{d\Phi}{dz}.$$

Hence, it follows immediately that $\frac{d^2\theta^*_{z,H}}{dz^2}>0$ if $\frac{d\Phi}{dz}\geq 0$.

Taking out the term $\frac{1}{1-k}$ from the expression for Φ, and substituting for $\bar{n}_S$, we can simplify this term as follows:

$$\Phi(\bar{n}_S)\equiv\frac{1}{1-k}\left[\frac{1}{\bar{n}_S}-\frac{\ln(\bar{n}_S)}{r_2-\alpha_S}+\frac{1}{r_2\bar{n}_S-\alpha_S}\right],$$

so $\frac{d\Phi}{dz}\geq 0$ whenever $\frac{d\Phi}{d\bar{n}_S}\leq 0$ (because $\frac{d\bar{n}_S}{dz}=-\frac{1}{1-k}<0$). Computing this derivative, we have:

$$\frac{d\Phi}{d\bar{n}_S}=\frac{1}{1-k}\left[-\frac{1}{\bar{n}_S^2}-\frac{1}{\bar{n}_S(r_2-\alpha_S)}-\frac{\ln(\bar{n}_S)}{(r_2-\alpha_S)^2}\frac{d\alpha_S}{d\bar{n}_S}-\frac{1}{(r_2\bar{n}_S-\alpha_S)^2}\left(r_2-\frac{d\alpha_S}{d\bar{n}_S}\right)\right]<0.$$

This (desired) sign arises for two reasons (and under a sufficient condition). First, consider the fourth term and note that $r_2-\frac{d\alpha_S}{d\bar{n}_S}>0\Leftrightarrow r_2>-\ln(\bar{n}_S)$ because $r_2>1-\ln(\bar{n}_S)$ from the definition of the failure threshold (a positive denominator). Thus, the fourth term has the desired sign.

Second, the third term has the opposing sign. Combining the first and the third term, a sufficient condition for $\frac{d\Phi}{d\bar{n}_S}<0$ is

$$-\frac{1}{\bar{n}_S^2}-\frac{\ln(\bar{n}_S)}{(r_2-\alpha_S)^2}\frac{d\alpha_S}{d\bar{n}_S}\leq 0\Leftrightarrow\bar{n}_S^2\ln(\bar{n}_S)^2\leq(r_2-\alpha_S)^2.$$

A sufficient condition for the latter inequality is $r_2-\alpha_S\geq -n_S\ln(\bar{n}_S)$. Since $r_2>1-\ln(\bar{n}_S)$ (positive denominator of failure threshold) and $-n_S\ln(\bar{n}_S)<n_S(1-\ln(\bar{n}_S))$, a sufficient condition for the

desired inequality is:

$$1-\ln(\overline{n}_S)-\overline{n}_S(1-\ln(\overline{n}_S)) \geq \overline{n}_S(1-\ln(\overline{n}_S)) \Leftrightarrow 1-\overline{n}_S \geq \overline{n}_S.$$

Since $z \geq 0$, a sufficient condition is $\ell_H \leq (1-k)/2$. This is quite restrictive, but we can then show that in the numerical example that the desired result holds more broadly. Taken together, under this condition, the failure threshold is convex in risk management effort.

Using Equation (B.11), we can rewrite the second sufficient condition as

$$\frac{r_2-\alpha_{z,H}}{r_2-\frac{\alpha_{z,H}}{\overline{n}_S}} \geq \overline{n}_S \ln(\overline{n}_S),$$

which always holds because the LHS is proportional to $\theta^*_{H,S}$ and thus positive, while the RHS is negative (because $\overline{n}_S < 1$). Finally, the convexity of the failure threshold arises from the sufficient condition stated in the proposition (as proven above). This completes the proof.