# Central Bank Digital Currency and Financial Stability*

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**Abstract**

We develop a model of financial intermediation with endogenous bank runs and remunerated Central Bank Digital Currency (CBDC) as consumers' alternative to bank deposits. On the one hand, higher CBDC remuneration raises bank fragility by increasing withdrawal incentives. On the other hand, it induces banks to offer more attractive deposit contracts to retain funding, which reduces fragility. We disentangle these two channels and show that higher CBDC remuneration can improve bank stability when banks have sufficient market power in the deposit market and are subject to panic runs.

*Keywords*: Central Bank Digital Currency, Bank Fragility, Financial Stability, CBDC Remuneration, Global Games.

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# 1 Introduction

Central banks worldwide are exploring the costs and benefits of introducing retail central bank digital currencies (CBDCs) in response to the declining use of cash and the growing prevalence of private digital currencies such as stablecoins. CBDCs are envisioned as public digital money designed to preserve monetary sovereignty while mitigating the risks posed by private alternatives (Kosse and Mattei, 2022). However, the introduction of CBDCs has raised concerns among policymakers about their potential adverse effects on the business model of traditional banks (Vives, 2019) and on the financial system (Ahnert et al., 2022).

One issue that has received particular attention is the effect of CBDC on financial stability (Bank for International Settlements, 2020). Its status as a safe asset and the potential for positive remuneration—a feature that differentiates it from physical cash—could render it an attractive store of value. This may give rise to an increased risk of bank runs during periods of financial stress. A central question is how CBDC remuneration policies could influence the stability of the banking system.[1]

This paper aims to inform this debate by incorporating remunerated CBDC into a global-games model of bank runs in the spirit of Goldstein and Pauzner (2005). In our paper, a monopolistic bank sets deposit rates in order to maximize profits. Importantly, when choosing the terms of the deposit contract, the bank must anticipate depositors' withdrawal decisions and account for depositors' outside option of a remunerated CBDC account (see e.g., Chiu et al., 2023; Andolfatto, 2021; Whited et al., 2022). By placing the interaction between CBDC remuneration, deposit rates, and the run probability at the center of analysis, our model offers an ideal framework to study how changes in CBDC remuneration affect bank stability.

In our setup, an increase in CBDC remuneration has two effects. First, a higher CBDC rate makes withdrawing funds at the interim stage more attractive, and thereby increases the likelihood of bank runs. This mechanism, which we refer to as *direct effect*, aligns with the concern that CBDCs could exacerbate bank fragility by offering a superior outside option for depositors (Keister and Monnet, 2022; Williamson, 2022). Second,

[1]The Chinese Central Bank (PBoC) started paying interest on its CBDC, the e-CNY, in January 2026 (Bloomberg, 2025).

higher CBDC remuneration improves consumers' outside option at the funding stage. This induces the bank to adjust its deposit rates to retain consumer funds, and thus creates an *indirect effect* that can amplify or mitigate the aforementioned direct effect. We provide sufficient conditions under which this indirect effect is beneficial and sufficiently strong so that an increase in CBDC remuneration decreases overall bank fragility. We also show via numerical examples that our main result holds for a wider set of parameters, and discuss how it is shaped by bank market power and the existence of panic runs.

Our model features a three-date economy populated by a bank and a continuum of risk-averse consumers subject to a liquidity shock. As is standard in the bank run literature (see e.g., Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005), consumers value liquidity insurance provided by the bank in the form of a higher repayment at the interim date. At the same time, a higher interim repayment increases the bank's exposure to a run. The bank's profit-maximizing choice of short-term and long-term repayments strikes a fine balance between these two effects and account for their overall effect on expected bank profits.

Importantly, the level of CBDC remuneration affects the terms of the deposit contract offered by the bank. Overall, a higher CBDC rate improves consumers' outside option and thus requires the bank to increase deposit rates to retain funding. This channel is consistent with empirical evidence that links bank deposit rates to the return on alternative investments (Drechsler et al., 2017).[2] However, the resulting impact on bank stability crucially depends on how the bank allocates this adjustment across the short-term and long term deposit rates. In particular, a higher short-term deposit rate increases the bank fragility by encouraging early withdrawals, while a higher long-term deposit rate reduces fragility by incentivizing patient consumers to keep their funds invested until maturity.

When CBDC remuneration is relatively low, the bank offers an interim repayment that is strictly below the liquidation value of investment. Since this implies that the bank does not provide any liquidity insurance, bank runs are only fundamental-driven. In this case, any marginal increase in CBDC remuneration induces the bank to increase

[2]More specifically, Drechsler et al. (2017) find that higher Fed Funds rates increase the return offered by asset classes such as bond or money market funds. As a result of these alternatives to deposits becoming more attractive, banks adjust their deposit rates to limit deposit outflows. Moreover, Martin et al. (2026) show that banks in distress raise deposit rates to attract new deposits, in line with our mechanism.

both short-term and long-term repayments by roughly similar amounts, which leads to an increase in bank fragility.

When CBDC remuneration is relatively high, it is optimal for the banks to provide liquidity insurance to depositors and set the short-term repayment strictly above the liquidation value. As a consequence, it becomes exposed to panic runs. In this case, it responds to a marginal increase in CBDC remuneration by improving the deposit contract mainly via an increase of the long-term repayment. We show that this adjustment of the long-term rate can be sufficiently strong to offset the detrimental effects of a higher CBDC remuneration (both the direct one and the indirect effect through a higher short-term repayment). As a result, bank fragility declines.

Our analysis identifies two key ingredients for the beneficial effect of CBDC on bank stability to emerge: (1) the bank's exposure to panic runs and (2) sufficient bank market power in the deposit market. Concerning the former, the presence of panic runs mitigates the bank's incentives for making the deposit contract more attractive by raising the short-term repayment and thus shifts the bulk of the adjustment onto the long-term rate. This creates the conditions for the beneficial indirect effect on the probability of a run to dominate. Concerning the latter, we extend the model the baseline model and assume that the deposit contract is determined by Nash bargaining between the bank and depositors. In this setting, we show that bank market power must be sufficiently large for a beneficial impact of CBDC remuneration on bank stability to materialize. Market power is important for two reasons. First, it allows the bank to have large enough profit margins to accommodate the increase in CBDC remuneration by increasing the long-term repayment. Second, higher market power implies that less weight is attached to the liquidity insurance provided by high short-term deposit rates. In the limiting case of perfect competition, we show analytically that a higher CBDC rate always leads to a reduction in the short-term deposit rate and an increase in the long-term deposit rate. However, this does not translate into lower bank fragility because the deposit rate is already relatively high and thus not very responsive to changes in CBDC remuneration.

Our paper makes three contributions to the literature. First, we provide a theoretical framework for analyzing the impact of CBDC remuneration on bank stability that highlights the role of deposit rate adjustments in shaping the equilibrium outcomes. Second, we inform the policy debate on CBDC design by identifying conditions under which

CBDC remuneration can improve bank stability. Third, we provide a methodological contribution to the broader literature on bank stability and global games by endogenizing the terms of the deposit contract and characterizing the optimal allocation of depositors' repayments by a profit-maximizing bank, in a context in which depositors are risk-averse and subject to idiosyncratic liquidity shocks.

**Literature.** Our paper is part of a fast-growing literature on CBDC. An overview of recent work is found in Ahnert et al. (2022). A key feature of our model is that the bank is not passive, but instead adjusts its behaviour (here its deposit rates) in response to the introduction to CBDC. This channel is also present in recent papers that study the effects of CBDC on credit supply and liquidity services (Brunnermeier and Niepelt, 2019; Andolfatto, 2021; Chiu et al., 2023; Keister and Sanches, 2022; Whited et al., 2023; Niepelt, 2023; Paul et al., 2024). In contrast, our focus is on financial stability.

Several other papers connect CBDC to financial stability. Using a Diamond and Dybvig (1983) model, Fernández-Villaverde et al. (2021) and Schilling et al. (2024) study the implications for bank runs. They show that the introduction of CBDC completely removes the risk of bank runs by fostering a flow of deposits from the banking system into the central bank (also featured in Skeie, 2020), but create a trade-off between efficiency and price stability. Keister and Monnet (2022) also consider the implications of CBDC for bank runs, but focus on the efficacy of government interventions. In their framework, CBDC allows the central bank to have more accurate information about the health of the banking sector and thus to intervene promptly to mitigate the risk of a run. In Williamson (2022), the fragility of banks induced by the introduction of CBDC is ex-ante efficient.

A key difference relative to these papers is our use of global-games methods to uniquely pin down the probability of a bank run. This approach allows us to study how CBDC design affects bank fragility, both directly via withdrawal incentives and indirectly via the bank's response in deposit rates. Global games were introduced by Carlsson and van Damme (1993), and have been widely applied to study run-like behavior (e.g. Rochet and Vives, 2004; Goldstein and Pauzner, 2005; Vives, 2014; Bouvard et al., 2015; Liu, 2016; Ahnert, 2016; Eisenbach, 2017; Ahnert et al., 2019; Liu, 2023; Carletti et al., 2023;

Schilling, 2023).[3]

Our contribution to the standard global-games bank-run model is twofold. First, we allow for consumers' portfolio choice. In particular, they can choose between CBDC and bank deposits at both the funding and the withdrawal stage. Second, by moving away from the assumption of perfect competition in the banking sector, we tackle the technically challenging problem of characterizing the bank's optimal choice concerning the terms of the deposit contract (i.e., both the short- and long-term rates) and how this choice is affected by CBDC remuneration.

# 2 Model

We build on Goldstein and Pauzner (2005), but make two key modifications. First, the bank exerts market power in the deposit market. Second, a remunerated CBDC is available as an outside option to consumers.

The economy extends over three dates $t = 0, 1, 2$ and is populated by a risk-neutral bank and a unit mass of risk-averse consumers indexed by $i \in [0, 1]$. There is a single divisible good for consumption and investment and no time discounting.

Consumers are endowed with one unit of funds at $t = 0$ only. Their preference over consumption is represented by a utility function $u(c)$ with $u(0) = 0$, $u'(c) > 0 > u''(c)$, and coefficient of relative risk aversion $RRA > 1$ for any $c > 1$. While consumers are identical ex-ante, a fraction $\lambda \in (0, 1)$ is hit by a preference shock at date 1 and must consume immediately, as in Diamond and Dybvig (1983). We refer to them as early or impatient. The remaining $1 - \lambda$ depositors are called late or patient, because they are indifferent between consuming at dates 1 and 2.

At date 0, the bank maximizes expected profits by raising funds in exchange for a demandable deposit contract $\{r_1, r_2\}$, where $r_1 \geq 0$ is the promised repayment upon withdrawal at the interim date and $r_2 \geq 0$ is the promised repayment upon withdrawal at the final date.[4] Thus, the deposit contract provides liquidity to depositors whenever

[3] Morris and Shin (2003) and Vives (2005) survey the theory and applications of global games.

[4] Uninsured deposits refer to any short-term or demandable debt instrument, including uninsured retail deposits and insured deposits when deposit insurance is not credible (Bonfim and Santos, 2023). Three quarters of U.S. commercial bank funding are deposits, half of which are uninsured (Egan et al.,

$r_1 > 1$.[5] The bank invests these funds in a risky project. The project can be liquidated at par at date 1 and yields a return $R$ with probability $\theta$ and zero otherwise at date 2. The variable $\theta \sim U[0,1]$ represents the economic fundamental and $R > 2$ is a constant that reflects the return from lending.

At $t = 1$, depositors choose whether to withdraw their funds early based on their preference shock and an imperfect signal about the fundamental. Each depositor receives a noisy private signal

$$s_i = \theta + \epsilon_i, \tag{1}$$

with $\epsilon_i \sim U[-\epsilon, +\epsilon]$. In addition to being informative about the profitability of the bank's investment project, it also provides information about the signals (and withdrawal actions) of other depositors. As is standard in much of the global-games literature, we assume vanishing noise, $\epsilon \to 0$, to simplify the analysis.[6]

The bank satisfies interim withdrawals by liquidating investment. Let $n \in [0,1]$ be the fraction of consumers who withdraw at $t = 1$. Whenever the bank is unable to make the promised repayment, either in the event of a run at the interim date or due to insolvency at the final date, all depositors receive a pro-rata share of available resources.[7]

As alternative to bank deposits, consumers can store their wealth in CBDC at date 0 and date 1. A deep-pocketed central bank offers consumers deposits with a per-period gross return $\omega \geq 1$.[8] Accordingly, $\omega = 1$ is equivalent to a model without CBDC, where cash offers a nominal return of 1.

Our main interpretation is that $\omega$ represents the remuneration of CBDC. However, it can also capture other CBDC-related benefits relative to cash such as the reduced risk of theft or the additional convenience derived from digital payment means, such as the ability to settle e-commerce transactions.[9] Our focus on CBDC remuneration (and the

2017).

[5] Feasibility requires $r_1 < 1/\lambda$.

[6] The assumption of vanishing private noise ensures a unique equilibrium and simplifies the analysis of the bank's choice of deposit rates at the initial date. In a similar framework, Vives (2014) studies the properties of the multiple equilibria that arise when this assumption is relaxed.

[7] This pro-rata assumption is a small technical deviation from the sequential-service constraint assumed in Goldstein and Pauzner (2005). It improves tractability without affecting the main properties of the model.

[8] We abstract from both raising funds (e.g. via taxation) and an investment choice of the central bank at $t = 0$.

[9] In principle, $\omega$ could also reflect an improved outside option arising from the existence of alternative

convenience of CBDC more broadly) reflects the main argument in the academic and policy debate on the impact of CBDC on financial stability. Henceforth, we refer to $\omega$ as CBDC remuneration but have these several interpretations in mind.

Relative to an economy with only bank deposits and cash, the introduction of CBDC has two effects. First, it improves the outside option of consumers deciding at $t = 0$ whether to deposit funds with the bank from $u(1)$ to $\lambda u(\omega) + (1 - \lambda)u(\omega^2)$, where the latter term reflects the compound return on CBDC over two periods. Second, it pays interest $\omega$ on funds withdrawn from the bank at $t = 1$ and re-deposited in CBDC. Table 1 summarizes the timeline of the economy.

| $t = 0$ | $t = 1$ | $t = 2$ |
|---|---|---|
| **1.** CBDC design | **1.** Fundamental shock | **1.** Investment matures |
| **2.** Bank sets rates | **2.** Private signals | **2.** Consumption |
| **3.** Consumers deposit | **3.** Withdrawal choice | |

Table 1: Timeline

# 3 Equilibrium

We solve backwards for the equilibrium. For a given deposit contract $\{r_1, r_2\}$ and CBDC remuneration $\omega$, we first characterize the bank failure threshold $\theta^F(r_1, r_2, \omega)$, which we refer to as our measure of bank fragility. Then, we solve for the bank's choice of deposit contract $(r_1^*(\omega), r_2^*(\omega))$. Finally, we study how changes in CBDC remuneration affect overall bank fragility.

## 3.1 Withdrawal stage

To obtain a unique equilibrium in the depositor withdrawal subgame, we apply global-games methods and build on Goldstein and Pauzner (2005). Some preliminary steps are useful for the analysis of individual withdrawal decisions. First, we can limit attention to deposit contracts that satisfy

$$r_2 > \omega r_1. \tag{2}$$

stores of value such as treasury bonds or money market funds. However, these do not have the means-of-payment function of cash, bank deposits, and CBDC.

To see why this must be the case, consider the converse, i.e. suppose $r_2 \leq \omega r_1$. Then, a depositor certain that the bank does not fail (in the high state when investment pays $R$) would still prefer to withdraw at $t = 1$ and deposit in CBDC because this would yield a higher return than keeping funds in the bank until $t = 2$. Thus, bank runs would occur with probability 1 and the bank would make zero expected profits, which cannot be optimal.

Next, we derive the usual dominance bounds that yield ranges of the fundamental $\theta$ for which depositors have a dominant strategy. First, consider the withdrawal decision of a patient consumer under the assumption that no other patient consumer withdraws (i.e. the bank is liquid at $t = 1$). Withdrawing and re-depositing in CBDC yields a payoff of $\omega r_1$ at date 2, while not withdrawing yields $r_2$ with probability $\theta$ and zero otherwise. Accordingly, he is indifferent for $u(\omega r_1) = \theta u(r_2)$, so that withdrawing funds is a dominant strategy for $\theta < \underline{\theta}$, with

$$\underline{\theta} = \frac{u(\omega r_1)}{u(r_2)} \in (0, 1). \tag{3}$$

We refer to runs occurring in the range $[0, \underline{\theta}]$ as fundamental runs because they are solely driven by a low realization of the economic fundamental $\theta$. Second, to establish an upper dominance region, we follow Goldstein and Pauzner (2005) and assume that there exists a threshold $\overline{\theta}$ such that the liquidation value improves to $R$ for $\theta > \overline{\theta}$.[10] In this range, it is optimal for a depositor not to withdraw irrespective of the withdrawal decision of all other depositors. The range $[\overline{\theta}, 1]$ identifies the upper dominance region, where the bound $\overline{\theta}$ can be arbitrarily close to (but strictly below) 1. For simplicity, we assume $\overline{\theta} \to 1$.

In the intermediate range $(\underline{\theta}, \overline{\theta})$, a depositor's decision to withdraw depends on what she expects the other depositors to do. The bank failure threshold $\theta^F$ is characterized in the following proposition.

**Proposition 1.** ***Failure threshold.*** *There exists a unique threshold equilibrium according to which depositors withdraw early if and only if $\theta < \theta^F$. The cutoff $\theta^F$ depends on the terms of the deposit contract, as follows:*

[10] This assumption can be viewed as a liquidation value of the bank's project that depends on the fundamental $\theta$. Specifically, it is equal to 1 for $\theta \in [0, \overline{\theta})$ and to $R$ for $\theta \in [\overline{\theta}, 1]$. An endogenous upper dominance region arises in Kashyap et al. (2024), where depositors receive signals about the asset's interim liquidation value instead.

1. *When $r_1 \leq 1$, only fundamental runs occur, so $\theta^F = \underline{\theta}$.*

2. *When $r_1 > 1$, both fundamental and panic runs occur, so $\theta^F > \underline{\theta}$. The relevant threshold is $\theta^F = \theta^*$, where*

$$\theta^* = \frac{\int_{\lambda}^{\overline{n}} u\left(\omega r_1\right) dn + \int_{\overline{n}}^{1} u\left(\frac{\omega}{n}\right) dn}{\int_{\lambda}^{\widehat{n}} u\left(r_2\right) dn + \int_{\widehat{n}}^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right) dn}. \tag{4}$$

3. *In both cases, the failure threshold $\theta^F$ increases in $r_1$ and $\omega$ and decreases in $r_2$.*

*Proof.* See Appendix A, which also states the definitions of $\overline{n}$, $\widehat{n}$, and the cross-partial derivatives of the failure threshold. □

The bank failure threshold is equal to the probability of a bank run, which we thus use as our measure of (ex-ante) bank fragility. When $r_1 \leq 1$, the bank can always raise enough resources at date 1 to meet withdrawals even if all depositors were to request an early repayment. As a result, any depositor deciding to wait until date 2 is certain to receive the promised repayment $r_2$ with probability $\theta$ (i.e., when the investment pays $R$). Since the promised payment is below the liquidation value of the investment ($r_1 \leq 1$), there are no strategic complementarities and runs only occur for weak fundamentals, that is when $\theta \leq \underline{\theta}$.

By contrast, strategic complementarities in depositors' withdrawal decisions arise for $r_1 > 1$. Sufficiently large withdrawals can force the bank to liquidate the entire investment and leave it without any funds to repay late depositors. As a result, panic runs occur and the relevant run threshold is $\theta^*$, with $\theta^* > \underline{\theta}$.

As shown in the Appendix, the comparative statics of $\theta^F$ with respect to CBDC remuneration $\omega$ and the deposit contract terms ($r_1$ and $r_2$) are qualitatively the same across these two cases despite the different nature of bank runs. First, a higher CBDC remuneration increases a patient depositor's expected payoff from withdrawing at the interim date, which raises bank fragility. This is the direct effect of CBDC remuneration and it always increases bank fragility, irrespective of whether runs are panic- or fundamental-driven. Second, a higher short-term deposit rate $r_1$ also raises bank fragility, as in Goldstein and Pauzner (2005). Finally, a higher long-term deposit rate $r_2$ reduces bank fragility, as it gives patient consumers better incentives not to withdraw at the interim date.

However, CBDC remuneration does not only affect bank fragility directly via the failure threshold $\theta^F$, but also indirectly by inducing the bank to change the deposit contract terms. Using total differentiation, we can express the overall effect of CBDC remuneration $\omega$ on bank fragility $\theta^F$ as the sum of the direct effect and two indirect effects,

$$\frac{d\theta^F}{d\omega} = \frac{\partial\theta^F}{\partial\omega} + \frac{\partial\theta^F}{\partial r_1}\frac{dr_1^*}{d\omega} + \frac{\partial\theta^F}{\partial r_2}\frac{dr_2^*}{d\omega}. \tag{5}$$

Hence, in order to assess the total effect, we need to characterize the equilibrium deposit contract $\{r_1^*, r_2^*\}$ and how it changes with CBDC remuneration $\omega$, which we turn to next.

## 3.2 The bank's choice of deposit rates

At $t = 0$, the bank chooses the deposit contract $\{r_1^*, r_2^*\}$ which maximizes its expected profits subject to consumers' participation constraint. Crucially, it does so by anticipating the withdrawal decisions of depositors at $t = 1$. Using the run threshold $\theta^F$ given in Proposition 1, the bank's problem can be compactly expressed as[11]

$$\max_{r_1,r_2} \Pi = \int_0^{\theta^F} \theta \cdot \max\left\{R\left(1-r_1\right), 0\right\} d\theta + \int_{\theta^F}^1 \theta\left[R\left(1-\lambda r_1\right) - \left(1-\lambda\right) r_2\right] d\theta \tag{6}$$

subject to

$$\int_0^{\theta^F} \left[\lambda \cdot u\left(\min\left\{1, r_1\right\}\right) + (1-\lambda) u\left(\omega \cdot \min\left\{1, r_1\right\}\right)\right] d\theta + \\ + \int_{\theta^F}^1 \left[\lambda u\left(r_1\right) + (1-\lambda)\theta \cdot u\left(r_2\right)\right] d\theta \geq U, \tag{7}$$

Expected bank profits $\Pi$ in (6) depend on whether a run occurs, as well as on whether it is fundamental- or panic-based. The two terms in (6) capture the instances when the bank expects to accrue positive profits at $t = 2$ upon the success of the project (i.e. with probability $\theta$). The first term represents the case in which a run occurs for $\theta \leq \theta^F$ and the bank still makes positive profits on the residual after having repaid all depositors, $(1 - r_1)$. This is possible only when $r_1 \leq 1$, so $\theta^F = \underline{\theta}$ in this case. The second term captures the situation of no run (for $\theta > \theta^F$). In this case, profits are equal

[11] In Appendix B, we state the problem separately for fundamental runs only ($r_1 \leq 1$) and for both fundamental and panic runs ($r_1 > 1$). The expressions in Appendix B are less compact but useful to build intuition.

to the return $R$ accrued on the $1 - \lambda r_1$ non-liquidated units net of repayment $r_2$ to all $(1 - \lambda)$ late depositors at $t = 2$.

Inequality (7) represents consumer's participation constraint. It requires the expected utility from a bank deposit on the left-hand side, which we label $EUD$, to weakly exceed consumers' outside option $U$, which is the utility derived from a CBDC deposit

$$U \equiv \lambda u(\omega) + (1 - \lambda)u(\omega^2). \tag{8}$$

By depositing in a bank, consumers expect to receive the minimum between the pro-rata share 1 and the promised repayment $r_1$ upon a run at $t = 1$ (when $\theta \leq \theta^F$). As shown in the first term of (7), early depositors consume it, while late types invest in CBDC between 1 and 2, thus accruing $\omega \cdot \min(1, r_1)$. In the case of no run, early depositors receive $r_1$, while late ones obtain $r_2$ with probability $\theta$.

The bank's problem is subject to two more constraints. First, the incentive compatibility constraint in (2) ensures that a bank run does not occur with probability 1, in which case the bank cannot make any positive expected profits (as already discussed above). Second, $\Pi \geq 0$ is the participation constraint of the bank, which ensures that it at least breaks even. We assume throughout that $R$ is high enough, so we can ignore this constraint going forward.

We are ready for our first result on the equilibrium deposit rates. Since the bank is able to exert market power, it sets deposit rates $r_1$ and $r_2$ that leave no rents to consumers.

**Lemma 1.** *In any equilibrium, consumers' participation constraint in* (7) *binds.*

*Proof.* See Appendix B. □

Next, we start to characterize the bank's choice of the deposit contract. As illustrated in Figure 1, the optimal deposit contract set by the bank $(r_1^*, r_2^*)$ is given by the tangency point between the highest iso-profit curve and the depositors' iso-utility curve corresponding to the binding participation constraint in (7).

The following result characterizes when the equilibrium features only fundamental runs, that is panic runs are absent.

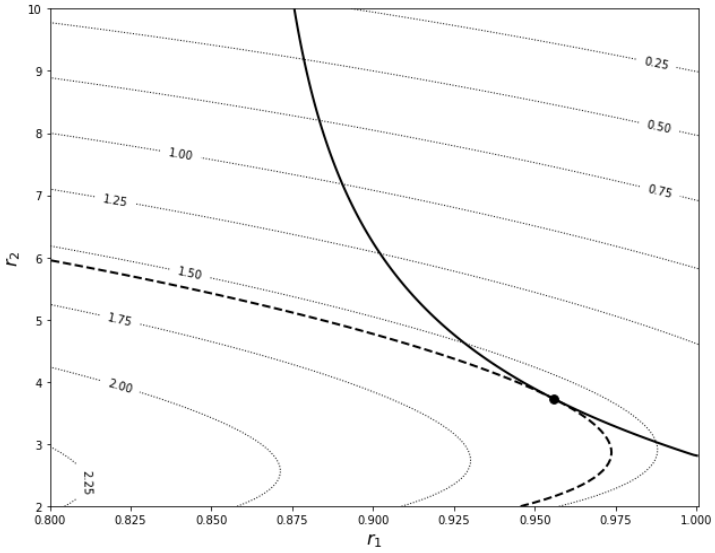


Figure 1: Equilibrium as the tangency point between the highest iso-profit curve (dashed) and consumers' iso-utility curve corresponding to a binding participation constraint (solid). The graph uses $u(c) = \frac{(c+f)^{1-\sigma}-f^{1-\sigma}}{1-\sigma}$ and parameter values $R = 10$, $\sigma = 4$, $f = 3$, $\lambda = 0.2$ and $\omega = 1.1$.

**Lemma 2.** *For sufficiently low $\omega$, the equilibrium features $r_1^* \leq 1$.*

*Proof.* See Appendix C. □

If CBDC remuneration is sufficiently low, the bank finds it optimal to offer an early repayment $r_1^* < 1$ and thus provides no liquidity insurance to risk-averse depositors exposed to liquidity shocks. This result contrasts with some canonical frameworks, such as Goldstein and Pauzner (2005). The difference arises from the difference in the bank's objective function. The bank maximizes its expected profits in our setup, rather than depositors' expected utility.[12]

We next turn to studying how the equilibrium deposit rates $r_1^*$ and $r_2^*$ vary with the level of CBDC remuneration. Going forward, we maintain the following assumption, which guarantees that an increase in CBDC remuneration $\omega$ reduces the attractiveness of bank deposits, ceteris paribus. This is the economically meaningful case in which the bank is forced to improve the deposit contract to retain deposits after an increase in $\omega$.

**Assumption 1.** $\lambda > \underline{\lambda}_0$.

[12] In Section 4 we allow the bank's market power to vary. In particular, we examine the case of perfect competition and show that $r_1^* > 1$ always emerges.

Under Assumption 1, an increase in CBDC remuneration $\omega$ tightens the participation constraint in (7). Moreover, all else equal, it also reduces bank profits by making runs more likely via increased withdrawal incentives. This implies that an increase in CBDC remuneration requires the bank to increase the attractiveness of the deposit contract by raising at least one of the two interest rates. The following Proposition characterizes the bank's adjustment.

**Proposition 2.** *Higher CBDC remuneration $\omega$ always leads to an increase in $r_2^*$. The impact on $r_1^*$ is as follows:*

*(i) When only fundamental runs are possible, the short-term deposit rate $r_1^*$ is non-decreasing in $\omega$;*

*(ii) When panic runs are possible, $r_1^*$ is non-decreasing in a right-neighborhood of $r_1^* = 1$, and non-increasing for all $\omega \geq \widetilde{\omega}$.*

*Proof.* See Appendix D, which also defines the bound $\widetilde{\omega}$. □

Higher deposit rates are costly to the bank because they reduce profits either directly through a higher cost of funds, or indirectly through heightened exposure to bank runs. The bank must therefore determine how to optimally allocate improvements in the deposit contract in response to a higher CBDC rate across both interest rates. Recall from Proposition 1 that $r_1$ and $r_2$ have opposing effects on the failure threshold. Proposition 2 shows that it always raises the long-term rate $r_2$, while its response via the short-term rate $r_1$ is more complex.

When CBDC remuneration $\omega$ is relatively low, panic runs are absent (i.e. $r_1^* < 1$, see Lemma 2) and the bank typically responds to higher CBDC remuneration by raising both deposit rates by roughly the same magnitude. While a higher long-term deposit rate $r_2^*$ unambiguously benefits depositors (through a lower run probability and higher utility), it nevertheless reduces bank profits when no run occurs. Since risk-averse depositors' prefer smooth consumption and fundamental runs are not very costly to the bank, it can contain the increase in the long-term rate $r_2^*$ required to maintain depositor participation by also raising the short-term rate $r_1^*$.

Things change, however, as $\omega$ grows large and the short-term rate enters the region

where panic runs can occur ($r_1^* > 1$). In particular, when a cutoff $\widetilde{\omega}$ is reached, additional increases in $r_1^*$ *reduce* depositors' expected utility through their detrimental effect on bank fragility and thus no longer serve the bank to retain deposit funding. Hence, from this point onward, the bank responds to higher CBDC remuneration by only increasing $r_2^*$.

Table 2 provides a numerical example that shows how both interest rates increase in lockstep for low values of $\omega$, while they diverge once $r_1$ rises above one as $\omega$ grows large. Moreover, the table highlights the role of risk aversion (driven by the parameter $\sigma$) and depositors' exposure to liquidity shocks $\lambda$ in shaping the bank's decision through the participation constraint. In particular, we see that increases in either parameter induce the bank to promise a higher early repayment $r_1^*$ across all values of $\omega$ considered. This is in line with standard intuition, because $r_1$ is a measure of the liquidity insurance offered by the bank to depositors. Higher risk-aversion and higher exposure to the liquidity shock make this insurance more valuable. Hence, an increase in $r_1$ has a large impact on depositors' expected utility, which allows the bank to economize on the date 2 repayment $r_2$ and thereby earn higher profits.

| | Baseline model ($\sigma = 4$; $\lambda = 0.2$ ) | | Higher risk aversion ($\sigma = 4.05$; $\lambda = 0.2$) | | More early consumers ($\sigma = 4$; $\lambda = 0.21$) | |
|---|---|---|---|---|---|---|
| $\omega$ | $r_1^*$ | $r_2^*$ | $r_1^*$ | $r_2^*$ | $r_1^*$ | $r_2^*$ |
| 1.000 | 0.8189271596 | 3.245083 | 0.8235798722 | 3.238934 | 0.8202671536 | 3.248900 |
| 1.025 | 0.8539170639 | 3.366287 | 0.8584411856 | 3.359390 | 0.8551788729 | 3.369810 |
| 1.050 | 0.8883676933 | 3.487871 | 0.8927620130 | 3.480213 | 0.8895565892 | 3.491121 |
| 1.075 | 0.9223008278 | 3.609850 | 0.9265649925 | 3.601423 | 0.9234218468 | 3.612847 |
| 1.100 | 0.9557393136 | 3.732249 | 0.9598737069 | 3.723045 | 0.9567972095 | 3.735009 |
| 1.125 | 0.9887065200 | 3.855095 | 0.9927121413 | 3.845111 | 0.9897057429 | 3.857635 |
| 1.150 | 1.0000000017 | 4.782325 | 1.0000000044 | 5.009943 | 1.0000000073 | 4.838733 |
| 1.175 | 1.0066677258 | 7.775006 | 1.0117389040 | 9.017712 | 1.0081857737 | 7.993763 |

Table 2: Equilibrium interest rates for different values of CBDC remuneration $\omega$. The numerical example uses the functional form $u(c) = \frac{(c+f)^{1-\sigma}}{1-\sigma} - \frac{f^{1-\sigma}}{1-\sigma}$ and parameter values $R = 10$ and $f = 3$. The parameter values for $\sigma$ and $\lambda$ are given in the column headings.

## 3.3 The total effect of CBDC remuneration on bank fragility

Having characterized the bank's choice of deposit rates and their reaction to a change in CBDC remuneration, we are now ready to describe its total effect on bank fragility.

Specifically, for $\omega < \widetilde{\omega}$, we have that

$$\frac{d\theta^F}{d\omega} = \underbrace{\frac{\partial \theta^F}{\partial \omega}}_{>0} + \underbrace{\frac{\partial \theta^F}{\partial r_1}\frac{dr_1^*}{d\omega}}_{\geq 0} + \underbrace{\frac{\partial \theta^F}{\partial r_2}\frac{dr_2^*}{d\omega}}_{<0}. \tag{9}$$

The above expression highlights that two indirect effects, with opposite signs, add to the direct effect. The next proposition contains our main result, which is the existence of a parameter space in which an increase in CBDC remuneration has a beneficial impact on financial stability.

**Proposition 3.** *There exists* $\widetilde{\lambda} < 1$ *such that for all* $\lambda > \widetilde{\lambda}$ *and* $\omega \geq \widetilde{\omega}$, $\frac{d\theta^F}{d\omega} < 0$.

*Proof.* See Appendix E, which also characterizes $\widetilde{\lambda}$. □

The above result shows that if both CBDC remuneration and the share of impatient consumers are sufficiently high, further increases in $\omega$ will cause a decrease in bank fragility. That is, the indirect effect through a higher long-term deposit rate $r_2$ is sufficiently strong to overturn the direct effect.

Table 3 tabulates equilibrium bank fragility for our numerical examples. Importantly, it shows that our result holds more broadly that suggested by the sufficient conditions in Proposition 3. In particular, recall from Table 2 that $r_1^*$ is increasing throughout all tabulated values of $\omega$, which implies (via Proposition 2) that the condition $\omega \geq \widetilde{\omega}$ is not satisfied. Moreover, note that the chosen parameter for $\lambda$ is rather low.

Table 3 also provides further evidence that the beneficial effect of CBDC on financial stability is robust to changes in the level of risk aversion and in the share of early consumers. As higher $\sigma$ and higher $\lambda$ induce the bank to increase $r_1^*$ (recall Table 2), across all values of $\omega$, $r_1^* > 1$ and panic runs emerge for lower levels of $\omega$ than in the baseline parametrization. Figure 2 also shows the non-monotonic relationship between CBDC remuneration and bank fragility.

In line with Equation (9), the numerical example also highlights that the beneficial effect of CBDC on stability emerges despite an indirect effect via $r_1^*$ that amplifies the direct one. In particular, a higher CBDC remuneration increases the short-term deposit rate $r_1^*$, which increases bank fragility. However, the bank responds by also increasing

| $\omega$ | Baseline model ($\sigma = 4$; $\lambda = 0.2$ ) $\theta^F$ | Higher risk aversion ($\sigma = 4.05$; $\lambda = 0.2$) $\theta^F$ | More early consumers ($\sigma = 4$; $\lambda = 0.21$) $\theta^F$ |
|---|---|---|---|
| 1.000 | 0.579461 | 0.585581 | 0.579902 |
| 1.025 | 0.598713 | 0.604781 | 0.599115 |
| 1.050 | 0.617138 | 0.623147 | 0.617505 |
| 1.075 | 0.634763 | 0.640705 | 0.635098 |
| 1.100 | 0.651613 | 0.657482 | 0.651919 |
| 1.125 | 0.667715 | 0.673505 | 0.667995 |
| 1.150 | 0.660047 | 0.661400 | 0.659189 |
| 1.175 | 0.651389 | 0.659689 | 0.652573 |

Table 3: Equilibrium bank fragility for different values of CBDC remuneration $\omega$. The numerical example uses the functional form $u(c) = \frac{(c+f)^{1-\sigma}}{1-\sigma} - \frac{f^{1-\sigma}}{1-\sigma}$ and parameter values $R = 10$ and $f = 3$. The parameter values for $\sigma$ and $\lambda$ are given in the column headings.

the long-term deposit rate $r_2^*$ in response to successive increases in CBDC remuneration, which reduces bank fragility. This second indirect effect is quantitatively large, so it dominates the other two effects, resulting in an eventual reduction in bank fragility and an overall non-monotonic relationship between CBDC remuneration and bank fragility.

Figure 3 decomposes the total effect of CBDC remuneration on bank fragility into its parts in order to illustrate how the magnitude of the direct and the two indirect effects change with CBDC remuneration $\omega$. Each line represents the counterfactual run threshold $\theta^*(\omega, r_1^*(\omega), r_2^*(\omega))$ that arises when we allow $\omega$ to only affect one specific channel only (direct, indirect via $r_1^*$, and indirect via $r_2^*$). Hence, the slope of each line can be interpreted as a measure of the magnitude of that particular channel. We note the following observations. First, the dashed line represents the indirect effect of $\omega$ through the change in the long-term repayment $r_2^*$, which is the only decreasing line. Second, the other two lines are steeper for small values of $\omega$, but flatter for large values of $\omega$. This suggests that the detrimental effects of a change in $\omega$ on bank fragility tend to be stronger than the beneficial one until the bank enters into the panic run region. We discuss in detail below how panic runs constitute an essential ingredient for the beneficial effect of higher CBDC remuneration.

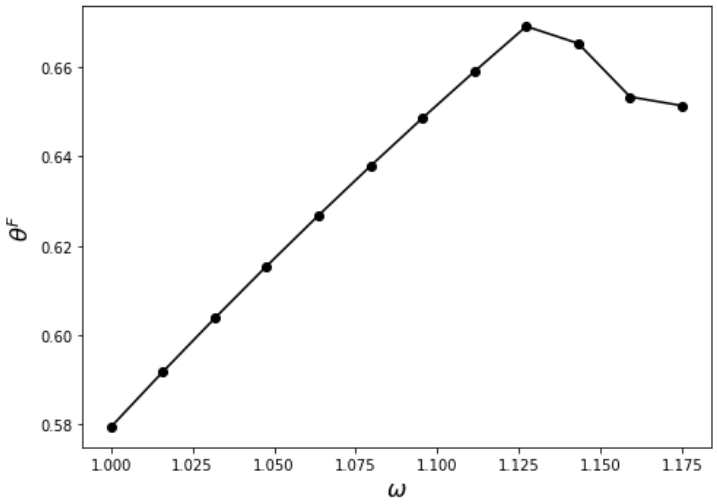


Figure 2: Equilibrium run threshold $\theta^F$ as a function of CBDC remuneration $\omega$. The numerical example assumes the utility function $u(c) = \frac{(c+f)^{1-\sigma}}{1-\sigma} - \frac{f^{1-\sigma}}{1-\sigma}$ and parameter values: $R = 10$, $\sigma = 4$, $f = 3$ and $\lambda = 0.2$. The curve is drawn for values of $\omega$ in the grid used in Table 2.

## 3.4 Panic runs are essential

To describe the economic forces underlying our main result, we argue that the beneficial impact on bank fragility does not occur when runs are only fundamental-driven. This result held up in all numerical examples, so we believe that it is universally true. However, it has been challenging to prove a general result analytically, so we obtain the following Proposition subject to a sufficient condition.

**Proposition 4.** *In the absence of panic runs, $\theta^F = \underline{\theta}$, an increase in CBDC remuneration increases bank fragility, $\frac{d\underline{\theta}}{d\omega} > 0$, for $\lambda \in \left(\underset{\sim}{\lambda}_0, \underset{\sim}{\lambda}_2\right)$.*

*Proof.* See Appendix F. □

Panic runs are an essential ingredient for the main result that an increase in CBDC remuneration can improve financial stability. Once the bank is induced to offer short-term deposit rate that gives rise to panic runs, the optimal breakdown across both repayments changes significantly. In this region, further increases in $r_1^*$ are harmful to the bank and only partially effective in increasing the attractiveness of deposits. This reflects the high sensitivity of the panic run threshold to changes in the short-term deposit rate. As a result, the bank tends to respond to an increase in CBDC remuneration by increasing the long-term repayment $r_2^*$ relatively more than the short-term repayment $r_1^*$. This

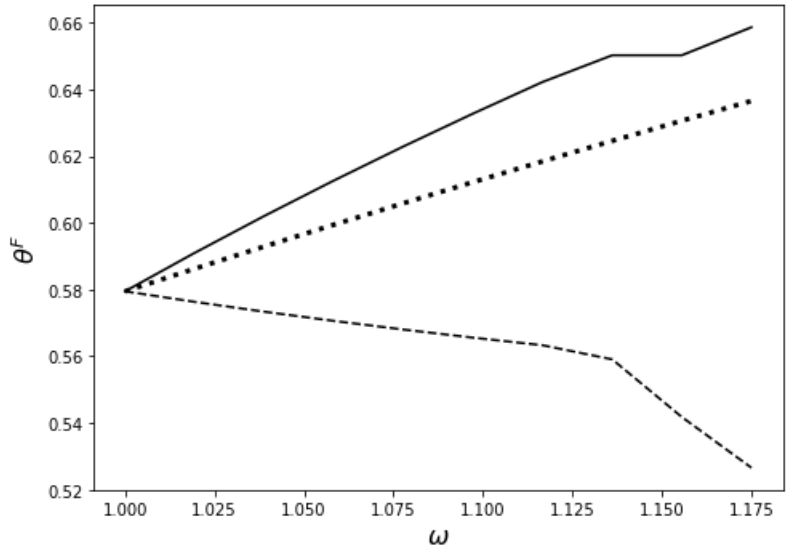


Figure 3: Decomposing the total effect of CBDC remuneration on bank fragility. The figure highlights the size of the three effects from an increase in CBDC remuneration. Specifically, the dotted line isolates the direct effect of CBDC remuneration on the run threshold, as it corresponds to $\theta^F(\omega, r_1^*(1), r_2^*(1))$, where deposit rates $r_1^*(1)$ and $r_2^*(1)$ are held constant at the optimal level for the initial level of $\omega = 1$. The dashed line, instead, captures the indirect effect via the long-term deposit rate $r_2^*$, as it corresponds to $\theta^F(1, r_1^*(1), r_2^*(\omega))$, with $r_2^*(\omega)$ being the equilibrium values of the long-term deposit rate in Table 2. Symmetrically, the solid line isolates the indirect effect via the short-term deposit rate $r_1^*$, as it corresponds to $\theta^F(1, r_1^*(\omega), r_2^*(1))$. Parameters are as in Figure 2.

translates into a stronger beneficial indirect effect (i.e., larger $\frac{\partial \theta^*}{\partial r_2}\frac{dr_2^*}{d\omega}$) and in a weaker detrimental indirect effect (i.e., smaller $\frac{\partial \theta^*}{\partial r_1}\frac{dr_1^*}{d\omega}$). As a result, as illustrated in Table 2 and Figure 3, the beneficial indirect effect can become sufficiently strong to offset the other two effects and drives an overall reduction in bank fragility upon an increase in CBDC remuneration.

# 4 Limited market power in the deposit market

So far, we have considered a bank that acts as a monopolist in the deposit market. In this subsection, we relax this assumption.[13] This approach is partly motivated by theoretical work on the effects of CBDC on bank credit supply, which reaches different conclusions depending on the level of competition in deposit markets (Keister and Sanches, 2022; Andolfatto, 2021; Chiu et al., 2023).

[13] A large literature documents imperfect competition in retail deposit markets, including Neumark and Sharpe (1992), Hannan and Berger (1997), and Drechsler et al. (2017).

More specifically, we assume that the deposit contract is determined by Nash bargaining between the bank and depositors. This approach is attractive because it allows us to model the degree of deposit market competition by varying the bank's bargaining power $\beta \in (0,1)$. Formally, the deposit contract is the solution to

$$\max_{r_1,r_2} (\Pi)^{\beta} (EUD - U)^{1-\beta}, \tag{10}$$

where bank profits $\Pi$ are given in (6) and depositor surplus from the deposit contract in excess of their outside option, $EUD - U$, is given in (7). We recover the baseline model in which the bank maximizes expected profits subject to depositors' participation constraint for $\beta \to 1$. As in the main text, the short-term repayment determines the nature of bank runs: $\theta^F \equiv \underline{\theta}$ when $r_1 \leq 1$ and $\theta^F \equiv \theta^*$ otherwise.

## 4.1 Perfect competition

The above specification for $\beta \to 0$ corresponds to a model with perfect competition: the deposit contract $\{r_1^*, r_2^*\}$ maximizes depositors' expected utility subject to non-negative bank profits. In this circumstance, the problem is as in Goldstein and Pauzner (2005), with late depositors being residual claimants of the bank's available resources:

$$r_2^*(r_1) = R\frac{1-\lambda r_1}{1-\lambda}, \tag{11}$$

and the bank making zero realized profits (and thus zero expected profits).

The following proposition summarizes the resulting implications of this polar case for the effect of CBDC remuneration on bank fragility.

**Proposition 5.** ***Perfect competition in the deposit market.*** *Consider $\beta \to 0$ and assume that $\sigma < \widehat{\sigma}$. Then, $r_1^* > 1$ and $\frac{\partial r_1^*}{\partial \omega} < 0$.*

*Proof.* See Appendix G, which also defines the cutoff $\widehat{\sigma}$. □

As in Goldstein and Pauzner (2005), the equilibrium deposit rate $r_1^*$ trades off the marginal cost associated with a higher run probability with the marginal benefit

due to better risk-sharing. This leads to $r_1^* > 1$ (liquidity provision) and, therefore, strategic complementarities in depositors' withdrawal decisions and panic runs. The proposition also states that the equilibrium deposit rate $r_1^*$ decreases in the level of CBDC remuneration $\omega$ when the level of risk aversion is sufficiently small. Because of the above mentioned negative relationship between interim and final deposit rates, the final deposit rate therefore increases in $\omega$, $\frac{dr_2^*}{d\omega} > 0$.

As in the baseline model, higher CBDC remuneration affects bank fragility both directly (via the failure threshold) and indirectly (via the deposit contract). The direct effect, captured by $\frac{\partial \theta^F}{\partial \omega}$ is positive, while the indirect effect, corresponding to $\frac{\partial \theta^F}{\partial r_1}\frac{dr_1^*}{d\omega}$, is negative. So, it remains to evaluate the total effect of higher CBDC remuneration on bank fragility.

A full analytical characterization of the general case is difficult, but further insights can be obtained from numerical examples. First, the numerical simulations we run show that (1) the result on the inverse relationship between the interim deposit rate $r_1^*$ and CBDC remuneration $\omega$ holds more broadly than the sufficient condition stated in the proposition; and (2) the direct effect always dominates under perfect competition.[14] Intuitively, deposit rates under perfect competition are already so high and little sensitive to changes in $\omega$ that the indirect effect is of second order, so that the overall effect of higher CBDC remuneration is to unambiguously increase bank fragility.

Furthermore, the numerical analysis also highlights the beneficial impact of the interim deposit rate adjustment on stability. Specifically, Figure 4 (left panel) shows that the interim deposit rate can decrease in CBDC remuneration and, accordingly, the final date deposit rate increases in CBDC remuneration.

In Figure 5, we decompose the total effect of $\omega$ on $\theta^*(r_1^*(\omega), \omega)$ into the direct and indirect effect to better illustrate their strength. The figure shows that if only the indirect effect were present, the run threshold would fall. This is captured by the decreasing solid line, which corresponds to the run threshold $\theta^*$ evaluated at $\omega = 1$ and $r_1^*(\omega)$. Conversely, if only the direct effect were present, the threshold $\theta^*$ would increase sharply, as captured by the increasing dashed line that corresponds to $\theta^*$ evaluated at $r_1^*(1)$ and $\omega$ varying

[14] An automated numerical search in the admissible parameter space for $\lambda$, $R$, $\sigma$ and $f$ found no combinations of parameters for which $\theta^*$ decreases in $\omega$ in equilibrium. The relevant Python code is included in the replication package.

between 1 and 1.18. Additionally, the figure highlights that the magnitude of the indirect effect (solid line) is smaller than that of the direct effect (dashed line), as indicated by the different slopes of the two curves.

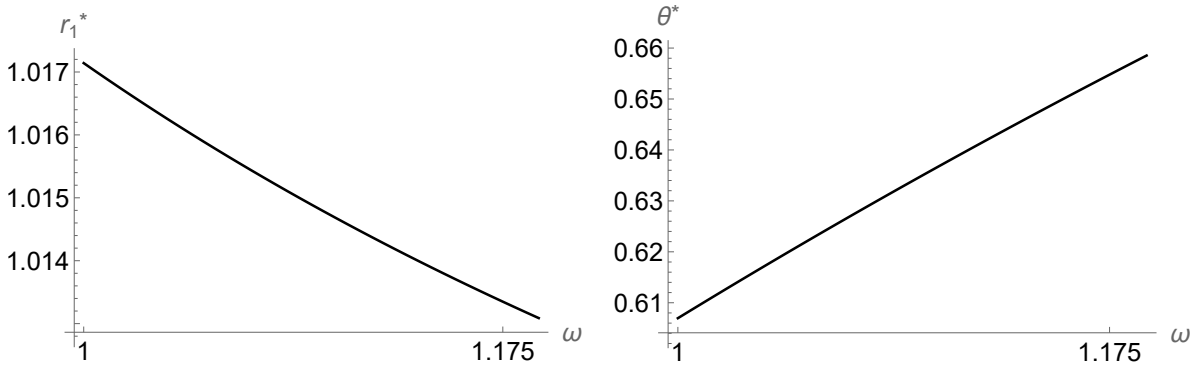


Figure 4: The case of perfect competition: the short-term deposit rate $r_1^*$ (left panel) and bank-run threshold $\theta^*$ (right panel) as a function of CBDC remuneration $\omega$. Parameters: $R = 10$, $\lambda = 0.2$, $u(c) = \frac{(c+f)^{1-\sigma}}{1-\sigma} - \frac{f^{1-\sigma}}{1-\sigma}$, $\sigma = 4$, $f = 3$. For these values, the cutoff is $\widehat{\sigma} \simeq 1.3$.

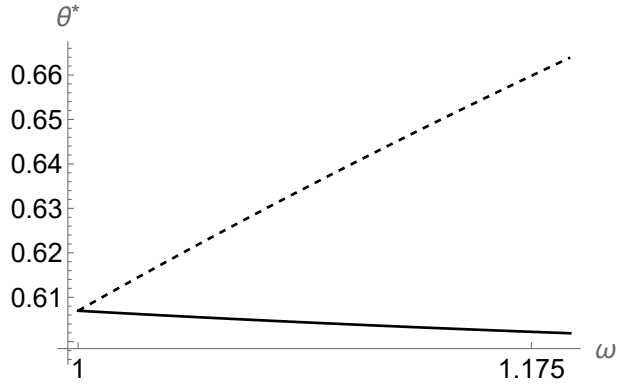


Figure 5: Decomposition of total effect of CBDC remuneration $\omega$ on bank fragility $\theta^*(r_1(\omega), \omega)$. The dashed line illustrates the direct effect as it corresponds to $\theta^*(r_1^*(1), \omega)$. The solid line shows the indirect effect as it corresponds to $\theta^*(1, r_1^*(\omega))$. The parameters are as in Figure 4.

## 4.2 Imperfect competition

We now turn to the intermediate case of limited market power, $0 < \beta < 1$. We obtain the following numerical result.

**Numerical Result 1.** ***Deposit rates with limited bank market power.*** *For a sufficiently high value of the bank's bargaining power $\beta$, the adjustment in the long-term deposit rate $r_2^*$ is strong enough such that bank fragility decreases in CBDC remuneration.*

| | $\beta = 1$ | | | $\beta = 0$ | | | $\beta = 0.97$ | | |
|---|---|---|---|---|---|---|---|---|---|
| $\omega$ | $r_1^*$ | $r_2^*$ | $\theta^F$ | $r_1^*$ | $r_2^*$ | $\theta^F$ | $r_1^*$ | $r_2^*$ | $\theta^F$ |
| 1.000 | 0.81892716 | 3.245083 | 0.579461 | 1.01714116 | 9.957147 | 0.606967 | 0.83593281 | 3.293709 | 0.585002 |
| 1.025 | 0.85391706 | 3.366287 | 0.598713 | 1.01649873 | 9.958753 | 0.614282 | 0.86931947 | 3.410645 | 0.603540 |
| 1.050 | 0.88836769 | 3.487871 | 0.617138 | 1.01589456 | 9.960264 | 0.621428 | 0.90223808 | 3.528111 | 0.621317 |
| 1.075 | 0.92230083 | 3.609850 | 0.634763 | 1.01532552 | 9.961686 | 0.628410 | 0.92303152 | 3.592432 | 0.635560 |
| 1.100 | 0.95573931 | 3.732249 | 0.651613 | 1.01478883 | 9.963028 | 0.635232 | 0.95573935 | 3.732249 | 0.651613 |
| 1.125 | 0.98870652 | 3.855095 | 0.667715 | 1.01428196 | 9.964295 | 0.641898 | 0.98899208 | 3.846887 | 0.668034 |
| 1.150 | 1.00000000 | 4.782325 | 0.660047 | 1.01380262 | 9.965493 | 0.648412 | 0.99999999 | 4.782327 | 0.660047 |
| 1.175 | 1.00666773 | 7.775006 | 0.651389 | 1.01334873 | 9.966628 | 0.654779 | 1.00704589 | 7.838390 | 0.651630 |

Table 4: Imperfect competition: equilibrium across different values of $\beta$.

Table 4 shows how deposit rates respond to increases in CBDC remuneration for different degrees of bank's bargaining power $\beta$. The main insight is that the beneficial effect of higher CBDC remuneration on bank fragility is preserved when the bank's bargaining power in the deposit market is sufficiently high, whereas a monotonically increasing relationship is obtained for more competitive deposit markets. This result highlights that sufficient bank market power is a necessary ingredient for our main result of a beneficial impact of CBDC remuneration on bank fragility.

Furthermore, a decline in the bank's bargaining power dampens the impact of CBDC remuneration on deposit rates. For high degrees of market power of the bank in the deposit market (high $\beta$), both the short-term and the long-term rates are fairly low and respond substantially to an increase in CBDC remuneration. Conversely, under perfect competition both rates start out from a much higher level and adjust relatively little in response to higher CBDC remuneration. In fact, the short-term deposit rate even decreases in the CBDC rate to dampen the impact on bank fragility.

# 5 Conclusion

We have examined the impact of CBDC remuneration on bank stability through the lens of a global-games model of financial intermediation and bank runs in which a remunerated CBDC provides consumers with an alternative to bank deposits (and cash). Consistent with widespread concerns, a higher CBDC remuneration raises bank fragility by increasing consumers' withdrawal incentives. However, it also induces the bank to offer more attractive deposit contracts in an effort to retain funding, which can lower fragility. In disentangling these two channels, we fully characterize the adjustment of the terms of

the deposit contract along two margins, the short- and the long-term deposit rate. This element, which hinges on the bank maximizing its profits rather than the consumers' expected utility consumers, constitutes a methodological contribution to the bank fragility literature.

We show that the occurrence of panic runs as well as sufficient bank market power vis-à-vis depositors are necessary to obtain a reduction in bank fragility in response to an increase in CBDC remuneration. Our results suggest that in contexts in which panic runs are not fully determined by the pricing of bank deposit contracts (e.g., in the presence of uncertain liquidation value or withdrawal demands), the introduction of a remunerated CBDC could reduce bank fragility.

Further extensions to our baseline model may generate additional insights on CBDC design. For example, one could consider the roles of bank equity and liquid reserves on the interaction between CBDC remuneration and financial stability. One could also examine how CBDC affects the bank's funding mix through the relative costs of retail and wholesale deposits, and the resulting implications for bank stability. Exploring general equilibrium effects via the response of asset prices and exchange rates after the introduction of CBDC may also be interesting. Finally, the role of CBDC design features (e.g., holding limits and contingent remuneration) as well as their interaction with traditional tools for mitigating run risk, such as lender of last resort policies, is an interesting avenue for further research.

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# A Proof of Proposition 1

We derive the failure thresholds and then state their partial and cross-partial derivatives.

## A.1 Derivation of the thresholds

The proof builds on Goldstein and Pauzner (2005), with the only difference that in our framework late depositors are promised a fixed repayment $r_2$ at the final date. Hence, the solvency condition at the final date, whereby the bank is able to repay the promised repayment, becomes relevant. The argument in the proof of Goldstein and Pauzner (2005) establish that there is a unique equilibrium in which depositors withdraw if and only if their private signal is below a common signal cutoff $s^*$. Since depositors' signals are independent and uniformly distributed over the interval $[\theta - \epsilon, \theta + \epsilon]$, the fraction of depositors withdrawing at $t = 1$, $n$, equals the probability of receiving a signal below $s^*$:

$$n(\theta, s^*) = \begin{cases} 1 & \text{if } \theta \leq s^* - \epsilon, \\ \lambda + (1-\lambda)\frac{s^* - \theta + \epsilon}{2\epsilon} & \text{if } s^* - \epsilon < \theta \leq s^* + \epsilon, \\ \lambda & \text{if } \theta > s^* + \epsilon. \end{cases} \tag{12}$$

Having characterized the proportion of agents withdrawing for any possible value of the fundamental $\theta$, we can compute the threshold signal $s^*$. To compute it, it is useful to first specify the payoff differential between waiting and withdrawing, $v(\theta, n)$, which equals:

$$v(\theta, n) = \begin{cases} \theta u(r_2) + (1-\theta)u(0) - u(\omega r_1) & \text{if } \lambda \leq n \leq \widehat{n}, \\ \theta u\left(R\frac{1-nr_1}{1-n}\right) + (1-\theta)u(0) - u(\omega r_1) & \text{if } \widehat{n} < n \leq \overline{n}, \\ \theta u(0) - u\left(\frac{\omega}{n}\right) & \text{if } \overline{n} < n \leq 1. \end{cases} \tag{13}$$

The cutoff $\overline{n}$ represents the proportion of early withdrawals at which the bank fully liquidates its assets at date 1 and it is equal to $\overline{n} = \frac{1}{r_1}$. We refer to it as the illiquidity threshold of the bank. Moreover, the threshold $\widehat{n}$ corresponds to the number of early withdrawals at which the bank is on the brink of insolvency at date 2. Formally, this corresponds to the solution to $R(1 - nr_1) - (1 - n)r_2 = 0$, so it is equal to $\widehat{n} = \frac{R - r_2}{Rr_1 - r_2}$. The inequality $\widehat{n} < \bar{n}$ follows directly from $r_1 > 1$.

Depositors' payoff differential $v(\theta, n)$ exhibits the same properties as the one in Goldstein and Pauzner (2005), including one-sided strategic complementarity. Specifically, $v(\theta, n)$ increases

in $\theta$, decreases in $n$ for $n < \overline{n}$ and increases in it for $n > \overline{n}$. It only crosses zero once, for $n < \overline{n}$, and stays negative for $n \geq \overline{n}$.

A patient depositor who receives the signal $s^*$ must be indifferent between withdrawing at date 1 and at date 2. The threshold $s^*$ is the solution to:

$$\int_{\lambda}^{\widehat{n}} \theta(n)\, u(r_2) dn + \int_{\widehat{n}}^{\overline{n}} \theta(n)\, u\left(R\frac{1-nr_1}{1-n}\right) dn = \int_{\lambda}^{\overline{n}} u(\omega r_1) dn + \int_{\overline{n}}^{1} u\left(\frac{\omega}{n}\right) dn. \tag{14}$$

where, from (12), $\theta(n) = s^* + \epsilon - 2\epsilon\frac{n-\lambda}{1-\lambda}$. Equation (14) follows from (13) and requires that a late depositor's expected utility from withdrawing at date 1 equals the expected payoff from waiting until date 2. In the limit of $\epsilon \to 0$, we have $\theta(n) \to s^*$ and we denote this limit as $\theta^*$. Solving (14) for $\theta^*$ gives the threshold stated in the proposition.

## A.2 Partial derivatives of the thresholds

**Case 1.** If $r_1 \leq 1$, then $\theta^F = \underline{\theta}$ and the partial derivatives are straightforward:

$$\frac{\partial \underline{\theta}}{\partial r_1} = \frac{\omega u'(\omega r_1)}{u(r_2)} > 0 \tag{15}$$

$$\frac{\partial \underline{\theta}}{\partial r_2} = -\frac{u(\omega r_1)u'(r_2)}{[u(r_2)]^2} = -\underline{\theta}\frac{u'(r_2)}{u(r_2)} < 0 \tag{16}$$

$$\frac{\partial \underline{\theta}}{\partial \omega} = \frac{r_1 u'(\omega r_1)}{u(r_2)} > 0 \tag{17}$$

As will be useful later, we compute the cross-partial derivatives as follows:

$$\frac{\partial^2 \underline{\theta}}{\partial r_1 \partial r_2} = -\frac{\omega u'(\omega r_1)u'(r_2)}{[u(r_2)]^2} = -\frac{u'(r_2)}{u(r_2)}\frac{\partial \underline{\theta}}{\partial r_1} < 0, \tag{18}$$

$$\frac{\partial^2 \underline{\theta}}{\partial r_1 \partial \omega} = \frac{u'(\omega r_1) + \omega r_1 u''(\omega r_1)}{u(r_2)}, \tag{19}$$

$$\frac{\partial^2 \underline{\theta}}{\partial r_2 \partial \omega} = -\frac{r_1 u'(\omega r_1)u'(r_2)}{[u(r_2)]^2} = -\frac{u'(r_2)}{u(r_2)}\frac{\partial \underline{\theta}}{\partial \omega} < 0. \tag{20}$$

**Case 2.** If $r_1 > 1$, then $\theta^F = \theta^*$ and its partial derivatives are:

$$\frac{\partial \theta^*}{\partial r_1} = \frac{\int_{\lambda}^{\overline{n}} u'(\omega r_1)\omega dn + \theta^* \int_{\widehat{n}}^{\overline{n}} u'\left(R\frac{1-nr_1}{1-n}\right)\left(R\frac{n}{1-n}\right) dn}{\int_{\lambda}^{\widehat{n}} u\left(r_2\right) dn + \int_{\widehat{n}}^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right) dn} > 0; \tag{21}$$

$$\frac{\partial \theta^*}{\partial r_2} = -\theta^* \frac{\int_{\lambda}^{\widehat{n}} u'(r_2) dn}{\int_{\lambda}^{\widehat{n}} u\left(r_2\right) dn + \int_{\widehat{n}}^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right) dn} < 0; \tag{22}$$

and

$$\frac{\partial\theta^*}{\partial\omega} = \frac{\int_{\lambda}^{\overline{n}} u'(\omega r_1) r_1 dn + \int_{\overline{n}}^{1} u'\left(\frac{\omega}{n}\right)\frac{1}{n} dn}{\int_{\lambda}^{\widehat{n}} u\left(r_2\right) dn + \int_{\widehat{n}}^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right) dn} > 0; \tag{23}$$

since the derivatives of the limits of the integrals cancel out and $u(0) = 0$. As above, we also compute some cross-partial derivatives:

$$\frac{\partial^2\theta^*}{\partial r_1 \partial r_2} = \frac{\partial^2\theta^*}{\partial r_2 \partial r_1} = \frac{u'(r_2)\theta^* \frac{\widehat{n}R}{Rr_1 - r_2} - \frac{\partial\theta^*}{\partial r_1}\int_{\lambda}^{\widehat{n}} u'\left(r_2\right) dn + \frac{\partial\theta^*}{\partial r_2}\int_{\widehat{n}}^{\overline{n}} u'\left(R\frac{1-nr_1}{1-n}\right)\left(\frac{n}{1-n}R\right) dn}{\int_{\lambda}^{\widehat{n}} u\left(r_2\right) dn + \int_{\widehat{n}}^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right) dn}, \tag{24}$$

which further simplifies to

$$\frac{\partial^2\theta^*}{\partial r_1 \partial r_2} = \frac{\partial^2\theta^*}{\partial r_2 \partial r_1} = \frac{\partial\theta^*}{\partial r_1}\frac{\partial\theta^*}{\partial r_2}\frac{1}{\theta^*} + \frac{u'(r_2)\theta^* \frac{\widehat{n}R}{Rr_1 - r_2} + \frac{\partial\theta^*}{\partial r_2}\int_{\widehat{n}}^{\overline{n}} u'\left(R\frac{1-nr_1}{1-n}\right)\left(\frac{n}{1-n}R\right) dn}{\int_{\lambda}^{\widehat{n}} u\left(r_2\right) dn + \int_{\widehat{n}}^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right) dn}. \tag{25}$$

Additionally, we can compute

$$\frac{\partial^2\theta^*}{\partial r_2 \partial\omega} = -\frac{\int_{\lambda}^{\widehat{n}} u'\left(r_2\right) dn}{\int_{\lambda}^{\widehat{n}} u\left(r_2\right) dn + \int_{\widehat{n}}^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right) dn}\frac{\partial\theta^*}{\partial\omega} < 0. \tag{26}$$

Finally,

$$\frac{\partial^2\theta^*}{\partial r_1 \partial\omega} = \frac{\int_{\lambda}^{\overline{n}} u'\left(\omega r_1\right) dn + \omega r_1 \int_{\lambda}^{\overline{n}} u''(\omega r_1) dn + \frac{\partial\theta^*}{\partial\omega}\int_{\widehat{n}}^{\overline{n}} u'\left(R\frac{1-nr_1}{1-n}\right)\left(\frac{n}{1-n}R\right) dn}{\int_{\lambda}^{\widehat{n}} u\left(r_2\right) dn + \int_{\widehat{n}}^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right) dn}, \tag{27}$$

where the third term of the numerator is positive and the sum of the first two terms is $\int_{\lambda}^{\overline{n}} u'\left(\omega r_1\right) dn + \omega r_1 \int_{\lambda}^{\overline{n}} u''(\omega r_1) dn = (\overline{n} - \lambda) u'\left(\omega r_1\right)\left[1 - RRA(\omega r_1)\right]$.

# B Proof of Lemma 1

**Payoffs.** To start, let us state the payoffs. For fundamental runs only, we have:

$$\Pi(\theta^F = \underline{\theta}) = \int_0^{\underline{\theta}} R\theta\big[(1 - r_1)\big] d\theta + \int_{\underline{\theta}}^{1} \theta\big[R(1 - \lambda r_1) - (1 - \lambda) r_2\big] d\theta, \tag{28}$$

$$PC(\theta^F = \underline{\theta}) = \int_0^{\underline{\theta}} \big[\lambda u(r_1) + (1 - \lambda) u(\omega r_1)\big] d\theta + \int_{\underline{\theta}}^{1} \big[\lambda u(r_1) + (1 - \lambda)\theta u(r_2)\big] d\theta - U, \tag{29}$$

where the participation constraint is the value of the deposit claim net of the outside option, $PC = EUD - U$, and $EUD$ is the LHS of (7).

For panic runs, we have

$$\Pi(\theta^F = \theta^*) = \int_{\theta^*}^{1} \theta\big[R(1-\lambda r_1) - (1-\lambda)r_2\big]d\theta, \tag{30}$$

$$PC(\theta^F = \theta^*) = \int_{0}^{\theta^*} \big[\lambda u(1) + (1-\lambda)u(\omega)\big]d\theta + \int_{\theta^*}^{1} \big[\lambda u(r_1) + (1-\lambda)\theta u(r_2)\big]d\theta - U. \tag{31}$$

**Constrained optimization.** Using a Kuhn-Tucker approach, let $\mu$ be the coefficient on the consumers' participation constraint. The Lagrangian is:

$$\mathcal{L} = \Pi + \mu\left(EUD - U\right).$$

Hence, the equilibrium pair $r_1^*, r_2^*$ and the Lagrange multiplier $\mu^*$ corresponds to the solution to the following system:

$$\frac{\partial \mathcal{L}}{\partial r_1} = \frac{\partial \Pi}{\partial r_1} + \mu^* \frac{\partial EUD}{\partial r_1} = 0; \tag{32}$$

$$\frac{\partial \mathcal{L}}{\partial r_2} = \frac{\partial \Pi}{\partial r_2} + \mu^* \frac{\partial EUD}{\partial r_2} = 0; \tag{33}$$

$$EUD - U \geq 0; \qquad \mu^*\left(EUD - U\right) = 0 \tag{34}$$

because of complementary slackness. Suppose that the PC is slack, $\mu = 0$. Re-arranging terms, we can re-write (32) as:

$$\frac{\partial \Pi}{\partial r_1} = -\frac{\partial \theta^*}{\partial r_1}\theta^*\left[R\left(1-\lambda r_1\right) - \left(1-\lambda\right)r_2\right] - \int_{\theta^*}^{1} \lambda R\theta d\theta < 0 \tag{35}$$

when $r_1 > 1$, and to

$$\begin{aligned}\frac{\partial \Pi}{\partial r_1} &= -\frac{\partial \underline{\theta}}{\partial r_1}\underline{\theta}\left[R\left(1-\lambda r_1\right) - \left(1-\lambda\right)r_2 - R\left(1-r_1\right)\right] - \int_0^{\underline{\theta}} R\theta d\theta - \int_{\underline{\theta}}^{1} \lambda R\theta d\theta \\ &= -\frac{\partial \underline{\theta}}{\partial r_1}\underline{\theta}\left(1-\lambda\right)\left[Rr_1 - r_2\right] - \int_0^{\underline{\theta}} R\theta d\theta - \int_{\underline{\theta}}^{1} \lambda R\theta d\theta < 0\end{aligned} \tag{36}$$

when $r_1 \leq 1$. To see that the expression in Equation (36) is negative, there are two independent approaches. First, and building on Allen and Gale (2000) (who study a similar technology but without investment risk), note that $\frac{r_2}{r_1} \leq R$ in any equilibrium. In words, the marginal rate of substitution between consumption at date 2 and consumption at date 1, as offered by the bank, is no larger than the marginal rate of transformation of the investment project when it succeeds. For the bank to be viable, it must offer a better trade-off than what an consumer can achieve in isolation. As a result, $Rr_1^* - r_2^* \geq 0$ in any equilibrium. Second, in any equilibrium, bank profits at date 2, $R\left(1-\lambda r_1\right) - \left(1-\lambda\right)r_2$, are larger than those accrued upon a run, $R\left(1-r_1\right)$.

If this were not the case, the bank would prefer a certain run on it, which always leads to asset liquidation, which is clearly worse than the outside option of consumers. Either argument leads us to conclude that expected bank profits strictly decrease in $r_1$.

Having established that the FOC for $r_1$ is always negative, the expression in (32) is negative. This implies that $r_1 = 0$. When $r_1 = 0$ and $\mu = 0$, it can be seen that $\theta^F = 0$ and $\frac{\partial \theta^F}{\partial r_2} = 0$ when evaluated at $r_1 = 0$. Hence, the expression for the second optimality condition in Equation (33) simplifies to $-\int_0^1 \theta(1-\lambda)d\theta < 0$. This, in turn, implies that the bank sets $r_2 = 0$. Evaluating $EUD$ at $r_1 = 0$ and $r_2 = 0$ gives $EUD = 0 < U$, violating the participation constraint of consumers. This contradiction implies that $\mu^* > 0$ must hold in any equilibrium. From Condition (34), this implies that $EUD = U$, so consumers' participation constraint binds.

# C Proof of Lemma 2

We construct the tangency argument of the main text and then prove the lemma. Using the result of Lemma 1, the bank's optimal choice of deposit rates is given by the triplet $\{r_1^*, r_2^*, \mu^*\}$ that solves Conditions (32) - (34) with a binding constraint. Solving for $\mu^*$ from (32) and substituting it into (33) yields

$$\frac{\frac{\partial \Pi}{\partial r_2}}{\frac{\partial \Pi}{\partial r_1}} = \frac{\frac{\partial EUD}{\partial r_2}}{\frac{\partial EUD}{\partial r_1}}, \tag{37}$$

where the LHS is equal to the inverse of the slope of the iso-profit curve, the RHS is that of the iso-utility curve. Lemma 1 implies that the iso-utility curve in question is the one for which depositing into a bank yields exactly the reservation utility $U$.

It remains to show that $r_1^* \leq 1$ for $\omega$ near 1. Consider $\omega = 1$. We observe that the participation constraint in (7) when evaluated at $\omega = 1 = r_1$ is slack and equal to:

$$(1-\lambda)\int_{\underline{\theta}}^1 \left[\theta u\left(r_2\right) - u(1)\right] d\theta > 0,$$

because $u(r_2)\,\underline{\theta}|_{r_1=1=\omega} = u(1)$ and, thus, $\theta u(r_2) > u(1)$ for $\theta > \underline{\theta}$. In fact, the participation constraint evaluated at $\omega = 1 = r_1$ can be further simplified to $(1-\lambda)\frac{(1-\underline{\theta})^2}{2\underline{\theta}}u(1) > 0$.

From Condition (36) we know that expected bank profits decrease in $r_1$ when evaluated at $r_1 = 1$. Taken together, we have that at $r_1 = 1$, bank's profits decrease in $r_1$ and consumers' participation constraint is slack, so the bank finds it optimal to set $r_1^* < 1$ when $\omega = 1$. The result stated in the proposition follows from continuity.

# D Proof of Proposition 2

The proof consists of four steps. First, we show that the profit function is supermodular in $r_1$ and $r_2$. Second, we show that an increase in $\omega$ makes the participation constraint more binding, so at least one deposit rate must increase. Third, we argue that $r_2^*$ has to increase strictly. Finally, we show that, for large values of $\omega$, $r_1^*$ is non-increasing in $\omega$. To ease the exposition, we separately consider fundamental runs only and panic runs.

**Fundamental runs only.** To prove supermodularity, we need to show that $\frac{\partial^2 \Pi}{\partial r_1 \partial r_2} > 0$. Differentiating (36) with respect to $r_2$, we obtain:

$$\begin{aligned}\frac{\partial^2 \Pi}{\partial r_1 \partial r_2} &= -\frac{\partial^2 \underline{\theta}}{\partial r_1 \partial r_2}\underline{\theta}(1-\lambda)(Rr_1 - r_2) - \frac{\partial \underline{\theta}}{\partial r_1}\frac{\partial \underline{\theta}}{\partial r_2}(1-\lambda)(Rr_1 - r_2) + \frac{\partial \underline{\theta}}{\partial r_1}\underline{\theta}(1-\lambda) - \frac{\partial \underline{\theta}}{\partial r_2}R\underline{\theta}(1-\lambda) \\ &= (1-\lambda)\underline{\theta}\left[\frac{\partial \underline{\theta}}{\partial r_1} - R\frac{\partial \underline{\theta}}{\partial r_2}\right] + 2(1-\lambda)\frac{\partial \underline{\theta}}{\partial r_1}\underline{\theta}\frac{u'(r_2)}{u(r_2)} > 0,\end{aligned}$$

using $\frac{\partial^2 \underline{\theta}}{\partial r_1 \partial r_2} = -\frac{u'(r_2)}{u(r_2)}\frac{\partial \underline{\theta}}{\partial r_1}$ and $\frac{\partial \underline{\theta}}{\partial r_2} = -\frac{u'(r_2)}{u(r_2)}\underline{\theta}$, the first two terms (on the first line) sum to zero.

Next, we study how the participation constraint changes with the terms of the deposit contract $\{r_1, r_2\}$ and CBDC remuneration $\omega$. Intuitively, higher deposit rates increase the value of the deposit claim $EUD$:

$$\begin{aligned}EUD_{r_1} &= \lambda\, u'(r_1) \;+\; (1-\lambda)\,\omega\,\underline{\theta}\,u'(\omega r_1) > 0, \\ EUD_{r_2} &= (1-\lambda)\frac{1-\underline{\theta}^2}{2}u'(r_2) > 0.\end{aligned}$$

Note that, at $\lambda = 1$, $EUD_{r_1} > EUD_{r_2}$, so by continuity this ranking also holds for large enough $\lambda$. In words, to ensure the participation of consumers, a smaller increase in $r_1$, relative to $r_2$, is required to offset an increase in $\omega$. Moving to the effect of $\omega$, we use the following definition, $PC \equiv EUD - U$. Then, we have that:

$$PC_\omega = (1-\lambda)\, r_1\, \underline{\theta}\, u'(\omega r_1) - \lambda\, u'(\omega) - (1-\lambda)\, 2\omega\, u'(\omega^2), \tag{38}$$

since the derivatives of the extreme of the integrals cancel out. At $\lambda \to 1$, $PC_\omega \to -u'(\omega) < 0$, so a value of $\underline{\lambda}_0 < 1$ exists by continuity such that $PC_\omega < 0$ for all $\lambda > \underline{\lambda}_0$.

From the effect that the deposit rates $\{r_1, r_2\}$ and CBDC remuneration $\omega$ have on consumers' participation constraint, we can rule out that both deposit rates decrease. Furthermore, supermodularity rules out that one deposit rate increases, while the other decreases. To see this,

assume by contradiction that after an increase in $\omega$, $r_2$ increases while $r_1$ decreases. Because of supermodularity, a reduction in $r_1$ weakly lowers the marginal benefit of increasing $r_2$. Thus, decreasing $r_1$ while increasing $r_2$ reduces the benefit of increasing $r_2$, while not increasing depositors' expected utility. In other words, the participation constraint can still be satisfied with $r_1$ held constant and $r_2$ adjusted upward. Therefore, an allocation with $r_1$ decreasing and $r_2$ increasing is weakly dominated by an allocation in which $r_1$ is held constant and $r_2$ increases, thus reaching a contradiction. It follows that the terms of the deposit contract are at least both weakly increasing in $\omega$, and at the optimum one must be strictly increasing in $\omega$.

To show that $r_2^*$ strictly increases in $\omega$, we rely on the fact that bank profit exhibits increasing differences in $(r_2, \omega)$:

$$\frac{\partial^2 \Pi}{\partial r_2 \partial \omega} = (1-\lambda)\left[-\frac{\partial^2 \underline{\theta}}{\partial r_2 \partial \omega}\underline{\theta}(Rr_1 - r_2) - \frac{\partial \underline{\theta}}{\partial r_2}\frac{\partial \underline{\theta}}{\partial \omega}(Rr_1 - r_2) + \frac{\partial \underline{\theta}}{\partial \omega}\underline{\theta}\right] > 0,$$

since $Rr_1^* \geq r_2^*$ in any equilibrium, as previously argued.

Consequently, a higher $\omega$ strictly raises the marginal benefit to increasing $r_2$. Hence, the bank strictly prefers to allocate the adjustment required by an increase in $\omega$ to $r_2$, so $r_2^*$ strictly increases in $\omega$, while $r_1^*$ is non-decreasing.

**Panic runs.** Consider now the case when $r_1 > 1$, so that the relevant run threshold is $\theta^*$. We mirror the logic of the $r_1 \leq 1$, thus following the same four steps. First, differentiating (35) with respect to $r_2$ to obtain:

$$\frac{\partial^2 \Pi}{\partial r_1 \partial r_2} = -\left(\frac{\partial^2 \theta^*}{\partial r_1 \partial r_2}\theta^* + \frac{\partial \theta^*}{\partial r_1}\frac{\partial \theta^*}{\partial r_2}\right)\Pi^{NR} + \frac{\partial \theta^*}{\partial r_2}\lambda R\theta^* + \frac{\partial \theta^*}{\partial r_1}\theta^*(1-\lambda), \tag{39}$$

where $\Pi^{NR} \equiv R(1-\lambda r_1) - (1-\lambda)r_2 \geq 0$ denotes bank profits when no run occurs. Using the decomposition of the cross-partial $\frac{\partial^2 \theta^*}{\partial r_1 \partial r_2}$ in (25), this can be rewritten as:

$$\frac{\partial^2 \Pi}{\partial r_1 \partial r_2} = \underbrace{-2\frac{\partial \theta^*}{\partial r_1}\frac{\partial \theta^*}{\partial r_2}\Pi^{NR}}_{>0} + \underbrace{\theta^*(1-\lambda)\frac{\partial \theta^*}{\partial r_1}}_{>0} + \underbrace{\lambda R\theta^*\frac{\partial \theta^*}{\partial r_2}}_{<0} - \Phi\,\theta^*\,\Pi^{NR}, \tag{40}$$

where

$$\Phi \equiv \frac{u'(r_2)\theta^*\frac{\widehat{n}R}{Rr_1 - r_2} + \frac{\partial \theta^*}{\partial r_2}\int_{\widehat{n}}^{\overline{n}} u'\left(R\frac{1-nr_1}{1-n}\right)\left(\frac{n}{1-n}R\right)dn}{\int_{\lambda}^{\widehat{n}} u\left(r_2\right)dn + \int_{\widehat{n}}^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right)dn}.$$

We now establish that (40) is positive for a $r_1$ strictly above but close to 1. In the limit, when $r_1 \to 1$, $\theta^* \to \underline{\theta}$, the run cost $\Delta \to 0$, and the insolvency threshold satisfies

$\widehat{n} \to \overline{n} = 1/r_1 \to 1$. As a result, the integral $\int_{\widehat{n}}^{\overline{n}}(\cdot)\,dn \to 0$ and the term $\Phi \to \frac{u'(r_2)\underline{\theta}R/(Rr_1-r_2)}{(\frac{1}{r_1}-\lambda)u(r_2)}$, which is finite and bounded. Furthermore, given the continuity of the run thresholds, in this limit (39) converges to the cross-partial computed in the case with only fundamental runs, which we have already proven to be positive. By continuity of $\frac{\partial^2\Pi}{\partial r_1 \partial r_2}$ in $r_1, r_2, \omega$, supermodularity is preserved in a neighborhood of $r_1 = 1$. Note that a numerical evaluation of (40) along the equilibrium path $(r_1^*(\omega), r_2^*(\omega))$ confirms that $\frac{\partial^2\Pi}{\partial r_1 \partial r_2} > 0$ for all admissible values of $\omega$ in our baseline parameterization.

We now move to the effect of a change in $\omega$ on depositors' participation constraint. Differentiating (31) with respect to $\omega$, $r_1$, and $r_2$ gives:

$$PC_\omega = (1-\lambda)\theta^* u'(\omega) - \frac{\partial \theta^*}{\partial \omega}\Delta - \lambda u'(\omega) - (1-\lambda)2\omega u'(\omega^2), \tag{41}$$

$$EUD_{r_1} = \lambda(1-\theta^*)u'(r_1) - \frac{\partial \theta^*}{\partial r_1}\Delta, \tag{42}$$

and

$$EUD_{r_2} = \int_{\theta^*}^{1}(1-\lambda)\theta u'(r_2)d\theta - \frac{\partial \theta^*}{\partial r_2}\Delta > 0, \tag{43}$$

where $\Delta$ represents the cost of a run and it is equal to

$$\Delta = [\lambda u(r_1) + (1-\lambda)\theta^* u(r_2) - \lambda u(1) - (1-\lambda)u(\omega)] > 0. \tag{44}$$

As in the case with only fundamental runs, we start by ruling out that the first positive term in $PC_\omega$ dominates. The least favorable case is $\theta^* = 1$, so the first and last term can be combined to $(1-\lambda)\left[u'(\omega) - 2\omega u'(\omega^2)\right] < 0$, where we used $RRA > 1$ for $\omega > 1$. Hence, as the positive term is smaller for any $\theta^* < 1$, $PC_\omega < 0$ always holds.

Similarly to the previous case, $EUD_{r_1} > 0$, $EUD_{r_2} > 0$, and $PC_\omega < 0$ imply at least one deposit rate must increase after a rise in $\omega$. Supermodularity of the profit function in $r_1$ and $r_2$ rules out a configuration in which one rate increases while the other decreases, by the same argument as in the fundamental-run case: if $r_2$ were to increase while $r_1$ decreases, supermodularity implies that a reduction in $r_1$ weakly lowers the marginal benefit of increasing $r_2$. Hence, a configuration with $r_1$ constant and $r_2$ adjusted upward weakly dominates. Thus, both deposit rates are non-decreasing in $\omega$.

Moreover, since bank profits decrease in $r_1$, as it can be seen from (35), the adjustment of $r_1$ only cannot be optimal. Hence, $r_2^*$ must strictly increase. This conclusion is reinforced by

increasing differences of profits in $(r_2, \omega)$:

$$\frac{\partial^2 \Pi}{\partial r_2 \partial \omega} = -\frac{\partial^2 \theta^*}{\partial r_2 \partial \omega}\theta^* \Pi^{NR} - \frac{\partial \theta^*}{\partial r_2}\frac{\partial \theta^*}{\partial \omega}\Pi^{NR} + \frac{\partial \theta^*}{\partial \omega}\theta^*(1-\lambda) > 0, \tag{45}$$

which is positive because $\frac{\partial^2 \theta^*}{\partial r_2 \partial \omega} < 0$, from (26), $\frac{\partial \theta^*}{\partial r_2} < 0$, $\frac{\partial \theta^*}{\partial \omega} > 0$, and $\Pi^{NR} > 0$.

The above analysis applies to the case of small $\omega$, as we have shown in Lemma 2 that when $\omega$ is close to 1, $r_1 \leq 1$ and $r_1$ weakly increases in $\omega$. Hence, for small values of $\omega$, the model exhibits the same properties as in the case when $r_1 \leq 1$ and, therefore, $r_1$ is non- decreasing in $\omega$ and $r_2$ strictly increasing with it.

To complete the proof, consider the case where $\omega$ is larger and so $r_1^* > 1$. For any $r_1 > 1$, there exists a value of $\omega$, which we denote as $\bar{\omega}$, such that $\theta^* \to 1$ when $\omega \to \bar{\omega}$. so the positive term $\lambda(1-\theta^*)u'(r_1)$ shrinks to zero while the negative term remains bounded away from zero (because $\frac{\partial \theta^*}{\partial r_1} > 0$ and $\Delta > 0$). By continuity, there exists at least one cutoff $\widetilde{\omega}$ such that

$$EUD_{r_1} \leq 0 \quad \text{for all } \omega \geq \widetilde{\omega}.$$

For $\omega \geq \widetilde{\omega}$, an increase in $r_1$ is detrimental to both the bank ($\partial \Pi / \partial r_1 < 0$) and depositors ($EUD_{r_1} \leq 0$). Hence $r_1^*$ is non-increasing in this region. Because $PC_\omega < 0$ and $EUD_{r_2} > 0$, restoring $PC = 0$ after a rise in $\omega$ requires a strict increase in $r_2^*$, which is again supported by the increasing differences in (45).

# E Proof of Proposition 3

For $\omega \geq \widetilde{\omega}$, Proposition 2 implies $r_1^* > 1$ and $\frac{dr_1^*}{d\omega} \leq 0$. Since $\frac{\partial \theta^*}{\partial r_1} > 0$, the indirect effect via $r_1^*$ is non-positive:

$$\frac{\partial \theta^*}{\partial r_1}\frac{dr_1^*}{d\omega} \leq 0.$$

Hence, a sufficient condition for $\frac{d\theta^*}{d\omega} < 0$ is that the remaining terms are negative when setting $\frac{dr_1^*}{d\omega} = 0$. In this case, the total derivative reduces to

$$\frac{d\theta^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega} + \frac{\partial \theta^*}{\partial r_2}\frac{dr_2^*}{d\omega}.$$

Since the participation constraint $PC = EUD - U = 0$ binds and $r_1^*$ is held fixed, implicit differentiation gives

$$\frac{dr_2^*}{d\omega} = -\frac{PC_\omega}{EUD_{r_2}} > 0,$$

where positivity follows from $PC_\omega < 0$, based on Assumption 1, and $EUD_{r_2} > 0$. Substituting and multiplying through by $EUD_{r_2} > 0$, we obtain:

$$\text{sign}\left(\frac{d\theta^*}{d\omega}\right) = \text{sign}\left(\frac{\partial\theta^*}{\partial\omega}\, EUD_{r_2} - \frac{\partial\theta^*}{\partial r_2}\, PC_\omega\right). \tag{46}$$

Expanding $PC_\omega = EUD_\omega - U_\omega$, it suffices to sign the expression

$$\Gamma(\lambda) \;\equiv\; \frac{\partial\theta^*}{\partial\omega}\, EUD_{r_2} + \frac{\partial\theta^*}{\partial r_2}\, U_\omega - \frac{\partial\theta^*}{\partial r_2}\, EUD_\omega. \tag{47}$$

In what follows, we try to establish the sign of $\Gamma(\lambda)$.

First, notice that all partial derivatives of $\theta^*$ share a common denominator $\mathcal{D}$, which is equal to

$$\mathcal{D} \equiv \int_\lambda^{\widehat{n}} u(r_2)\, dn + \int_{\widehat{n}}^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right) dn > 0$$

To keep the notation compact, also denote the two numerators as $N_\omega$ and $N_{r_2}$, which are equal to:

$$\begin{aligned} N_\omega &\equiv \int_\lambda^{\overline{n}} u'(\omega r_1)\, r_1\, dn + \int_{\overline{n}}^{1} u'\Big(\frac{\omega}{n}\Big)\, \frac{1}{n}\, dn > 0, \\ N_{r_2} &\equiv (\widehat{n} - \lambda)\, u'(r_2) > 0, \end{aligned}$$

respectively, so that $\frac{\partial\theta^*}{\partial\omega} \equiv \frac{N_\omega}{\mathcal{D}}$ and $\frac{\partial\theta^*}{\partial r_2} \equiv -\frac{\theta^* N_{r_2}}{\mathcal{D}}$.

This allows us to rearrange (47) as follows:

$$\mathcal{D}\cdot\Gamma(\lambda) = N_\omega \cdot EUD_{r_2} + \theta^* N_{r_2} \cdot EUD_\omega - \theta^* N_{r_2} \cdot U_\omega. \tag{48}$$

Hence, given that $\mathcal{D} > 0$, we have $\text{sign}(\Gamma) = \text{sign}(\mathcal{D}\cdot\Gamma)$, We can also rearrange the expressions for $EUD_{r_2}$ and $EUD_\omega$, as follows:

$$\begin{aligned} EUD_{r_2} &= (1-\lambda)\frac{1-(\theta^*)^2}{2}\, u'(r_2) + \frac{\theta^* N_{r_2}}{\mathcal{D}}\, \Delta, \\ EUD_\omega &= (1-\lambda)\theta^*\, u'(\omega) - \frac{N_\omega}{\mathcal{D}}\, \Delta. \end{aligned}$$

Substituting into (48) and expanding:

$$\begin{aligned} N_\omega \cdot EUD_{r_2} &= (1-\lambda)\frac{1-(\theta^*)^2}{2}\, u'(r_2)\, N_\omega + \frac{\theta^* N_{r_2} N_\omega}{\mathcal{D}}\, \Delta, \\ \theta^* N_{r_2} \cdot EUD_\omega &= (1-\lambda)(\theta^*)^2 N_{r_2}\, u'(\omega) - \frac{\theta^* N_{r_2} N_\omega}{\mathcal{D}}\, \Delta. \end{aligned}$$

Summing these two expressions, the terms involving $\Delta$ cancel exactly:

$$N_\omega \cdot EUD_{r_2} + \theta^* N_{r_2} \cdot EUD_\omega = (1-\lambda)\frac{1-(\theta^*)^2}{2}\, u'(r_2)\, N_\omega + (1-\lambda)(\theta^*)^2 N_{r_2}\, u'(\omega).$$

Using $U_\omega = \lambda\, u'(\omega) + (1-\lambda)\, 2\omega\, u'(\omega^2)$ and substituting back into (48), we obtain:

$$\mathcal{D} \cdot \Gamma(\lambda) = (1-\lambda)\, \Psi - \lambda\, \theta^* N_{r_2}\, u'(\omega), \tag{49}$$

where

$$\Psi \equiv N_\omega\, \frac{1-(\theta^*)^2}{2}\, u'(r_2) + (\theta^*)^2 N_{r_2}\, u'(\omega) - \theta^* N_{r_2} \cdot 2\omega\, u'(\omega^2). \tag{50}$$

The last term in (49), $-\lambda\, \theta^* N_{r_2}\, u'(\omega)$, is always strictly negative. The sign of $\Psi$ is, instead, potentially ambiguous. Assume the worst for our desired result, i.e., $\Psi > 0$. In this case, then $\mathcal{D} \cdot \Gamma(\lambda) < 0$ if and only if

$$\lambda > \frac{\Psi}{\Psi + \theta^* N_{r_2}\, u'(\omega)} \equiv \widetilde{\lambda}(\omega). \tag{51}$$

The bound $\widetilde{\lambda}(\omega)$ is strictly less than 1 for all admissible parameter values, since the denominator exceeds the numerator by $\theta^* N_{r_2}\, u'(\omega) > 0$.

Define $\widetilde{\lambda} \equiv \max\left\{\underline{\lambda}_0,\ \sup_{\omega \geq \widetilde{\omega}} \widetilde{\lambda}(\omega)\right\}$. Since $\widetilde{\lambda}(\omega) < 1$ for all $\omega$, we have $\widetilde{\lambda} < 1$. For any $\lambda > \widetilde{\lambda}$ and $\omega \geq \widetilde{\omega}$, both Assumption 1 holds and $\Gamma(\lambda) < 0$. It follows that $\frac{d\theta^*}{d\omega} < 0$, which completes the proof.

# F Proof of Proposition 4

When $r_1 \leq 1$, the failure threshold is uniquely pinned down by the dominance condition equating the utility from early withdrawal and from waiting until the final date:

$$u(\omega r_1^*) = \underline{\theta}\, u(r_2^*). \tag{52}$$

Since $u(\cdot)$ is strictly increasing, $r_1^* = r_1^*(\omega)$, and $r_2^* = r_2^*(\omega)$, Equation (52) thus defines a unique failure threshold, $\underline{\theta}(\omega)$. Total differentiation with respect to $\omega$ yields

$$\frac{d\underline{\theta}}{d\omega} = \frac{u'(\omega r_1^*)\left(r_1^* + \omega \frac{dr_1^*}{d\omega}\right) - \underline{\theta}\, u'(r_2^*)\frac{dr_2^*}{d\omega}}{u(r_2^*)}. \tag{53}$$

We know that $r_1^*$ does not fall as $\omega$ increases (Proposition 2). Since an increase in $r_1^*$ upon an increase in $\omega$ makes it more likely for Equation (53) to be positive, a sufficient condition for our

desired result, $\frac{d\underline{\theta}}{d\omega}$, can be derived by assuming $\frac{dr_1^*}{d\omega} = 0$. Intuitively, this is the worst case for our desired result because (1) we get rid of one positive indirect effect and (2) $r_2^*$ must increase a lot to guarantee a binding participation constraint of consumers. This means that, when $r_1^*$ stays constant, $\frac{dr_2^*}{d\omega}$ is maximal. In particular, $\frac{dr_2^*}{d\omega}$ is entirely pinned down by the change in the participation constraint when $\frac{dr_1^*}{d\omega} = 0$:

$$\frac{dr_2^*}{d\omega} = -\frac{PC_\omega}{EUD_{r_2}}$$

Using the expression above for $PC_\omega$ and $EUD_{r_2}$, we can rewrite the numerator of $\frac{d\underline{\theta}}{d\omega}$ as:

$$u'(\omega r_1^*)r_1^* + \underline{\theta}u'(r_2^*)\frac{(1-\lambda)\underline{\theta}r_1^*u'(r_1\omega) - \lambda u'(\omega) - (1-\lambda)2\omega u'(\omega^2)}{(1-\lambda)\frac{1-\underline{\theta}^2}{2}u'(r_2^*)}.$$

Rearranging the terms slightly, given that the denominator is positive, the sign of $\frac{d\underline{\theta}}{d\omega}$ is equal to the sign of

$$(1-\lambda)u'(\omega r_1^*)r_1^*\left(\frac{1-\underline{\theta}^2}{2} + \underline{\theta}^2\right) - \lambda\underline{\theta}u'(\omega) - (1-\lambda)2\omega u'(\omega^2) \tag{54}$$

$$= (1-\lambda)u'(\omega r_1^*)r_1^*\left(\frac{1+\underline{\theta}^2}{2}\right) - \lambda\underline{\theta}u'(\omega) - (1-\lambda)2\omega u'(\omega^2) \tag{55}$$

$$= (1-\lambda)\left[u'(\omega r_1^*)r_1^*\left(\frac{1+\underline{\theta}^2}{2}\right) - 2\omega u'(\omega^2)\right] - \lambda\underline{\theta}u'(\omega). \tag{56}$$

We can see that the above expression evaluated at $\underline{\lambda}_0$, which makes $PC_\omega = 0$, is positive because $\frac{1+\underline{\theta}^2}{2} > \underline{\theta}$. Note that $\lambda$ does not enter directly in the expression of $\underline{\theta}$, so the above expression is continuous and strictly decreasing in $\lambda$. To see this, differentiate it with respect to $\lambda$. We obtain:

$$-u'(\omega r_1^*)\, r_1^*\, \frac{1+\underline{\theta}^2}{2} - \underline{\theta}\, u'(\omega) + 2\omega\, u'(\omega^2) < 0,$$

where negativity follows because $u'(\omega r_1^*)\, r_1^*\, \frac{1+\underline{\theta}^2}{2} > 2\omega\, u'(\omega^2)$. This last inequality follows from the positive sign established when $\lambda = \underline{\lambda}_0$ noticing that $PC_\omega = 0$ means

$$(1-\underline{\lambda}_0)\, r_1^*\, \underline{\theta}\, u'(\omega r_1^*) \;=\; \underline{\lambda}_0\, u'(\omega) \;+\; (1-\underline{\lambda}_0)\, 2\omega\, u'(\omega^2).$$

We now identify a cutoff $\underline{\lambda}_0 < \underline{\lambda}_2 < 1$, as the level at which the expression in (54) is zero.

$$\underline{\lambda}_2 \equiv \frac{u'(\omega r_1^*)r_1^*\left(\frac{1+\underline{\theta}^2}{2}\right) - 2\omega u'(\omega^2)}{u'(\omega r_1^*)r_1^*\left(\frac{1+\underline{\theta}^2}{2}\right) - 2\omega u'(\omega^2) + \underline{\theta}u'(\omega)} < 1. \tag{57}$$

It follows that for any $\lambda < \underset{\sim}{\lambda}_2$, the expression in (54) is positive. The inequality $\underset{\sim}{\lambda}_0 < \underset{\sim}{\lambda}_2$ follows directly from the definition of $\underset{\sim}{\lambda}_2$, which guarantees that the interval $(\underset{\sim}{\lambda}_0, \underset{\sim}{\lambda}_2)$ is non-empty and well-defined.

Allowing adjustment of $r_1$ makes it even more likely, so $\underset{\sim}{\lambda}_0 < \lambda < \underset{\sim}{\lambda}_2$ is sufficient for the desired result. Therefore, $\frac{d\theta}{d\omega} > 0$ for all $\lambda \in (\underset{\sim}{\lambda}_0, \underset{\sim}{\lambda}_2)$ and the proposition follows.

# G Proof of Proposition 5

The proof proceeds in steps. First, we derive the equilibrium deposit rate $r_1^*$. Then, we move on to study the effect that a change in $\omega$ has on the terms of the deposit contract.

As in Goldstein and Pauzner (2005), we assume that the contract features $r_1^* > 1$ and then we show that this holds true in equilibrium. As it will become useful later, we rewrite the expression for the panic run threshold $\theta^*$ substituting $r_2 = \frac{1-nr_1}{1-n}R$ in (4), which gives:

$$\theta^* = \frac{\int_\lambda^{\overline{n}} u\left(\omega r_1\right) dn + \int_{\overline{n}}^1 u\left(\frac{\omega}{n}\right) dn}{\int_\lambda^{\overline{n}} u\left(\frac{1-nr_1}{1-n}R\right) dn} \tag{58}$$

Hence, the derivatives of the threshold $\theta^*$ with respect to $r_1$ and $\omega$ are, respectively:

$$\frac{\partial \theta^*}{\partial r_1} = \frac{\int_\lambda^{\overline{n}} u'\left(\omega r_1\right) \omega dn + \theta^* \int_\lambda^{\overline{n}} u'\left(\frac{1-nr_1}{1-n}R\right) \frac{n}{1-n} R dn}{\int_\lambda^{\overline{n}} u\left(\frac{1-nr_1}{1-n}R\right) dn} > 0, \tag{59}$$

$$\frac{\partial \theta^*}{\partial \omega} = \frac{\int_\lambda^{\overline{n}} u'\left(\omega r_1\right) r_1 dn + \int_{\overline{n}}^1 u'\left(\frac{\omega}{n}\right) \frac{1}{n} dn}{\int_\lambda^{\overline{n}} u\left(\frac{1-nr_1}{1-n}R\right) dn} > 0, \tag{60}$$

since $u(0) = 0$ and $\frac{1-\overline{n}r_1}{1-\overline{n}}R = 0$. When $r_1 > 1$, $\theta^F = \theta^*$ and the equilibrium deposit rate $r_1^*$ solves the following first-order condition

$$\begin{aligned} FOC \equiv & -\frac{\partial \theta^*}{\partial r_1}\left[\lambda u(r_1) + (1-\lambda)\theta^* u\left(\frac{1-\lambda r_1}{1-\lambda}R\right) - \lambda u(1) - (1-\lambda)u(\omega)\right] \\ & + \lambda \int_{\theta^*}^1 \left[u'(r_1) - \theta R u'\left(\frac{1-\lambda r_1}{1-\lambda}R\right)\right] d\theta = 0, \end{aligned} \tag{61}$$

where $\left[\lambda u(r_1) + (1-\lambda)\theta^* u\left(\frac{1-\lambda r_1}{1-\lambda}R\right) - \lambda u(1) - (1-\lambda)u(\omega)\right] > 0$ is the cost of a run. Evaluating the FOC at $r_1 = 1$ gives:

$$\lambda \int_{\underline{\theta}}^1 \left[u'\left(1\right) - \theta R u'\left(R\right)\right] d\theta > 0,$$

because of $RRA > 1$ and so $cu'(c)$ decreases in $c$ and the fact that the term in the square bracket is equal to zero when $\theta^F = \underline{\theta}$ and $r_1 = 1$. Hence, $FOC_{r_1=1} > 0$, so $r_1^* > 1$ in equilibrium.

We now move on to study the effect of a change in $\omega$ on the run threshold $\theta^*$. Recall that any changes in $r_1$ immediately translates into a decrease in $r_2$ since depositors are residual claimants of bank available resources. This negative relationship between interim and final deposit rates is already accounted for in the derivative of the run threshold with respect to $r_1$, as given in (59), thus we can express the total derivative (5) of the run threshold with respect to $\omega$ as follows

$$\frac{d\theta^*}{d\omega} = \frac{\partial\theta^*}{\partial\omega} + \frac{\partial\theta^*}{\partial r_1}\frac{dr_1^*}{d\omega},$$

where $\frac{\partial\theta^*}{\partial r_1}$ also account for effect of a change in the repayment expected by depositors when they wait until date 2.

In order to complete the proof, we need to show that $r_1^*$ decreases in $\omega$. To do so, we can use the FOC for $r_1$, the IFT, and $SOC < 0$ (as $r_1^*$ is a local maximum). We derive this partial derivative next; it has three parts:

$$\begin{aligned}\frac{\partial FOC}{\partial\omega} &= -\frac{\partial^2\theta^*}{\partial r_1\partial\omega}\left[\lambda u(r_1) + (1-\lambda)\theta^* u(c_{2\lambda}) - \lambda u(1) - (1-\lambda)u(\omega)\right] \\ &\quad + \frac{\partial\theta^*}{\partial r_1}(1-\lambda)\left[u'(\omega) - \frac{\partial\theta^*}{\partial\omega}u(c_{2\lambda})\right] - \lambda\left[u'(r_1) - \theta^* R u'(c_{2\lambda})\right]\frac{\partial\theta^*}{\partial\omega},\end{aligned} \tag{62}$$

where $c_{2\lambda} \equiv R\frac{1-\lambda r_1}{1-\lambda}$. The third term is negative (the bracket is positive) because of the risk-sharing benefit of liquidity provision.

The first term is negative whenever the cross-partial $\frac{\partial^2\theta^*}{\partial r_1\partial\omega}$ is positive, which is the case since it is equal to:

$$\frac{\partial^2\theta^*}{\partial r_1\partial\omega} = \frac{\int_\lambda^{\overline{n}}\left[u''(r_1\omega)\,\omega\,r_1 + u'(r_1\omega)\right]dn}{\int_\lambda^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right)dn} + \frac{\partial\theta^*}{\partial\omega}\frac{\int_\lambda^{\overline{n}} u'\left(R\frac{1-nr_1}{1-n}\right)R\frac{n}{1-n}dn}{\int_\lambda^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right)dn}$$

Using (60), one can see that the second term of this cross-partial is positive, while its first one can be rearranged as follows:

$$\frac{1 - RRA(\omega r_1^*)}{\int_\lambda^{\overline{n}} u\left(R\frac{1-nr_1}{1-n}\right)dn}\int_\lambda^{\overline{n}} u'(r_1\omega)dn,$$

where RRA is the relative risk aversion. Note that RRA monotonically increases in the level of absolute risk aversion $\sigma$ and in the level of consumption. Hence, we denote as $\widehat{\sigma}$ the level of risk aversion such that, for $\sigma < \widehat{\sigma}$, the RRA when evaluated at $\omega r_1 \to R$, which is the upper bounds

of $\omega r_1^*$, approaches 1. Thus, for any $\sigma < \widehat{\sigma}$, the first term in the cross-partial is dominated by the second one. As a result of the positive cross-partial derivative, the first term in (62) is also negative.

Thus, it only remains to be shown that the second term in (62) is also negative. This is the case when $\frac{\partial \theta^*}{\partial \omega} u(c_{2\lambda}) - u'(\omega) > 0$.

Using the expression for $\frac{\partial \theta^*}{\partial \omega}$ and noting that $\left(\frac{1-nr_1}{1-n}R\right) \leq \left(\frac{1-\lambda r_1}{1-\lambda}R\right)$ for all $n \geq \lambda$, a sufficient condition for this inequality to hold is $u'(\omega) \leq \frac{1}{\overline{n}-\lambda}\left[\int_\lambda^{\overline{n}} u'(\omega r_1)\, r_1 dn + \int_{\overline{n}}^1 u'\left(\frac{\omega}{n}\right)\frac{1}{n} dn\right]$. Multiplying both sides by $\omega$ yields

$$\omega u'(\omega) \leq \omega r_1 u'(\omega r_1) + \frac{1}{\overline{n}-\lambda}\int_{\overline{n}}^1 u'\left(\frac{\omega}{n}\right)\frac{\omega}{n} dn. \tag{63}$$

Note that the term on LHS is bigger than the first term on the RHS. A lower bound for the second term on the RHS is $\frac{1-\overline{n}}{\overline{n}-\lambda}\omega r_1 u'(\omega r_1)$. Thus, the RHS of Condition (63) is at least $\frac{1-\lambda}{\overline{n}-\lambda}\omega r_1 u'(\omega r_1)$, so a simple sufficient condition is

$$\omega u'(\omega) \leq \frac{1-\lambda}{\overline{n}-\lambda}\omega r_1 u'(\omega r_1). \tag{64}$$

One can show that this condition is always satisfied. Substituting $\overline{n} = \frac{1}{r_1}$ and dividing by $\omega$ each side, the inequality above can be rewritten as follows:

$$u'(\omega) \leq \frac{(1-\lambda)}{1-\lambda r_1} r_1^2 u'(\omega r_1).$$

It is immediate that the inequality is satisfied for $r_1 = 1$. It is also satisfied for $r_1 \to \frac{1}{\lambda}$ Hence, the inequality must hold true for all $r_1$ in the admissible range, unless the RHS is not monotonically changing with $r_1$. To exclude this case, we take the derivative:

$$\frac{\partial RHS}{\partial r_1} = \frac{(1-\lambda)r_1}{(1-\lambda r_1)^2}\left[2u'(\omega r_1)(1-\lambda r_1) + r_1 u''(\omega r_1)\omega(1-\lambda r_1) + \lambda r_1 u'(\omega r_1)\right],$$

which, using the definition of RRA, can be rearranged as follows:

$$\frac{\partial RHS}{\partial r_1} = \frac{(1-\lambda)r_1 u'(\omega r_1)}{(1-\lambda r_1)^2}\left[1 + (1-\lambda r_1)(1 - RRA(\omega r_1^*))\right].$$

For any $\sigma < \widehat{\sigma}$, the RHS increases monotonically in $r_1$ and so the fact that the inequality holds when $r_1 = 1$ establishes that it is always satisfied. This completes the proof.