A funding liquidity risk channel for monetary policy transmission*

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Abstract

We study the borrowing of a bank subject to rollover risk and derive implications for monetary policy transmission and financial stability. The optimal borrowing balances the benefit of additional profitable investments with the cost of higher bank fragility. This trade-off leads to a funding liquidity risk channel for monetary policy transmission. While a tighter policy decreases borrowing, its effect on financial stability depends on the interaction between a price effect and an opposing scale effect. In particular, the price effect dominates whenever the risk-free rate is low, impairing financial stability. Thus, an exit from a low-interest rate environment may increase bank fragility. We derive several testable implications for monetary transmission and bank fragility.

Keywords: rollover risk, monetary policy, fragility, global games, funding liquidity risk channel.

JEL classifications: G01, G21, G28.

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1 Introduction

Banks are important conduits for the transmission of monetary policy from central banks to the real economy. Much of the prevailing literature suggests that banks respond to monetary policy actions by adjusting either the amount or riskiness of their lending (e.g., Bernanke, 2007; Dell'Ariccia and Marquez, 2013; Martinez-Miera and Repullo, 2017). Banks are also inherently fragile institutions that are prone to runs on their debt claims. Despite this fragility, banks heavily rely on funding that is subject to rollover risk (e.g., short-term unsecured wholesale debt and interbank loans) to finance investments as these markets are deeper than those for retail deposits or equity (Adrian and Shin, 2011).[1]

These observations raise several important questions. First, how does rollover risk impact a bank's borrowing and investing decisions? Second, how do monetary policy actions influence bank borrowing and investment in the presence of rollover risk, and what is the impact on bank fragility? And third, how do variations in bank equity affect the transmission of monetary policy?

In this paper, we address these questions by proposing a positive theory of the *funding liquidity risk channel* of monetary policy transmission. We consider a purposefully simple model to illustrate this channel. Our analysis builds on the bank-run model of Rochet and Vives (2004) and Vives (2014). A bank issues uninsured demandable debt to finance profitable investment that is costly to liquidate.[2] Following

[1] Hanson et al. (2015) document that deposits represent over three quarters of funding of U.S. commercial banks. Moreover, in larger commercial banks, approximately half of deposits are uninsured. Certificates of deposits (CDs) are a common form of unsecured short-term debt routinely used by European banks. The market for CDs is roughly the same size as that for repos in Europe and about ten times as large as the unsecured interbank market (Perignon et al., 2018).

[2] Consistent with much evidence, unsecured bank debt is assumed to be demandable. Demandability is a feature of optimal debt contracts in models with uncertain liquidity needs (Diamond and Dybvig, 1983), agency conflicts (Calomiris and Kahn, 1991; Diamond and Rajan, 2001), or expro-

a shock to the bank's balance sheet, concerns over the future viability of the bank can precipitate withdrawals, requiring the bank to liquidate investments. Bank failure is thus driven by both the exogenous balance sheet shock and losses from the costly liquidation of investments to serve endogenous withdrawals. Moreover, individual withdrawal incentives are further sparked by concerns about the withdrawals of other investors, so the run on the bank is also a consequence of a coordination failure.

Using global game techniques (Carlsson and van Damme, 1993; Morris and Shin, 2003; Vives, 2005), we derive the unique equilibrium in which the bank fails if the exogenous shock exceeds a certain threshold. This fragility threshold decreases in the face value of debt, while the impact of greater borrowing is, in general, ambiguous. On the one hand, when the bank borrows more, it becomes exposed to a higher risk of a run. On the other hand, it also increases profitable investments and thereby raises the resources available to serve its debt. The former effect dominates if the required return of investors – and thus the equilibrium face value of debt – is sufficiently high.

In equilibrium, the bank chooses how much debt to issue (and thus its investment scale) to maximize the expected equity value, taking into account its impact on the failure threshold and the face value of debt. This gives rise to the following trade-off at the heart of the funding liquidity risk channel. On the one hand, by scaling up its investments by issuing debt, the bank earns the intermediation margin (the difference between the investment return and the face value of debt). On the other hand, the bank is more likely to fail because greater debt increases the banks' susceptibility to a run. The risk of such a run leads the bank to scale down its borrowing and investment compared to a situation without coordination failure. (We consider this benchmark without coordination failure throughout the paper.)

priation risk (Ahnert and Perotti, 2021). Accordingly, in what follows, we use "uninsured deposits" to refer to short-term or demandable debt including uninsured retail deposits or wholesale funding.

We consider how changes in monetary policy, i.e. the risk-free rate (a measure for conventional monetary policy) and the degree of asset market liquidity (a measure for unconventional policy), affect bank borrowing and its fragility. First, we show that a lower risk-free rate and greater market liquidity are expansionary in the sense that they lead to greater borrowing and investment. These results are in line with empirical evidence of the standard bank lending channel (e.g., Bernanke and Blinder, 1992; Stein and Kashyap, 2000). However, the mechanism in our model differs from other, observationally equivalent theories (e.g., Peek and Rosengreen, 2013). In particular, changes in the bank's borrowing and investment scale constitute an optimal response to a policy-induced change in the risk of experiencing a run. Section 6 discusses the cross-sectional testable implications of this mechanism in more detail.

Second, we consider how changes in monetary policy stance affect financial stability. We use the bank's fragility threshold as our measure of financial stability. In general, changes in monetary policy exert three effects on the fragility threshold. First, there is a direct *price effect*. Expansionary policy reduces funding costs or increases asset valuations and thereby help to reduce fragility. Second, there is an indirect *scale effect* whereby expansionary policy that induces the bank to increase its borrowing and investment leads to an increase in fragility. Finally, there is an *amplification effect*, which captures the feedback between the risk-adjustment in the price of debt and the bank's default risk. This amplification effect alters the magnitude, but not the sign, of the total effect of monetary policy on bank fragility.

To understand how monetary policy influences financial stability, we must compare the price and scale effects. In general, the scale effect tends to dominate the price effect when the level of the risk-free interest is high. Thus, in a *high interest rate environment*, expansionary monetary policy increases fragility. By contrast, in a *low interest rate environment*, expansionary monetary policy tends to reduce fragility.

This latter result contrasts with the implications of the standard risk-taking channel of monetary policy that is regularly emphasized in the policy debate (e.g., Cunliffe, 2019). In particular, our funding liquidity risk channel suggests that due to the dominating price effect, an 'exit' from a low interest environment can be associated with adverse consequences for financial stability.

We consider two extensions of our baseline model to probe its robustness. First, we consider the bank to be endowed with equity. Since equity absorbs losses, it exerts a catalytic effect on borrowing and investment: equity mitigates concerns about the bank's future solvency, thus reducing rollover risk and shifting bank borrowing and lending closer to the benchmark of perfect coordination.[3] Second, we study the consequences of having a multiplicative or scale-dependent shock, which introduces a new risk-taking incentive for the bank. Our baseline model, in contrast, assumes an additive or scale-invariant shock to the balance sheet, as in Ahnert et al. (2019), which allows us to better isolate the funding liquidity risk channel. We show that our insights are robust to the introduction of risk-taking on the asset side.

Finally, our model generates empirical predictions on how rollover risk and monetary policy influence bank borrowing and investment and financial stability. While several of these are consistent with existing empirical findings, we also provide new predictions on the cross-sectional impact of monetary policy on banks. In particular, we argue that banks that are subject to more rollover risk, e.g., have lower Net Stable Funding Ratios, are less sensitive to changes in monetary policy. Moreover, we discuss how one might test the impact of monetary policy on bank fragility.

[3] This catalytic effect is similar to that explored by Morris and Shin (2006) in an international finance context. In their model, official sector assistance to a country in the midst of a financial crisis can spur debtor to lend more, thereby alleviating the crisis.

Literature. Our paper relates to several strands of the literature. The traditional literature on the bank lending channel posits that a monetary policy tightening leads to a shortfall of banks' deposits and reduces lending (Bernanke and Gertler, 1995; Stein and Kashyap, 1995; Stein, 1998; Boivin et al., 2010; Peek and Rosengreen, 2013). Recently, Drechsler et al. (2017) suggest that banks respond to monetary policy shifts by exercising their market power in deposit markets. We complement this literature by arguing that banks' responses are also shaped by rollover risk.

We also contribute to the literature on bank runs and global games. In the unique equilibrium, the run is a consequence of a coordination failure following a large shock to the bank's balance sheet. In particular, we build on Rochet and Vives (2004) where investors delegate their rollover decisions to professional fund managers, so the decisions to roll over are global strategic complements.[4] We contribute to this literature by showing how, via the endogenous borrowing choice and face value of debt, the effects of monetary policy on fragility engender countervailing indirect effects, which are absent in other studies with exogenous borrowing and investment (e.g., Vives, 2014; König, 2015; Bebchuk and Goldstein, 2011).

Our paper also connects to the literature on the risk-taking channel of monetary policy and its implications for financial stability. Dell'Ariccia et al. (2014) study the risk-shifting incentives of banks and argue that expansionary monetary policy increases bank leverage and risk-taking. Martinez-Miera and Repullo (2017) consider a 'search-for-yield' mechanism: when monetary policy reduces yields on safer assets

[4] Goldstein and Pauzner (2005) study one-sided strategic complementarity due to the sequential service constraint of banks (Diamond and Dybvig, 1983). Liu (2016) studies how runs on banks interact with freezes in the interbank market, giving rise to amplification. Eisenbach (2017) shows how rollover risk from demandable debt effectively disciplines banks for idiosyncratic shocks, while a two-sided inefficiency arises for aggregate shocks. Ahnert et al. (2019) consider how the introduction of senior secured debt influences rollover risk and banks' borrowing and investment choices. Carletti et al. (2020) study the role of liquidity regulation and its interaction with capital requirements. Li and Ma (2021) study the interaction between bank runs and falling asset prices in secondary markets and show the desirability of a committed liquidity support by a regulator.

compared to the interest on their long-term liabilities, financial institutions rebalance their asset portfolios towards riskier short-term assets. Our model focuses instead on banks' "*risk-taking on the liability side*" i.e. higher failure risk stemming from funding balance sheet expansions by increased borrowing.

2 Model

The model builds on Rochet and Vives (2004) and Vives (2014). There are three dates $t = 0, 1, 2$, a single good for consumption an investment, a continuum $\omega > 0$ of investors and a representative bank owner / manager (henceforth banker for short). Each investor is risk-neutral, endowed with one unit of funds at $t = 0$ and is indifferent between consuming at $t = 1$ and $t = 2$. Investors can store their funds at $t = 0$ to earn a (gross) risk-free return $r > 0$ at $t = 2$. The banker is penniless but has access to illiquid investments that return $R > r$ at $t = 2$ per unit invested at $t = 0$. However, selling investments at $t = 1$ yields only a fraction $\psi \in (0, 1)$ of the investment's return. To finance investments, the banker issues $D \geq 0$ of demandable debt to investors at $t = 0$ with face value $F > 0$. The bank's initial balance sheet is $I \equiv D$, where I is the amount invested.[5]

At $t = 1$, investors either redeem their claims against the bank or roll them over until $t = 2$. Each investor delegates the rollover decision to a professional fund manager who is rewarded for making the right decision: if the bank does not fail, a manager's payoff difference between withdrawing and rolling over is a cost $c > 0$; if the bank fails, the differential payoff is a benefit $b > 0$.[6] The conservatism ratio,

[5] For an extension with equity on the bank's balance sheet at $t = 0$, see section 5.

[6] As an example, assume the cost of withdrawal is c; the benefit from getting the money back or withdrawing when the bank fails is $b + c$; the payoff for rolling over when the bank fails is zero.

$\gamma \equiv \frac{b}{b+c} \in (0,1)$, summarizes these payoffs, with more conservative managers (higher γ) being less inclined to roll over.[7] For simplicity, we assume that the face value of debt, F, is independent of the withdrawal date.

The banker is protected by limited liability and is subject to a shock A at $t = 2$. This shock may improve the bank's balance sheet, $A < 0$, but operational, market, or legal risks may require writedowns, $A > 0$. The shock is drawn at $t = 1$ from a continuous probability distribution with a decreasing reverse hazard rate, $\frac{d}{dA}\frac{g(A)}{G(A)} < 0$, where $G(A)$ denotes the cumulative distribution and $g(A)$ the probability density.[8]

Assets	Liabilities
$RI - A$	FD
	$E_2(A)$

Table 1: Balance sheet at $t = 2$ after a small shock and all debt rolled over.

Table 1 shows the balance sheet at $t = 2$ for a small shock and when all debt is rolled over. In this case, the value of bank equity is given by $E_2(A) \equiv \max\{0, RI - A - FD\}$ and the bank fails whenever $A > \overline{A} \equiv RI - FD$. If, however, a fraction $\ell \in [0,1]$ of debt is withdrawn at $t = 1$, the banker liquidates a share $\ell \frac{FD}{\psi RI}$ of the investment to repay debt holders. As a result, the banker has insufficient resources to repay the outstanding debt at $t = 2$ and fails whenever

$$RI - A - \ell\frac{FD}{\psi} < (1-\ell)FD\,, \tag{1}$$

[7]Reviewing debt markets during the financial crisis, Krishnamurthy (2010) argues that investor conservatism was an important determinant of short-term lending behavior. See also Vives (2014).

[8]This property ensures a unique borrowing choice of the bank. It holds, for instance, for the exponential, (log-)normal, and uniform distributions.

or $A > RI - FD\,[1 + \ell z]$, where $z \equiv \frac{1}{\psi} - 1 > 0$ is the per-unit cost of liquidation.[9] We assume zero recovery upon bank failure at the final date.[10]

At $t = 1$, fund managers base their withdrawal decisions on a noisy private signal (Morris and Shin, 2003):

$$x_i = A + \epsilon_i, \tag{2}$$

where ϵ_i is a mean-zero noise term that is independent of the shock A and identically and independently distributed across fund managers according to a continuous distribution H with support $[-\epsilon, \epsilon]$ for $\epsilon > 0$. Table 2 summarizes the timeline.

$t = 0$	$t = 1$	$t = 2$
1. Debt issuance	1. Balance sheet shock realizes	1. Investment matures
2. Investment	2. Private signals about shock	2. Shock materializes
	3. Withdrawals	3. Debt repayment
	4. Consumption	4. Consumption

Table 2: Timeline of events.

3 Equilibrium

We solve for the symmetric pure-strategy perfect Bayesian equilibrium in which fund managers at $t = 1$ use threshold strategies, i.e. rolling over if and only if their signal indicates a healthy balance sheet, $x_i \leq x^*$, and the bank fails if and only if $A > A^*$.[11]

[9] The bank closes early if it runs out of funds at $t = 1$, $RI - A < \ell \frac{FD}{\psi}$. The relevant failure condition, however, is equation (1), as in Rochet and Vives (2004) or Vives (2014).

[10] Our results are qualitatively unchanged for positive recovery values.

[11] Since we assume that private information is sufficiently precise, the equilibrium is unique (Morris and Shin, 2003). It is also an extremal equilibrium that is in monotone strategies (Vives, 2005). Since the rollover decision is binary, our focus on threshold strategies is without loss of generality.

Definition 1. *The symmetric pure-strategy perfect Bayesian equilibrium comprises thresholds, x^* and A^*, borrowing volume D^*, and face value of debt F^* such that*

a. at $t = 0$, given the thresholds (x^*, A^*), the banker maximizes his expected equity value by choosing borrowing D and the face value F, subject to a participation constraint of investors that ensures them an expected return of at least r;

b. at $t = 1$, given the banker's choices (D^*, F^*), the threshold strategy, x^*, maximizes fund managers' expected payoffs, given that the banker fails for $A > A^*$.

We solve the model backwards in two steps. First, we pin down x^* and A^* at the rollover stage at $t = 1$. Second, we characterize the banker's borrowing choice at $t = 0$, where the banker takes into account how his choices affect rollover risk.

3.1 Rollover risk of debt

For the solution of the rollover subgame at date $t = 1$, we focus on the global game solution of vanishing private noise about the balance sheet shock, $\epsilon \to 0$ (Eisenbach, 2017; Ahnert et al., 2019; Carletti et al., 2020). As a result, the rollover threshold converges to the failure threshold, $x^* \to A^*$. In this case, fund managers face only strategic uncertainty about the behavior of other managers at $t = 1$ because the fundamental uncertainty about the magnitude of the shock vanishes.

Proposition 1. Failure threshold. *There exists a unique failure threshold A^*. All fund managers refuse to roll over and the banker defaults if and only if*

$$A > A^* \equiv RI - FD\,[1 + \gamma z]\,. \tag{3}$$

Proof. See Appendix A1. ■

Proposition 1 pins down the unique incidence of a run on bank debt. The failure threshold can be expressed as $A^* = \overline{A} - \gamma zFD$, so rollover risk causes a deviation of the failure threshold A^* from the perfect-coordination benchmark, $\overline{A}$. In particular, rollover risk is a consequence of the combination of fund manager conservatism, $\gamma > 0$, and asset illiquidity, $\psi < 1$ (or $z > 0$). Without rollover risk, $\gamma z \to 0$, the failure threshold thus converges to its perfect-coordination benchmark, $A^* \to \overline{A}$. We henceforth use A^* as our measure of bank fragility. An increase of A^* reduces fragility as it decreases the range of shocks (A^*, ∞) for which the bank fails.

Lemma 1. Borrowing volume and fragility. *An increase in bank borrowing increases fragility, $\frac{\partial A^*}{\partial D} < 0$, if and only if debt is sufficiently expensive, $F > \frac{R}{1+\gamma z}$.*

Proof. See Appendix A1. ■

Greater borrowing induces two opposing effects on the failure threshold. First, greater borrowing allows the bank to increase the scale of its profitable investments, $\frac{\partial I}{\partial D} = 1$, which reduces fragility because it provides more resources that can be used to repay debt. Second, because the additional borrowing may be withdrawn, it increases the bank's exposure to a debt run, thus heightening fragility. Note that the effective per-unit cost of debt that impacts the bank's fragility at $t = 1$ exceeds the face value F due to the possibility that assets are liquidated at a cost when the debt is withdrawn. This additional per-unit cost of debt is given by γzF. As shown in Lemma 1, the second effect dominates the first if the total effective per-unit cost of debt at $t = 1$, $F(1 + \gamma z)$, exceeds the per-unit return from the bank's investment. In this case, more borrowing unambiguously increases fragility. Note further, that a higher liquidation value mitigates the influence of fund manager conservatism and reduces fragility, $\frac{\partial A^*}{\partial \psi} > 0$ (Rochet and Vives, 2004). Finally, with a higher face value of debt, the bank must liquidate a larger share of assets to meet early withdrawals, $\frac{\partial A^*}{\partial F} < 0$. A higher risk-free rate does not directly affect fragility, $\frac{\partial A^*}{\partial r} = 0$.

3.2 Optimal borrowing and investment

The banker chooses the level of borrowing D and offers a face value F to investors in order to maximize his expected equity value at date $t = 2$, subject to the investor participation constraint in funding markets, the balance sheet identity, and the failure threshold $A^*(D, F) \equiv A^*$. The banker's problem can be expressed as

$$\max_{D,\,F} \ \pi \quad \equiv \quad \int_{-\infty}^{\infty} E_2(A)dG(A) = \int_{-\infty}^{A^*(D,F)} \big[RI - A - FD\big]\, dG(A) \tag{4}$$

$$\text{s.t.} \quad F\,G(A^*(F,D)) \geq r, \tag{5}$$

where the participation constraint in (5) states that investors' expected return from funding the bank must be at least the risk-free return earned on the outside option. Since the banker's profit function is strictly decreasing in F for all values of D, profit maximization implies that (5) must hold with equality. Proposition 2 characterizes the interior solution to the constrained optimization problem.

Proposition 2. Bank borrowing and debt pricing. *There exists an interval $\mathcal{R} \equiv (\underset{\sim}{r}, \tilde{r})$ such that for any $r \in \mathcal{R}$, there exists a unique interior equilibrium with borrowing volume $D^* \in (0, \omega)$ and face value $F^* \in (\frac{R}{1+\gamma z}, R)$.*

Proof. See Appendix A2. ■

To understand the intuition behind Proposition 2, note that, for a given face value F, a marginal increase in borrowing exerts two effects on the bank's expected equity value, π. First, an increase in D alters the failure threshold, A^*, and the set of shock realizations where the bank fails. As shown in Lemma 1, borrowing lowers the failure threshold if and only if the face value is sufficiently large, $F > R/(1+\gamma z)$. Second, for a given realization of the shock, an increase in D changes the equity value conditional on the bank not failing, $(R-F)D - A$. In particular, greater borrowing increases the bank's equity value if and only if $F < R$. Otherwise, debt is prohibitively expensive and the bank incurs a loss from borrowing any amount.

Now consider the bank's optimal borrowing whenever the face value F falls outside of the bounds specified in Proposition 2. If debt is sufficiently cheap, $F < R/(1+\gamma z) < R$, the failure threshold and the equity value are both strictly increasing in D. Greater borrowing increases both the likelihood that the bank does not fail and the value of equity for any realization of the shock. In this case, the banker strictly prefers to borrow as much as possible, $D^* = \omega$. Second, if debt is very expensive, $F > R$, then the failure threshold and the equity value are both decreasing in D and the bank completely abstains from any borrowing, $D^* = 0$.

It follows that for an intermediate face value, $R/(1+\gamma z) < F < R$, the banker's equity value strictly increases in D, while the failure threshold strictly decreases. Hence, the banker's optimal borrowing choice trades off the marginal benefit from increasing the equity value against the marginal cost which stems from the higher expected loss due to the increased likelihood of failure. For given F, the optimal choice of D satisfies

$$R - F = -(1+\gamma z)FD\frac{g(A^*)}{G(A^*)}\frac{dA^*}{dD}. \tag{6}$$

The left-hand side of equation (6) corresponds to the net marginal benefit from an additional unit of borrowing and investing. Conditional on not failing, the banker obtains an additional return R and has to pay out the face value F, leaving him with a net marginal benefit of $R - F$, the intermediation margin. The right-hand side of equation (6) is the expected loss due to a marginal change in the threshold A^*. The term $\frac{g(A^*)}{G(A^*)} \frac{dA^*}{dD}$ measures the likelihood of default following the total change in the failure threshold conditional on not failing. The banker internalizes, via the creditors' participation constraint, how the marginal increase in borrowing impacts the face value of debt, F, and thereby the failure threshold, A^*. With a slight abuse of notation, the total effect of greater borrowing on the failure threshold is given by

$$\frac{dA^*}{dD} = \frac{\partial A^*}{\partial D} + \frac{\partial A^*}{\partial F} \frac{dF}{dD}. \tag{7}$$

The losses in the event of failure on the right-hand side of (6) include the bank's equity value, $E_2(A^*) = \gamma z F D$, as well as the losses to creditors, FD, that the banker internalizes through investor's participation constraint.

Conditions (5) and (6) together determine the banker's choice of borrowing and the face value of debt. The interval $\mathcal{R}$ in which an interior optimum exists is derived by using the creditors' participation constraint to obtain thresholds such that, at $r = \underaccent{\tilde}{r}$, there is no borrowing, $D^* = 0$, and at $r = \tilde{r}$, the banker borrows $D^* = \omega$.

Figure 1 shows the equilibrium. The solid line is the borrowing schedule $D^*(F)$ derived from Equation (6) and the dashed line, $F^*(D)$, is the participation constraint (5). Given $r \in \mathcal{R}$, the two curves intersect such that $F^* \in (\frac{R}{1+\gamma z}, R)$ and $D^* \in (0, \omega)$.

4 Implications of monetary policy

In this section, we study the transmission of monetary policy to the banker's investment and borrowing decisions as well as financial fragility. For simplicity, we assume

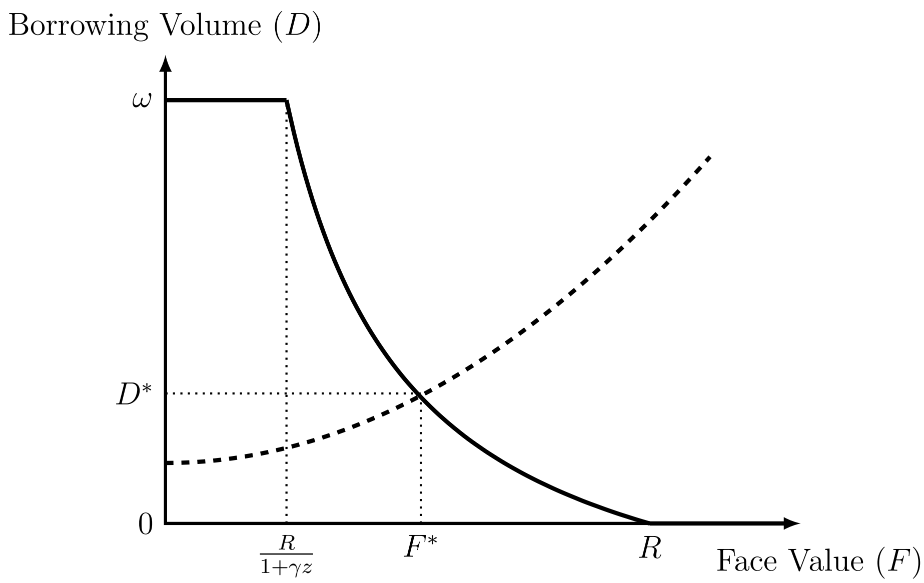

Figure 1: The unique equilibrium borrowing volume and debt face value are determined at the intersection of the creditors' participation constraint (dashed line) and the bank's demand for funds (solid line).

that monetary policy can directly affect the risk-free return, r, and the secondary market value of bank assets, ψ. We refer to changes in the risk-free return as stemming from 'conventional' monetary policy and to changes in the secondary market value as being driven by 'unconventional' policy. An increase in the risk-free return is said to follow from a tightening of conventional monetary policy, while an increase in the secondary market value of assets is attributed to a looser unconventional policy. We assume that monetary policy determines r and ψ at date $t = 0$, before the bank makes its borrowing and investment choices.

4.1 Benchmark: Monetary policy transmission in the absence of rollover risk

When $\gamma = 0$, fund managers are no longer subject to strategic complementarity in their rollover decisions and inefficient runs are eliminated. Consequently, the bank fails at $t = 2$ whenever $A > \overline{A}$. Moreover, the bank borrows the maximal feasible amount, $D^* = \omega$, as long as $F^* < R$, i.e., the net marginal benefit is positive.

Proposition 3. Monetary policy in the absence of rollover risk. *In the absence of rollover risk, $\gamma = 0$, monetary policy generically has no effect on the bank's borrowing and investment scale.*

In the absence of rollover risk, conventional monetary policy has no effect on the intensive margin of the banker's borrowing and investment scale. However, by influencing the face value of debt, a change in the risk-free return can impact the extensive margin of bank borrowing. As F^* increases and crosses R, following a tightening of conventional monetary policy, the bank stops borrowing. Unconventional policy has no effect since the bank does not require additional liquidity to serve withdrawals.

4.2 Monetary policy in the presence of rollover risk: the funding liquidity risk channel

When $\gamma > 0$, the rollover decisions are subject to strategic complementarity. As a result, the bank fails whenever $A > A^*$. Moreover, for any $F \in (R/(1 + \gamma z), R)$, the banker optimally responds to an increase in the risk of failure by borrowing and investing less. In other words, the presence of rollover risk gives rise to a 'scale effect' such that $D^* < \omega$. This, in turn, induces the optimal borrowing and investment level to respond to changes in monetary policy. As such, we refer to this channel as the *funding liquidity risk channel* for the transmission of monetary policy.

Proposition 4. Monetary policy and the funding liquidity risk channel.

1. *Conventional monetary policy: An increase in the risk-free return, r, increases the face value of debt, $\frac{dF^*}{dr} > 0$, and reduces borrowing, $\frac{dD^*}{dr} < 0$.*

2. *Unconventional monetary policy: An increase in ψ raises borrowing, $\frac{dD^*}{d\psi} > 0$. Moreover, there exists a bound, $\overline{r} \in (\underset{\sim}{r}, \tilde{r})$, such that an increase in the liquidation value ψ reduces the face value of debt, $\frac{dF^*}{d\psi} < 0$, if and only if $r < \overline{r}$.*

Proof. See Appendix A3. ■

An increase in the risk-free return implies that investors require a higher face value in order to supply funding to the bank. A higher face value, in turn, raises the marginal cost of borrowing by increasing the likelihood of the bank failing due to run risk. Moreover, the bank must forego greater equity value in the event of failure. The banker, thus, responds by reducing the amount of borrowing and investment, thereby counteracting the effect of the higher face value on the risk of a run.

Figure 2 shows the scale effect and funding liquidity risk channel of monetary policy. It plots the banker's optimal borrowing against the risk-free return, r. The dashed line depicts the perfect-coordination benchmark in the absence of coordination risk ($\gamma = 0$), while the solid line shows the case with rollover risk ($\gamma > 0$). For values of r that are either sufficiently small ($r < \underset{\sim}{r}$) or sufficiently large ($r > \tilde{r}$), rollover risk has no effect on the bank's borrowing and investment scale. For intermediate values $r \in (\underset{\sim}{r}, \tilde{r})$, however, the scale effect is present and is given by the distance between the dashed line and solid curve. Due to the scale effect, an increase in the risk-free return is associated with lower borrowing as the bank trades off the net marginal benefit against the increased likelihood of failure due to a higher borrowing scale.

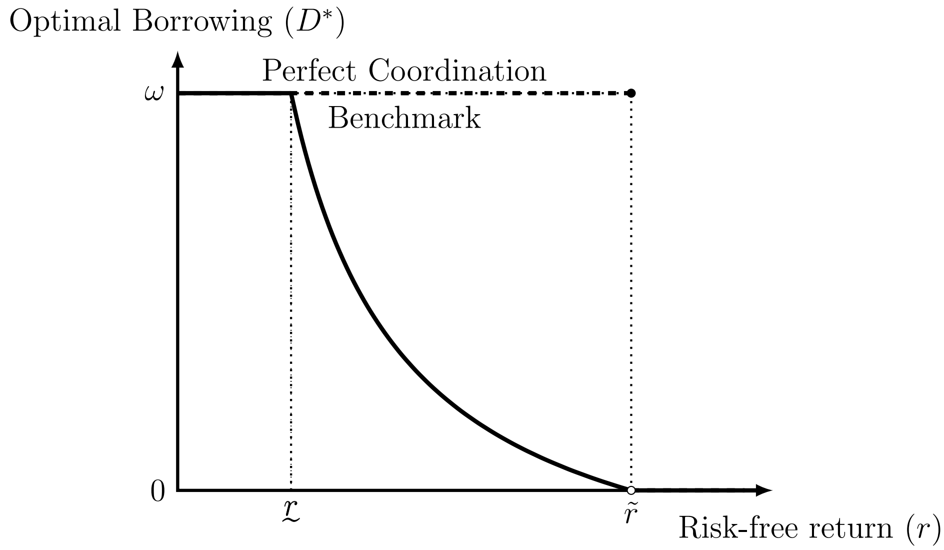

Figure 2: Rollover risk reduces borrowing relative to a perfect-coordination benchmark. Borrowing volume with (solid) and without rollover risk (dashed).

4.3 Monetary policy and financial stability

We next turn to the financial stability implications of monetary policy. We measure financial stability by the bank's failure probability. Since the ex-ante failure probability is monotonically decreasing in the failure threshold A^*, to study the effects of monetary policy on financial stability, it suffices to consider its effects on A^*.

Unlike conventional monetary policy, which operates via the investor participation constraint, unconventional monetary policy exerts a direct effect on the bank's failure threshold, A^*. An increase in ψ improves the market liquidity of bank assets, which reduces the likelihood of a run. As a consequence, for any face value of debt, the marginal cost of borrowing decreases, and so the banker increases his borrowing and investment level for any value of F. The increase in borrowing, however, tends to counteract the effect of improved market liquidity on the failure threshold. Hence, in equilibrium, the total effect of ψ on the face value can be ambiguous. This ambiguity leads to an interesting interaction between the *level* of the risk-free return and the effect of changes in market liquidity. If the risk-free return is sufficiently large, then the indirect negative effect on A^* due to the increase in borrowing dominates and the direct positive effect, which increases the equilibrium face value of debt.

The results in Proposition 4 imply that the total effects of monetary policy on financial stability are a priori not clear. In particular, it follows from Lemma 1 that for $F \in (R/(1+\gamma z), R)$, the failure threshold decreases in both F and D. But since changes in the risk-free return or the liquidation value of the assets can have opposing effects on the face value of debt and the bank's borrowing volume, the total effects on the failure threshold are ambiguous.

To see this, we decompose the total effect of a marginal increase in the risk-free return r on the failure threshold into three separate terms:

$$\frac{dA^*}{dr} = \underbrace{\frac{1}{1+\frac{g(A^*)}{G(A^*)}\frac{\partial A^*}{\partial F}F^*}}_{\text{Amplification effect }(+)} \times \left(\underbrace{\frac{\partial A^*}{\partial D}\frac{dD^*}{dr}}_{\text{Scale effect }(+)} + \overbrace{\frac{1}{G(A^*)}\frac{\partial A^*}{\partial F}}^{\text{Price effect }(-)} \right). \tag{8}$$

The first term in brackets is a scale effect, which affects fragility via the total adjustment in borrowing scale. Since $\frac{dD^*}{dr} < 0$ and $\frac{\partial A^*}{\partial D} < 0$, it follows that the scale effect reduces fragility. The second effect in brackets, which we refer to as the price effect, captures the contribution of the higher face value of debt to the total effect on A^*. Since $\frac{\partial A^*}{\partial F} < 0$, the price effect raises the bank's fragility. Finally, the third effect (that multiplies into the previous two effects) is an amplification that accounts for the fact that an increase in fragility leads, in itself, to an increase in the face value which, in turn, further increases fragility.

For unconventional monetary policy shifts (changes in ψ), we obtain a similar decomposition:

$$\frac{dA^*}{d\psi} = \underbrace{\frac{1}{1+\frac{\partial A^*}{\partial F}\frac{g(A^*)}{G(A^*)}F^*}}_{\text{Amplification effect }(+)} \times \left(\underbrace{\frac{\partial A^*}{\partial D}\frac{dD^*}{d\psi}}_{\text{Scale effect }(-)} + \overbrace{\frac{\partial A^*}{\partial \psi}}^{\text{Price effect }(+)} \right). \tag{9}$$

In contrast to conventional monetary policy, the scale effect now increases fragility since $\frac{dD^*}{d\psi} > 0$, while the price effect reduces fragility because $\frac{\partial A^*}{\partial \psi} > 0$. The net effect is again amplified via the failure-risk-adjustment of the face value.

As Proposition 5 shows, the impact of monetary policy on financial stability depends on the relative magnitudes of the opposing scale and price effects. These are, in turn, shaped by the level of the risk-free return.

Proposition 5. Monetary policy and financial stability.

1. *Conventional monetary policy: There exists a unique* $\underline{r} \in (\underset{\sim}{r}, \tilde{r})$*, where* $\underline{r} < \bar{r}$*. A tightening of policy reduces fragility,* $\frac{dA^*}{dr} > 0$*, if and only if* $r > \underline{r}$*.*

2. *Unconventional monetary policy: A higher liquidation value reduces fragility,* $\frac{dA^*}{d\psi} > 0$*, if and only if* $r < \bar{r}$*.*

Proof. See Appendix A3. ■

The intuition for the impact of conventional monetary policy on financial fragility is as follow. First note that the failure threshold, A^*, is proportional to the bank's total funding cost, FD. Thus, at any interior equilibrium, the impact of the scale effect in reducing fragility increases with the face value of debt since $\partial A^*/\partial D = R - (1 + \gamma z)F < 0$. At the same time, the negative impact of the price effect increases with the amount of bank borrowing since $\partial A^*/\partial F = -(1 + \gamma z)D$. Thus, whenever the risk-free return is sufficiently small, the face value of debt is low, while bank borrowing is relatively high. In such a low interest rate environment, the price effect dominates and financial fragility worsens following a tightening of conventional monetary policy.

Conversely, in a high interest rate environment, the face value of debt, and thus the magnitude of the scale effect, are large compared to the borrowing volume and price effect. Consequently, an increase in the risk-free rate reduces financial fragility. At the threshold $\underline{r}$, the face value is such that price and scale effects wash out.

A similar intuition holds for the effect of unconventional monetary policy on financial fragility. However, in this case, the price effect is proportional to the total funding costs since $\partial A^*/\partial \psi = -\gamma FD/\psi^2$. As a consequence, the magnitude of the price effect of unconventional monetary policy is scaled up when compared to that of conventional policy. This scaling-up, therefore, implies that a larger face value of debt is required to render the scale and price effect equal. As such, we obtain a larger critical threshold, $\bar{r} > \underline{r}$, for the impact of unconventional monetary policy on financial fragility.

Discussion. Rochet and Vives (2004) and Vives (2014) study the rollover subgame at $t = 1$, where the face value of debt and amount of borrowing are fixed. In particular, they show that an improvement in market liquidity conditions unambiguously reduce fragility. Extending their analysis, we also account for how an increase in ψ affects both the face value and the volume of debt and analyze the total effect of ψ on bank fragility. We, thus, generalize their results to show when the direct effect may be dominated by adjustments in the volume and price of debt.

The financial stability implications of our funding liquidity risk channel can be related to those of the standard 'risk-taking channel' of monetary policy (Dell'Ariccia et al., 2014, 2017). Under the risk-taking channel, a reduction in interest rates leads to a compression of banks' charter values, which increases their incentives to invest in riskier loans, independent of the prevailing level of the interest rate. In our derivation of the funding liquidity risk channel, we deliberately abstracted from such risk-taking on the asset side of banks' balance sheet and focused on the default risk emanating from the liability side in order to isolate the funding liquidity risk channel.

Figure 3 illustrates the financial stability implications emanating from the funding liquidity risk channel. Expansionary policies (a decrease in r or an increase in ψ) raise bank fragility via the scaling channel whenever $r > \overline{r}$. Thus, for 'high interest environments' the financial stability consequences of our funding liquidity risk channel are in line with those of the standard risk-taking channel. However, in a 'low interest rate environment', i.e. for $r < \underline{r}$, expansionary monetary policy measures exert a stabilizing influence in contrast to that from the risk-taking channel.

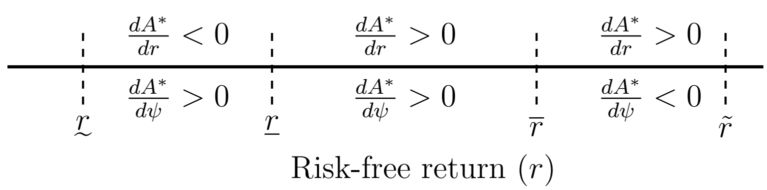

Figure 3: Effects of conventional and unconventional monetary policy on bank fragility for different monetary policy stances (levels of interest rates).

To further illustrate this point, consider a monetary policy 'exit' from a low interest environment. While the standard risk-taking channel would predict that financial stability would improve, our funding liquidity risk channel implies the opposite, i.e., a worsening due to the dominating price effect.

5 Extensions

In this section we explore two important extensions to our baseline model. First, we consider how the presence of equity influences the bank's decision to issue additional debt. And second, we show that our analysis is robust when we consider a scale-dependent shock and additional asset-side risk.

5.1 Equity

In this section, we assume that the banker is endowed with initial equity $E > 0$ at $t = 0$. This endowment has no effect on the analysis of the subgame equilibrium at $t = 1$ except for the fact that the balance sheet identity at $t = 0$ now reads $I \equiv D + E$ and, after substituting into equation (3), the failure threshold changes to

$$A^* \equiv R(D + E) - F D\,[1 + \gamma z]\,. \tag{10}$$

Since the banker invests all funds into profitable investments, the additional resources $R\,E > 0$ at $t = 2$ can also be used to serve debt. Hence, equity allows the banker to withstand larger shocks and lowers fragility, $\frac{\partial A^*}{\partial E} = R > 0$. Moreover, the presence of equity requires to distinguish between the level of borrowing, D^*, and the level of investment, $I^* = D^* + E$. Proposition 6 states the resulting optimal borrowing and investment of the banker, generalizing our previous result.

Proposition 6. Bank equity. *There exist values $\underset{\sim}{r}_E > \underset{\sim}{r}$ and $\tilde{r}_E > \tilde{r}$ such that the bank's optimal borrowing is given by Equation (6), with $\underset{\sim}{r}$ being replaced by $\underset{\sim}{r}_E$ and $\tilde{r}$ replaced by $\tilde{r}_E$. Greater equity increases borrowing, $dD^*/dE > 0$, makes debt cheaper, $dF^*/dE < 0$, and decreases fragility, $dA^*/dE > 0$.*

Proof. See Appendix A4. ■

Bank equity exerts a catalytic effect on bank scale as Figure 4 illustrates. That is, a marginal increase in equity induces the banker to increase borrowing and investment. Along the extensive margin, greater equity shifts the bounds $\underset{\sim}{r}_E$ and $\tilde{r}_E$ outwards. The increase in the lower bound implies that additional equity enables the banker to borrow the maximum $D^* = \omega$ for a larger range of risk-free returns. The increase in the upper bound implies that the range of risk-free returns where borrowing becomes prohibitively costly shrinks. And, on the intensive margin, for any given $r \in (\underset{\sim}{r}_E, \tilde{r}_E)$, an increase in equity increases bank borrowing further, $dD^*/dE > 0$.

How does the presence of bank equity influence the pass-through of monetary policy changes via the funding liquidity risk channel? Figure 4 conveys the main insights. First, while both bounds, $\underset{\sim}{r}_E$ and $\tilde{r}_E$ increase following a marginal increase in bank capital, the upper bound increases by more. Thus the gap, $\tilde{r}_E - \underset{\sim}{r}_E$, also increases. Second, since $dD^*/dE > 0$ for all $r \in (\underset{\sim}{r}_E, \tilde{r}_E)$, this suggests that, typically, the borrowing schedule $D^*(r)$ becomes flatter. This implies that marginal changes in the monetary policy rate have a smaller impact on bank borrowing when banks hold more equity. This result follows since more bank equity reduces rollover risk. Hence, changes in monetary policy have a smaller impact on the bank's fragility and

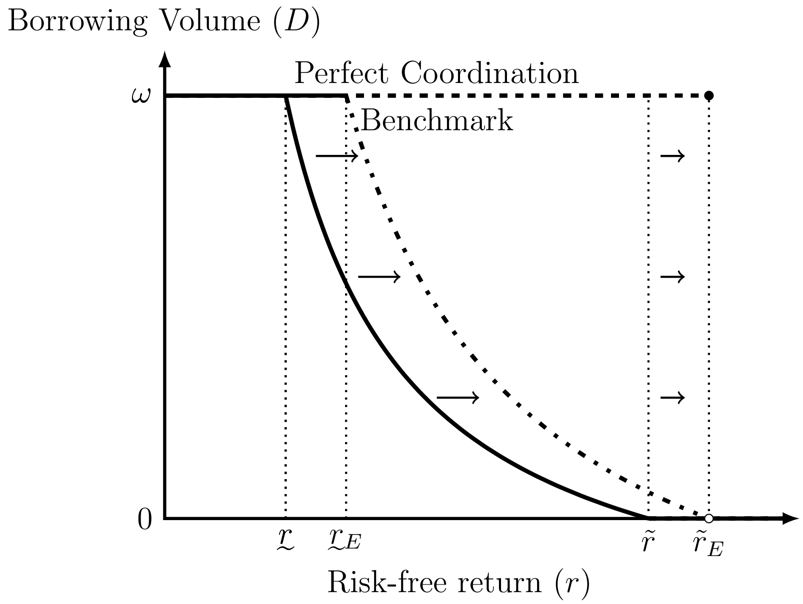

Figure 4: Equity increases the range where over which maximum borrowing takes place and also shifts the borrowed amount outwards (dash-dotted curve). Borrowing volume with (solid curve) and without rollover risk (dashed line).

therefore dampen the bank's corresponding adjustment of borrowing and investment. In Section 6, we discuss how this result may be empirically tested.

5.2 Scale-dependent shock and asset-side risk

Our preceding analysis assumed a scale-invariant (or additive) shock to the banker's balance sheet. This allowed us to derive the key implications of the funding liquidity risk channel by fully abstracting from the effect of the bank's decisions on asset-side risk. In this section, we touch upon the robustness of our analysis when we relax this assumption and introduce a scale-dependent (or multiplicative) shock whose impact depends on the scale of the banker's borrowing. Specifically, we suppose that the investment return, R, is a random variable with distribution $G(R)$ (density $g(R)$)

with an increasing hazard rate. Thus, the size of the shock experienced by the banker depends on the scale of his investments and hence on the banker's choice of borrowing.

Allowing for equity $E > 0$ on the banker's balance sheet, the failure condition is given by $RI - \ell \frac{FD}{\psi} < (1 - \ell)FD$. The global games analysis continues to apply, giving rise to a threshold such that the bank fails if and only if $R < R^*$. As in Rochet and Vives (2004), this threshold becomes

$$R^* = \frac{DF}{D + E}(1 + \gamma z), \tag{11}$$

where we substituted the balance sheet identity $I \equiv D + E$ into the denominator on the right-hand side. More borrowing increases leverage and unambiguously increases bank fragility, $\frac{\partial R^*}{\partial D} > 0$, but at a diminishing rate, $\frac{\partial^2 R^*}{\partial D^2} > 0$.[12]

The banker's optimization problem at $t = 0$ is

$$\max_{F,D} \int_{R^*}^{\infty} [R(D + E) - FD] \, dG(R) \quad \text{s.t.} \quad F(1 - G(R^*)) = r. \tag{12}$$

The analogue of condition (6) is now

$$\mathbf{E}\left[\tilde{R} \middle| R \geq R^* \right] - F = (1 + \gamma z) F D H(R^*) \frac{dR^*}{dD} \tag{13}$$

where $H(x) \equiv \frac{g(x)}{1 - G(x)}$ is the hazard rate of the investment return and $dR^*/dD = \partial R^*/\partial D + \partial R^*/\partial F \times dF/dD$. The key trade-off underlying the funding liquidity risk channel, i.e. balancing the benefit of scaling up investments with the cost of more expensive debt and losing equity due to higher rollover risk, remains largely unchanged. What is different, however, is that the benefit (left-hand side) now also

[12] The concavity of the threshold is a consequence of the scale-dependency of the shock. In contrast, the scale-invariant threshold A^* decreased linearly in D only if the the face value of debt was sufficiently large (i.e. $r > \underline{r}$).

depends on the scale of the bank's investment, I via the dependency of the left-hand side on R^*. In particular, since in this case the investment itself is the source of risk, the benefit from borrowing more is the expected return conditional on the bank not failing. As such, the left-hand side of equation (13) also depends on the failure threshold, R^*.

What is the influence of this additional effect on the benefits? Note that the expected return on investment increases when bank fragility increases, i.e. $\frac{d\mathbf{E}\left[\tilde{R}\middle|R\geq R^*\right]}{dR^*} > 0$, which strengthens the bank's incentives to increase its borrowing. This suggests that our earlier analysis with a scale-invariant shock provided a lower bound for the effects on bank borrowing. Moreover, it allowed us to study the funding liquidity risk channel in isolation without having to account for asset-side risk adjustments arising from limited liability.

6 Testable implications

In this section, we present several empirical predictions from our model. First, we briefly summarize the underlying mechanism behind the prediction. Next, for Predictions 1-3, we present supporting evidence from the existing empirical literature. Prediction 4, on the cross-sectional implications of monetary policy, and Prediction 5, on fragility and monetary policy, have, to the best of our knowledge, not yet been subject to empirical scrutiny and may inform future work. To this end, we comment on possible testing strategies.

Prediction 1. *Greater rollover risk reduces bank borrowing and reduces bank lending.*

Mechanism. Coordination failure and asset illiquidity give rise to rollover risk in our model and engender bank fragility. In particular, the greater the scope for coordination failure, the greater is bank fragility. From an ex-ante perspective, the prospect of greater bank fragility leads to an increase in the face value of debt. The bank, in turn, responds by reducing the amount it borrows.

Supporting evidence. Prediction 1 is consistent with a number of recent empirical findings. Jasova et al. (2018) document that a reduction in rollover risk due to the ECB's very long-term refinancing operations increases bank lending. Iyer et al. (2013) find that bank lending to firms is reduced if a larger share of its funding comes from the unsecured interbank market in which it is exposed to rollover risk. de Haas and van Lelyfeld (2014) find that lending growth of banks slows more after a shock for banks with higher reliance on wholesale funding. Ivashina and Scharfstein (2010) and Cornett et al. (2011) document a larger reduction in domestic credit following the financial crisis of 2007-08 for U.S. banks with a higher reliance on wholesale funding.

Prediction 2. *A monetary policy tightening decreases bank borrowing and lending by increasing the cost of funding.*

Mechanism. Our prediction on the impact of a monetary policy tightening is a variant of the standard bank lending channel. However, in contrast to much of the literature that focuses on the asset-side of banks' balance sheets, our model emphasises the role of banks' liabilities, in particular that of (uninsured) funding. As such, a monetary policy tightening increases creditors' outside option and thereby the cost of funding which, in turn, increases the bank's failure risk. The bank counteracts the increased failure risk by optimally reducing the amounts it borrows and lends.

Supporting evidence. Choi and Choi (2021) document how, following a monetary policy tightening, U.S. banks substitute low-interest retail deposits with high-interest wholesale deposits. But, as the substitution is not perfect, there is an overall reduction of bank borrowing, which translates into a reduction of bank lending. Drechsler et al. (2017) document a related result whereby a monetary policy tightening lead to increases in the deposit rate and outflows, thereby leading to a decrease in bank lending. Girotti (2021) also finds a similar result.

Prediction 3. *The borrowing and lending by banks with greater equity is less sensitive to a monetary policy change.*

Mechanism. Greater bank equity increases the interval of risk-free rates over which bank borrowing and lending is strictly positive. Moreover, the size of the interval grows as bank equity increases. As a consequence, the bank's borrowing becomes less responsive. Thus, a marginal increase in the monetary policy rate has a smaller impact on better-capitalized banks.

Supporting evidence. van den Heuvel (2012) finds that output growth of banking sectors in U.S. states that have low bank capital-to-asset ratios are more sensitive to changes in the Federal funds rate. Jiménez et al. (2012) show that monetary policy tightening episodes have larger negative effects on bank lending for Spanish banks with lower capital. Using a different identification strategy, Jiménez et al. (2014) show that monetary policy cuts have a more pronounced impact on lowly capitalised banks to lend more. Acharya et al. (2020) demonstrate that 'full allotment' liquidity provision by the ECB resulted in a lowering of deposit spreads and a further extension of credit by highly capitalised banks to their borrowers. Both these results are consistent with the predictions of our model wherein an increase in bank capital leads to a reduction in the face value of debt claims and an increase in lending.

Prediction 4. *Banks that are subject more to rollover risk, respond less to changes in monetary policy.*

Mechanism. This prediction stems from the following two observations. First, an increase in rollover risk leads to an increase in the range of the risk-free rate where bank borrowing is positive and below the perfect-coordination benchmark. Graphically, an increase in rollover risk leads to a decrease in the lower bound, $\underset{\sim}{r}$, while the upper bound, $\tilde{r}$, remains unchanged in Figure 2. And second, since the total supply of funding is fixed, the schedule for the bank's borrowing becomes, generically, flatter. Consequently, marginal changes in the risk-free rate elicit weaker responses in bank borrowing.

Testing strategy. Prediction 4 provides cross-sectional implications for the effects of monetary policy on bank borrowing and investment. It may be tested in, for example, an empirical specification where bank borrowing is regressed on measures of monetary policy interest rates. In particular, one can interacting the dependent variable with proxies for banks' shares of uninsured and unsecured wholesale debt (e.g., Net Stable Funding Ratios). Our theory predicts that the respective coefficient should be more negative when the share of debt that is subject to rollover risk is larger.

Prediction 5. *In a low interest rate environment, a monetary policy tightening increases bank fragility.*

Mechanism. In a low-interest rate environment, the bank's funding cost is low and the amount of borrowing is high. Thus, following a tightening of monetary policy, the price effect dominates the scale effect, leading to higher financial fragility. Conversely, in a high-interest rate environment, funding costs tend to be larger. This, in turn, amplifies the impact of the scale effect, relative to the price effect, from a tightening of monetary policy. And so financial fragility reduces.

Testing strategy. Bank fragility is, in general, difficult to measure as it incorporates both solvency risk and market illiquidity. However, one possible avenue may be to leverage on the firm-level Liquidity Mismatch Index (LMI) proposed by Brunnermeier et al. (2014). The LMI measures the mismatch between the market liquidity of assets and the funding liquidity of liabilities. Bai et al. (2018) construct the LMI index for approximately 3000 bank holding companies in the U.S., and use it to investigate their fragility. As such, one may attempt to test our prediction by regressing the LMI against measures of monetary policy.

7 Conclusion

In this paper, we propose a funding liquidity risk channel for the transmission of monetary policy. In a parsimonious model, banks' uninsured debt is subject to rollover risk. In choosing how much debt to issue, the bank balances the benefit of earning the intermediation margin with the cost of failing due to a higher run risk.

Monetary policy operates by influencing either the opportunity cost of lending to the bank or the liquidation value of the bank's assets. We argue that the effects of monetary policy on bank fragility are, in general, ambiguous and depend the prevailing interest rate environment. In particular, we show that an exit from unconventional monetary policy can impair financial stability, resulting in higher bank fragility. Finally, we derive several testable implications of our model that are consistent with existing evidence or may inform future empirical work.

To isolate our main channel of monetary transmission, we made several simplifying assumptions with respect to the dead-weight losses borne by creditors and the term-structure of debt. Nevertheless, one may relax these assumptions without impacting the core trade-off. For example, in the event of bank failure, one can assume that creditors obtain a pro-rata share of the bank's liquidation value. The qualitative effects for the transmission of monetary policy via the funding liquidity risk channel, however, would remain unchanged.

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A Proofs

A1 Proof of Proposition 1 and Lemma 1

The proof is in two steps. First, we show that the failure threshold based on condition (1) implies the existence of well-defined dominance regions. In these regions, fund managers have a dominant action to roll over or withdraw irrespective of the actions of other managers. Suppose that all debt is rolled over, $\ell = 0$. The bank still fails if $A > \overline{A} = RI - FD$, so withdrawing is a dominant strategy for fund managers for $x > \overline{A} + \epsilon$. Similarly, suppose that no debt is rolled over, $\ell = 1$. The bank still does not fail if $A < \underline{A} \equiv RI - \frac{FD}{\psi}$. Thus, rolling over is a dominant strategy for $x < \underline{A} - \epsilon$. For $A \in [\underline{A}, \overline{A}]$, whether the bank fails depends on the withdrawal proportion of fund managers. If the realization of A was common knowledge, the model would exhibit multiple self-fulfilling equilibria (Figure 5).

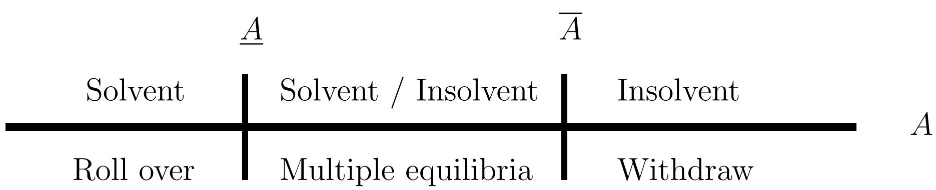

Figure 5: Tripartite classification of the balance sheet shock

Second, we characterize the equilibrium under incomplete information. Suppose fund managers use a symmetric threshold strategy around x^*. For a given realization $A \in [\,\underline{A}, \overline{A}\,]$, the proportion of fund managers who do not roll over debt is $\ell\Big(A, x^*\Big) = \text{Prob}\left(x_i > x^* \middle| A\right) = \text{Prob}\left(\epsilon_i > x^* - A\right) = 1 - H\Big(x^* - A\Big)$. Using (1), the failure threshold A^* solves

$$A^* = RI - \big(1 + z\ell\big(A^*, x^*\big)\big)FD. \tag{A1}$$

For given x^*, the left-hand side of (A1) is strictly increasing in A^* and rises from $-\infty$ to ∞. The right-hand side is strictly decreasing in A^* and is bounded above and below by $(1 + z)FD$ and FD, respectively. Hence, there exists a unique failure threshold A^*.

The posterior distribution of the shock conditional on the private signal can be derived using Bayes' rule. The threshold x^* is pinned down by indifference between rolling over and withdrawing of a fund manager who observes $x_i = x^*$: $1-\gamma = \Pr(A > A^* | x_i = x^*)$. For small ϵ, the latter can be written as $\gamma = \Pr(A < A^* | x_i = x^*) = 1 - H\Big(x^* - A^*\Big)$. The indifference condition therefore implies $x^* - A^* = H^{-1}\big(1-\gamma\big)$. Inserting it into $\ell\big(A^*, x^*\big)$, the withdrawal proportion at the threshold A^* becomes $\ell\Big(A^*, x^*\Big) = 1 - H\Big(x^* - A^*\Big) = 1 - H\Big(H^{-1}\Big(1-\gamma\Big)\Big) = \gamma$. The failure threshold A^* stated in Proposition 1 follows immediately.

The fact that there are no other equilibria in non-threshold strategies follows from standard arguments (Morris and Shin, 2003; Vives, 2005). It exploits the fact that the regions above $\overline{A}$ and below $\underline{A}$ admit equilibria in strictly dominated strategies. Finally, the partial derivatives of A^* are immediate: $\frac{\partial A^*}{dF} = -(1+z\gamma)D < 0$ and $\frac{\partial A^*}{dD} = R - (1+z\gamma)F$.

A2 Proof of Proposition 2

The bank's problem is given by (4). Denote by μ the Lagrange multiplier of the investor participation constraint. The first-order conditions for an interior optimum are:

$$D: \quad \frac{\partial \mathcal{L}}{\partial D} = (R-F)G(A^*) + (\mu F + FD\gamma z)g(A^*)\frac{\partial A^*}{\partial D} = 0 \tag{A2}$$

$$F: \quad \frac{\partial \mathcal{L}}{\partial F} = (\mu - D)G(A^*) + (\mu F + FD\gamma z)g(A^*)\frac{\partial A^*}{\partial F} = 0 \tag{A3}$$

$$\mu: \quad \frac{\partial \mathcal{L}}{\partial \mu} = -r + FG(A^*) = 0 \tag{A4}$$

Combining equations (A2) and (A3) yields $\mu^* = D + \frac{(R-F)\frac{\partial A^*}{\partial F}}{\frac{\partial A^*}{\partial D}}$. Substituting μ^* into (A2) yields

$$\frac{G(A^*)}{g(A^*)} = \frac{\gamma z F D(-\frac{\partial A^*}{\partial D} + R)}{R-F} = \frac{\gamma z(1+\gamma z)F^2 D}{R-F}. \tag{A5}$$

Equations (A4) and (A5) pin down the interior critical point (F^*, D^*). An interior optimum requires $R/(1+\gamma z) < F < R$. Note that (A3) becomes strictly negative if $F \geq R$. Similarly, $\frac{\partial A^*}{\partial D} > 0$ if $F < R/(1+\gamma z)$, implying that (A2) becomes strictly positive.

The critical point (F^*, D^*) constitutes a maximum as the bordered Hessian, evaluated at (F^*, D^*), is strictly positive. To see this, note that the second derivatives are given by

$$\frac{\partial^2 \mathcal{L}}{\partial D \partial F} = (\mu - D)\frac{\partial G(A^*)/g(A^*)}{\partial A^*}\frac{\partial A^*}{\partial D} - \frac{G(A^*)}{g(A^*)} + (\mu + \gamma z D)F\frac{\partial^2 A^*}{\partial F \partial D} + \gamma z F\frac{\partial A^*}{\partial F} \quad \text{(A6)}$$

$$\frac{\partial^2 \mathcal{L}}{\partial D^2} = (R - F)\frac{\partial G(A^*)/g(A^*)}{\partial A^*}\frac{\partial A^*}{\partial D} + \gamma z F\frac{\partial A^*}{\partial D} \quad \text{(A7)}$$

$$\frac{\partial^2 \mathcal{L}}{\partial F^2} = (\mu - D)\frac{\partial G(A^*)/g(A^*)}{\partial A^*}\frac{\partial A^*}{\partial F} + (\mu + \gamma z D)\frac{\partial A^*}{\partial F} \quad \text{(A8)}$$

Since $\mu^* > D$ and $\frac{R}{1+\gamma z} < F < R$, equations (A6) to (A8) are all negative. Using that the participation constraint is $V(F, D) = FG(A^*) - r$, we further obtain:

$$\frac{\partial V}{\partial F} = G(A^*) + Fg(A^*)\frac{\partial A^*}{\partial F}, \quad \frac{\partial V}{\partial D} = Fg(A^*)\frac{\partial A^*}{\partial D}. \quad \text{(A9)}$$

Thus, $\frac{\partial V}{\partial D}$ is negative for $F > R/(1+\gamma z)$, while $\frac{\partial V}{\partial F}$ is positive when evaluated at (F^*, D^*). To see this, we can express $\frac{\partial V}{\partial F} = g(A^*)\left[\frac{G(A^*)}{g(A^*)} + F\frac{\partial A^*}{\partial F}\right]$ and substitute (A5) for G/g to get:

$$\frac{\partial V}{\partial F} = g(A^*)\left[\frac{FD(1+\gamma z)}{R - F}\left(F(1+\gamma z) - R\right)\right] > 0$$

since $F > R/(1+\gamma z)$. Hence, the bordered Hessian matrix is

$$H = \begin{vmatrix} 0 & \frac{\partial V}{\partial F} & \frac{\partial V}{\partial D} \\ \frac{\partial V}{\partial F} & \frac{\partial^2 \mathcal{L}}{\partial D^2} & \frac{\partial^2 \mathcal{L}}{\partial D \partial F} \\ \frac{\partial V}{\partial D} & \frac{\partial^2 \mathcal{L}}{\partial D \partial F} & \frac{\partial^2 \mathcal{L}}{\partial F^2} \end{vmatrix} > 0.$$

Finally, note that for $F^* \to R$, the left-hand side of (A5) converges to $+\infty$ while the right-hand side remains bounded, implying that $D^* \to 0$. For $F < R/(1+\gamma z)$, the first-order necessary conditions imply that $D^* = \omega$. Using the participation constraint, we can derive bounds on r, given by $\underset{\sim}{r} \equiv RG(0)/(1+\gamma z)$ and $\tilde{r} \equiv RG(0)$, such that for $r \in (\underset{\sim}{r}, \tilde{r})$, it must be that $F \in (R/(1+\gamma z), R)$. Clearly, if $r > \tilde{r}$, then $F > R$ and if $r < \underset{\sim}{r}$, then $F < R/(1+\gamma z)$. This completes the characterization of the bank's optimal choice.

A3 Proof of Propositions 4 – 5

Let the vector of parameters be $\mathbf{v} = (r, \psi)$. The first-order condition for borrowing is

$$\phi(D, F; \mathbf{v}) \equiv \frac{G(A^*)}{g(A^*)}(R - F) \, - \, \gamma z F D \left(-\frac{\partial A^*}{\partial D} + R \right) \, = \, 0, \tag{A10}$$

with

$$\frac{\partial \phi}{\partial D} = \frac{\partial A^*}{\partial D} \frac{d\,(G/g)}{dA^*}(R - F) + \left(\frac{\partial A^*}{\partial D} - R \right) \gamma z F < 0$$

and

$$\frac{\partial \phi}{\partial F} = \frac{\partial A^*}{\partial F} \left(\frac{d\,(G/g)}{dA^*}(R - F) + 2\gamma z F \right) - \frac{G}{g} < 0.$$

We apply the implicit function theorem (IFT) to $\phi(D^*, F^*; \mathbf{v}) = 0$ and the participation constraint $V(D^*, F^*; \mathbf{v}) = 0$. The Jacobian of this system of equations is

$$\mathbf{J} = \begin{pmatrix} \frac{\partial V(D^*, F^*; \mathbf{v})}{\partial F} & \frac{\partial V(D^*, F^*; \mathbf{v})}{\partial D} \\ \frac{\partial \phi(D^*, F^*; \mathbf{v})}{\partial F} & \frac{\partial \phi(D^*, F^*; \mathbf{v})}{\partial D} \end{pmatrix}. \tag{A11}$$

At the equilibrium point (D^*, F^*), the determinant of the Jacobian is negative, $|\mathbf{J}| < 0$, since all entries are negative except for $\frac{\partial V(D^*, F; \mathbf{v})}{\partial F} > 0$. Henceforth, we evaluate all expressions at the equilibrium but suppress the arguments of the derivatives for brevity.

Risk-free return. With $\partial V / \partial r = -1$ and $\frac{\partial \phi}{\partial r} = 0$, the IFT yields

$$\frac{dD^*}{dr} = -\frac{\begin{vmatrix} \frac{\partial V}{\partial F} & -1 \\ \frac{\partial \phi}{\partial F} & 0 \end{vmatrix}}{|\mathbf{J}|} < 0\,, \qquad \frac{dF^*}{dr} = -\frac{\begin{vmatrix} -1 & \frac{\partial V}{\partial D} \\ 0 & \frac{\partial \phi}{\partial D} \end{vmatrix}}{|\mathbf{J}|} > 0\,.$$

Liquidation value. The parameter ψ appears via z, i.e., $z \equiv \frac{1-\psi}{\psi}$ with $dz/d\psi < 0$. Since $\frac{\partial A^*}{\partial z} = -\gamma F D < 0$ and $\frac{\partial^2 A^*}{\partial D \partial z} = -\gamma F < 0$, we have $\frac{\partial V}{\partial z} = Fg\frac{\partial A^*}{\partial z} < 0$ and $\frac{\partial \phi}{\partial z} =$

$(R-F)\frac{d\left(\frac{G}{g}\right)}{dA^*}\frac{\partial A^*}{\partial z} + \gamma FD(\frac{\partial A^*}{\partial D} - R) + \gamma zF\frac{\partial A^*}{\partial z} < 0$. The IFT yields $\frac{dD^*}{dz} < 0$ and because z depends negatively on ψ, we have $\frac{dD^*}{d\psi} > 0$.

The sign of $\frac{dF^*}{dz}$ is ambiguous:

$$\frac{dF^*}{dz} = -\frac{\left(\frac{\partial V}{\partial z}\frac{\partial \phi}{\partial D} - \frac{\partial V}{\partial D}\frac{\partial \phi}{\partial z}\right)}{|\mathbf{J}|} = \frac{g(A^*)(F^*)^2(1+\gamma z)^2\frac{\partial A^*}{\partial z}\left(\frac{R(1+2\gamma z)}{(1+\gamma z)^2} - F^*\right)}{|\mathbf{J}|} \gtreqless 0 \Leftrightarrow F^* \lesseqgtr \overline{F} \tag{A12}$$

where

$$\overline{F} \equiv \frac{R(1+2\gamma z)}{(1+\gamma z)^2}. \tag{A13}$$

Observe that $\overline{F} \in \left(\frac{R}{1+\gamma z}, R\right)$. Evaluating (A5) at $\overline{F}$ yields the corresponding debt level $\overline{D}$. Evaluating the participation constraint at $(\overline{F}, \overline{D})$ yields $\overline{r} \in (\underset{\sim}{r}, \tilde{r})$ which solves $V(\overline{D}, \overline{F}; \overline{r}) = 0$. $\overline{F}$ and $\overline{D}$ are independent of r. Since F^* strictly increases in r, it follows that $\frac{dF^*}{d\gamma} < 0$ if and only if $r > \overline{r}$, where $\overline{r} > \underline{r}$. Since z depends negatively on ψ, we have $\frac{dF^*}{d\psi} \gtreqless 0 \Leftrightarrow r \gtreqless \overline{r}$.

Total effects on fragility. The total effects of $\mathbf{v}$ on fragility can be computed directly from the participation constraint $FG(A^*) - r = 0$. We have

$$\frac{dA^*}{d\psi} = -\frac{G(A^*)}{g(A^*)}\frac{\frac{dF^*}{d\psi}}{F^*} \quad \text{and} \quad \frac{dA^*}{dr} = -\frac{G(A^*)}{g(A^*)}\frac{\frac{dF^*}{dr} - \frac{1}{G(A^*)}}{F^*}.$$

It follows immediately from (A12) and the previous result that

$$\frac{dA^*}{d\psi} > 0 \quad \Leftrightarrow \quad r < \overline{r}.$$

Next, consider $\frac{dA^*}{dr}$. Substituting for dF^*/dr (using the previous results) and re-arranging, we obtain

$$\begin{aligned}
\frac{dA^*}{dr} &\propto \frac{\partial \phi}{\partial D} - \frac{|\mathbf{J}|}{G(A^*)} \;=\; -\frac{\partial A^*}{\partial F}\frac{\partial \phi}{\partial D} + \frac{\partial A^*}{\partial D}\frac{\partial \phi}{\partial F} \\
&= \frac{\partial A^*}{\partial F}\left(-\frac{\partial A^*}{\partial D} + R\right)\gamma z F^* \;+\; 2\gamma z F^* \frac{\partial A^*}{\partial F}\frac{\partial A^*}{\partial D} - \gamma z F^* D^* \frac{\partial A^*}{\partial D}\frac{-\frac{\partial A^*}{\partial D} + R}{R - F^*} \\
&= \frac{\gamma z F^*}{R - F^*}\left(-\frac{\partial A^*}{\partial D} + R\right)\left(\frac{\partial A^*}{\partial F}(R - F^*) - D^*\frac{\partial A^*}{\partial D}\right) \;+\; 2\gamma z F^* \frac{\partial A^*}{\partial D}\frac{\partial A^*}{\partial F} \\
&= \frac{\gamma z F^*(1+\gamma z)F^*}{R - F^*}\left(\frac{\partial A^*}{\partial F}(R - F^*) - D^*\frac{\partial A^*}{\partial D} - \frac{2D^*(R - F^*)}{F^*}\frac{\partial A^*}{\partial D}\right) \\
&\propto RD^*\left(F^* + (1+\gamma z)F^* - 2R\right) \;=\; (2+\gamma z)RD^*(F^* - \underline{F}),
\end{aligned}$$

where

$$\underline{F} \;\equiv\; \frac{R}{1 + \frac{\gamma z}{2}}. \tag{A14}$$

Note that the first line follows by substituting the Jacobian, the second line follows from using optimality condition (A5) to substitute for $G(A^*)/g(A^*)$ and substituting the expressions for $\partial\phi/\partial F$ and $\partial\phi/\partial D$ and the fifth line follows after substituting the expressions for $\partial A^*/\partial D$ and $\partial A^*/\partial F$ and re-arranging.

Observe that $\underline{F} \in (R/(1+\gamma z), R)$. Evaluating (A5) at $\underline{F}$ yields the corresponding debt level $\underline{D}$. Evaluating the participation constraint at $(\underline{F}, \underline{D})$, we obtain a critical value $\underline{r} \in (\underset{\sim}{r}, \tilde{r})$ that solves $V(\underline{D}, \underline{F}; \underline{r}) = 0$. Observe further that $\underline{F}$ and $\underline{D}$ are independent of r. Since F^* strictly increases in r, it follows that $F^* > \underline{F}$ and therefore $\frac{dA^*}{dr} > 0$ if and only if $r > \underline{r}$.

A4 Proof of Proposition 6

The failure threshold follows from substituting $I = D + E$ in equation 3. Moreover, the proof of Proposition 2 continues to apply except for the generalized bounds $\underset{\sim}{r}_E$ and $\tilde{r}_E$ because E enters A^* and E_2 linearly, so the first-order conditions in Appendix A2 hold. To calculate the bounds $\underset{\sim}{r}_E$ and $\tilde{r}_E$, observe that for $F = \frac{R}{1+\gamma z}$, the first-order conditions still imply

$D^* = \omega$ and thus $A^* = RE$. The investor participation constraint implies $\underaccent{\tilde}{r}_E = \frac{RG(RE)}{1+\gamma z}$. By the same logic, for $F \geq R$, the FOCs imply $D^* = 0$ and thus $\tilde{r}_E = RG(RE)$. Hence $\underaccent{\tilde}{r} < \underaccent{\tilde}{r}_E$ and $\tilde{r} < \tilde{r}_E$ because $G(\cdot)$ monotonically increases. The comparative statics of these bounds with respect to equity follow immediately.

To compute the comparative statics of (D^*, F^*, A^*) w.r.t. equity, note that E enters only via A^*, so equations (A10) and (A11) still apply. The partial derivatives with respect to E are given by $\frac{\partial \phi}{\partial E} = \frac{d(G/g)}{dA^*} \frac{\partial A^*}{\partial E}(R - F)$ and $\frac{\partial V}{\partial E} = gF \frac{\partial A^*}{\partial E}$. Hence:

$$\frac{dD^*}{dE} = -\frac{\begin{vmatrix} \frac{\partial V}{\partial F} & \frac{\partial V}{\partial E} \\ \frac{\partial \phi}{\partial F} & \frac{\partial \phi}{\partial E} \end{vmatrix}}{|\mathbf{J}|} > 0, \qquad \frac{dF^*}{dE} = -\frac{\begin{vmatrix} \frac{\partial V}{\partial E} & \frac{\partial V}{\partial D} \\ \frac{\partial \phi}{\partial E} & \frac{\partial \phi}{\partial D} \end{vmatrix}}{|\mathbf{J}|} < 0,$$

where the sign of the latter expressions follows from $\frac{\partial V}{\partial E} \frac{\partial \phi}{\partial D} - \frac{\partial V}{\partial D} \frac{\partial \phi}{\partial E} = -g(F^*)^2 \gamma z(1+\gamma z)R < 0$. Finally, as E enters the participation constraint only via A^*, we have $\frac{dA^*}{dE} = -\frac{G}{gF} \frac{dF^*}{dE} > 0$.