Should Bank Capital Regulation Be Risk Sensitive?*

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Abstract

We present a simple model to study the risk sensitivity of capital regulation. A banker funds investment with uninsured deposits and costly capital, where capital resolves a moral hazard problem in the banker's choice of risk. Investors are uninformed about investment quality, but a regulator receives a signal about it and imposes minimum capital requirements. With a perfect signal, capital requirements are risk sensitive and achieve the first-best levels of risk and intermediation: safer banks attract cheaper deposit funding and require less capital. With a noisy signal, risk-sensitive capital regulation can implement a separating equilibrium in which low-quality banks do not participate. We show that the degree of risk sensitivity is non-monotone in the precision of the signal and in investment characteristics. Without a signal, a leverage ratio still induces the efficient risk choice but leads to excessive or insufficient intermediation.

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1 Introduction

Bank capital regulation gained formal international acceptance with the Basel I framework in 1988 and remains at the core of banking regulation today. The appropriate degree of risk sensitivity of capital regulation has been a central part of the debate and reforms over the last few decades. On the one hand, increased risk sensitivity improves the efficient use of scarce capital, ensures the appropriate pricing of risk, and provides incentives for holding low-risk assets. On the other hand, these advantages of risk sensitivity may be hampered due to imperfect information about the risk of bank assets, increased complexity of regulation, and less comparability across banks (Basel Committee, 2013b). These concerns have led some to call for increased reliance on the leverage ratio, which does not vary with the risk of bank assets (Haldane, 2012). We briefly review the history of bank capital regulation in section 2.

In this paper, we propose a parsimonious model to examine the optimal risk sensitivity of bank capital regulation. For the model to shed light on this issue, it requires (i) a role for capital, (ii) heterogeneous bank assets, and (iii) a role for regulation. First, a banker can invest in a risky project using his own funds (equity capital) or borrowed funds (uninsured deposits). Capital is assumed to be costly, so the banker wishes to borrow as much as possible from competitive investors. But capital is essential for resolving a moral hazard problem of the banker's choice of risk (Holmström and Tirole, 1997). When the banker chooses the probability of default (PD), enough skin in the game provides the banker with incentives to control risk.

Second, project quality is uncertain. Its scrap value defines the loss given default (LGD) – a key determinant of Basel capital requirements. The banker observes the scrap value – his type – but investors do not. Third, a role for regulation arises from an information advantage of the regulator, perhaps due to its role as supervisor. The regulator receives a signal about the banker's type and imposes minimum capital requirements to maximize utilitarian welfare. Our rationale of bank regulation is in line with the "representation hypothesis" proposed by Dewatripont and Tirole (1994): bank depositors may not have the expertise, resources, or information to monitor banks, so the regulator represents them in collecting information about the bank.[1]

Using our simple model, we examine the risk sensitivity of capital regulation for three information structures. To relate our results to the policy and practitioner literature, we use the term "risk sensitivity." To be clear, the sensitivity of bank capital regulation in the model is with respect to the different realizations of the project's scrap value (that is, with respect to LGD), not the probability of default (PD).

In the first information structure, the regulator observes the banker's type. The regulator sets a menu of minimum capital requirements to disseminate information about investment quality to investors. Risk-sensitive capital regulation achieves the first-best allocation of both risk and intermediation levels. A banker whose project has a low scrap value (a low type) faces a high funding cost and tight capital requirements in order to control the banker's risk choice. Since capital is costly, low types choose not to participate. Among the participating banker types, those with a higher type receive cheaper funding, so a lower capital requirement suffices to ensure that the banker exerts risk management effort. One interpretation of such risk-sensitive capital requirements in practice is different risk weights of assets with different quality.

In the second information structure, banker types are unobserved. The regulator can only impose a leverage ratio invariant to banker types. In the resulting

[1] A private solution may include demandable debt with a sequential service constraint in the bank's capital structure to provide monitoring incentives for investors (Calomiris and Kahn, 1991).

pooling equilibrium, the cost of funding is risk insensitive, but the leverage ratio still provides incentives for the banker to provide an efficient level of effort. Moreover, a two-sided inefficiency arises on the extensive margin of intermediation: all banker types participate when the average type is high (excessive intermediation), while none participates when the average type is low (insufficient intermediation).

The third information structure is the intermediate case of an informative but noisy signal. Motivated by the efficient allocation, we study a separating equilibrium in which only high banker types participate. This allocation is attained with a menu of risk-sensitive capital requirements, one requirement for each possible signal. Signal precision shapes the equilibrium. For a sufficiently precise signal – so the regulator is good at differentiating types – an interior menu of capital requirements exists in which the banker is funded with both deposits and equity. In particular, the capital requirement after the regulator receives a high signal is at the efficient level, while the capital requirement after a low signal is set high enough to deter participation of low banker types. For an imprecise signal, in contrast, the regulator imposes full equity funding after a low signal, and the capital requirement after a high signal also increases above the efficient level. Deterring low banker types is costly when the signal is imprecise due to the distortion of the capital requirement after a high signal.

In our model, the risk sensitivity of bank capital regulation is the difference between the capital requirement after a low signal and after a high signal. The degree of risk sensitivity is non-monotonic in the precision of regulatory information, which arises as the separating equilibrium shifts between the two regions described above. When the signal is fairly precise, risk sensitivity decreases in signal precision. This result contrasts with some policy debate that argues for less risk sensitivity as information becomes less precise. In our model, less precise information makes the regulator worse at telling banker types apart, so a higher capital requirement after a low signal is needed to deter participation of low banker types.

When the signal is fairly imprecise, however, risk sensitivity increases in signal precision. In this case, the regulator imposes full equity funding on the banker after a low signal, so deterring participation of low banker types requires a lower capital requirement after a high signal, increasing risk sensitivity. We also show that the degree of risk sensitivity is non-monotone in investment characteristics such as the return on investment, the success probability when effort is exerted, and the scrap value of high banker types. In sum, our analysis has uncovered several non-monotonicities of the optimal risk sensitivity in a simple and linear model.

Our paper is closely related to Repullo (2013), who offers a parsimonious model to rationalize cyclical adjustment of risk-sensitive bank capital requirements. Both papers motivate bank capital with a moral hazard problem in the banker's risk choice. There are two main differences, however. First, the motivation for regulation arises from a social cost of bank failure in his model, while we consider an informational advantage of the regulator. Second, the banker type is observed throughout in his paper, while we derive rich implications from considering different information structures about the banker type.

Other theoretical work on the risk sensitivity of bank capital regulation includes Colliard (2018), who studies a bank's strategic use of internal models where a bank trades off the benefit of lower capital requirements from reporting lower risk with the cost of fines after an audit by its supervisor. Work on the choice between Basel's standardized approach (SA) and internal ratings based (IRB) approach includes Repullo and Suarez (2004), Hakenes and Schnabel (2011), and Feess and Hege (2012).

The rest of the paper is organized as follows. Section 2 briefly reviews the history of bank capital regulation under Basel rules. Section 3 describes the model. The next three sections characterize the equilibrium for different information structures: sections 4 and 5 for observed and unobserved types, respectively, and section 6 for a noisy signal. Section 7 extends our results to partial deposit insurance and discusses our timing. Section 8 concludes. All proofs are in the Appendix.

2 History of risk-sensitive capital regulation

The first capital adequacy framework developed by the Basel Committee in 1988 focused on credit risk and recognized the benefits of allowing differentiation in the regulatory capital. However, the mechanisms were relatively simple and did not allow for significant differentiation across banks. For instance, risks associated with trading activities (e.g., market risks) and operations (e.g., legal, business) were not considered. Even with regards to credit risk, assets were grouped into only five risk categories (0%, 10%, 20%, 50%, and 100%) based on negotiated levels rather than on a formal assessment of the risk of each asset. Capital was set at 8% of assets and adjusted by a loan's credit risk weight. As a result, required capital did not necessarily reflect differences in bank risk and business models (Basel Committee, 1988).

Subsequent reforms greatly increased the scope for increased risk sensitivity, starting with a modification in 1996 that opened the door to the use of banks' internal models to determine the amount of capital required for market risk. The rationale for allowing internal models was that standardized models could not capture the complexity of trading and derivatives activities (Basel Committee, 2013b). The use of internal models was extended to the measurement of credit risk in 2004 (under Basel II). A major motivation of this new risk-based capital regime was to improve incentives for banks to better measure and manage risks and to reduce the incentives for regulatory arbitrage. Basel II also strengthened disclosure requirements to support market discipline and thereby reinforce banks' incentives to manage their risks.

This risk sensitivity remains at the core of the new Basel III capital framework, with a continued role for internal models and extension of the framework to better manage a wider range of risks, such as liquidity risk, risks related to off-balance-sheet activities, and too big to fail (Gomes et al., 2017). The Basel Committee on Banking Supervision nonetheless chose to recognize the limits to risk sensitivity and, therefore, imposed a leverage requirement. The purpose of the non–risk-based leverage ratio was to act as a credible supplementary measure to the risk-based capital requirements (Basel Committee, 2014).

Even with this new backstop, interest in limiting risk sensitivity grew further, particularly related to the use of internal models. In finalizing Basel III, the Basel Committee commissioned a study about whether the framework has gone too far in terms of risk sensitivity, with too much given up in terms of simplicity and comparability (Basel Committee, 2013b). The rationale for limits to risk sensitivity were largely related to imperfect information, including the accuracy and completeness of the data, limitations of risk models, and the nature of risk itself. These concerns were reinforced by evidence about large divergences in measures of risk-weighted assets across banks (for instance, for credit risk in the banking book) unexplained by differences in underlying risks of the exposure (Basel Committee, 2013a). Hence, the Committee adjusted the risk sensitivity of capital requirements in a number of dimensions. For instance, the standardized approach to credit risk was made more risk sensitive, while internal models were subjected to capital floors (Basel Committee, 2017). Haldane (2012) argues that capital regulation should rely primarily on the leverage ratio rather than using it as a backstop, because information about risk is too noisy and taking risk into account introduces too much complexity.

3 Model

There are two dates $t = 0, 1$, a single good for consumption and investment, and three types of risk-neutral agents: a banker, a regulator, and many competitive investors. The banker has a unit endowment at $t = 0$ and prefers early over late consumption, $u^B(c_0, c_1) = z\,c_o + c_1$, where c_t is consumption at date t and $z > 1$ measures the banker's impatience. Investors have a unit endowment at $t = 0$ and are indifferent between consuming at either date, $u^I(c_0, c_1) = c_0 + c_1$, so they are willing to fund the banker as long as they receive a unit return in expectation.[2] Impatience makes it costly for the banker to use his own funds (equity) compared to borrowed funds (debt).[3]

At $t = 0$, the banker has access to a risky project that requires one unit of investment. Its return at $t = 1$ is R with endogenous probability a or a scrap value S interpreted as the quality of investment ("bank risk"). At $t = 0$, S is drawn from a cumulative distribution function G with support $[S_L, S_H]$, where $0 \leq S_L < S_H \leq 1$. The banker knows S, his type, at $t = 0$, but investors learn it only at $t = 1$.

The banker can exert (risk management) effort by monitoring investment (Holmström and Tirole, 1997). Effort takes place after the banker has learned his type and funding for investment has been raised. Specifically, the banker chooses the success probability $a \in \{a_L, a_H\}$ where $0 < a_L < a_H < 1$. The banker receives a private non-pecuniary benefit $B > 0$ from not exerting effort at $t = 1$. The effort choice and the private benefit are unobservable, so the funding contract between the banker and investors is never contingent on effort. This contract may, however, depend on the

[2] Normalizing the required return of investors to one is without loss of generality. What matters for our results is that the banker is more impatient than investors to generate gains from intermediation.

[3] Other reasons for costly equity include a tax advantage of debt or a deposit insurance subsidy. See also the discussion in Repullo and Suarez (2004).

regulator's information about the banker's type S.

The regulator maximizes utilitarian welfare. Endowed with a supervision technology, the regulator learns about investment quality. It receives a private signal x about S on which it bases minimum capital requirements k_x that stipulate the level of his own funds the banker must invest. The signal may be imprecise, where q_S is the probability that the regulator's signal is correct, conditional on S. The regulator can commit to a menu of minimum capital requirements $\{k_x\}$ at the beginning of $t = 0$.

We make several assumptions about the parameters characterizing investment. First, risk management effort is efficient irrespective of the scrap value of investment. The condition $a_H(R - S) > a_L(R - S) + B$ is toughest to hold for $S = S_H$, yielding

$$b \equiv \frac{B}{a_H - a_L} < R - S_H. \tag{1}$$

Second, the banker is impatient enough not to fund investment purely with his own funds even if its scrap value is high and risk management effort is exerted:

$$a_H R + (1 - a_H) S_H < z. \tag{2}$$

These two assumptions provide a fundamental role for bank capital. The moral hazard problem requires a minimum level of capital (skin in the game) for incentives, while the impatience of the banker imposes a maximum level of capital for participation.

The banker offers investors a risky debt contract (uninsured deposits) with face value D at $t = 1$ that may depend on the regulator's information. To simplify the analysis, we assume that investment has a positive net present value (from the perspective of investors) if and only if effort is exerted, irrespective of the scrap value:

$$a_H R + (1 - a_H) S_L > 1 > a_L R + (1 - a_L) S_H. \tag{3}$$

Table 1 summarizes the timeline.[4]

Initial date $(t = 0)$	**Final date** $(t = 1)$
0. Regulator commits to a menu $\{k_x\}$	
1. Banker observes his type S	1. Investment payoff realized
2. Banker chooses whether to participate	2. Banker repays / defaults
3. Regulator receives signal x and imposes k_x	3. Consumption
4. Banker issues debt $1 - k_x$ at face value D_x	
5. Banker invests and exerts effort a	
6. Consumption	

Table 1: Timeline of events.

4 Observed types

We start with a regulator who observes (O) the type, $x = S$ for all S. The regulator imposes a capital requirement, $k_O \geq 0$, the minimum proportion of the banker's own funds invested. This requirement reveals S to investors who require a face value D_O on debt.[5] We solve for the equilibrium capital ratios, effort choices, and funding costs.

The impatient banker chooses to invest as little possible and consumes $1 - k_O$ at $t = 0$. By limited liability, debt is risky as only the proceeds from investment can

[4]For a discussion and generalization of the timeline, see Section 7.2.

[5]For investors to infer the quality of investment from the capital requirement, the requirement must be strongly monotone in quality. This condition holds in equilibrium, as we show below.

be used to repay investors at $t = 1$. Capital is required to provide the banker with incentives for exerting effort, $a^* = a_H$. Since the project has negative net present value without effort, investors can break even only if effort is induced by a minimum capital requirement, $k \geq k_O$, so the banker has enough of his own funds invested. Specifically, it requires the expected payoff from exerting effort to exceed that of not exerting effort, $z[1-k_O] + a_H[R - D_O(1-k_O)] \geq z[1-k_O] + a_L[R - D_O(1-k_O)] + B$:

$$k \geq k_O(D_O) \equiv 1 - \frac{R-b}{D_O}. \tag{4}$$

We turn to the pricing of debt. When investment succeeds, investors are fully repaid, D_O, or else they receive an equal share of the scrap value, $\frac{S}{1-k_O}$ (partial default). For investors to break even conditional on high effort, the face value of debt is

$$D_O(k_O) \equiv \frac{1-(1-a_H)\frac{S}{1-k_O}}{a_H}. \tag{5}$$

Solving for the minimum capital requirement yields $k_O^* = 1 - a_H(R-b) - (1-a_H)S$ and the face value of debt is $D_O^* = \frac{R-b}{a_H(R-b)+(1-a_H)S}$. It is easy to verify that full repayment occurs when investment succeeds, $R \geq D_O^*(1-k_O^*)$, and partial default occurs otherwise, $S < D_O^*(1-k_O^*)$, because exerting effort is efficient (condition 6).

Given this solution to the incentive problem, we turn to the participation of the banker. He receives the information rent $R - D_O^*(1-k_O^*) = b$ upon successful investment and zero otherwise. Participation requires the return on equity, the discounted expected information rent divided by bank capital, to exceed the opportunity cost of capital, $RoE(S) \equiv a_H \frac{b}{k_O^*} \geq z$. It places an upper bound on bank capital as the return on equity from intermediation increases in project quality, $\frac{dRoE}{dS} > 0$. Throughout the paper, we assume that partial intermediation is efficient (see also Figure 1):

$$RoE(S_L) < z < RoE(S_H). \tag{6}$$

Proposition 1. ***Observable types.*** *If the regulator observes banker types, $x = S$, the first-best allocation is partial intermediation. A low-quality banker, $S < \bar{S}$, does not participate, while a high-quality banker, $S \geq \bar{S}$, attracts funding and invests. This allocation is attained by risk-sensitive minimum capital requirements and risky debt, which induces high effort, $a^* = a_H$. Capital levels and funding costs are*

$$k_O^* = 1 - a_H(R - b) - (1 - a_H)S, \qquad D_O^* = \frac{R - b}{a_H(R - b) + (1 - a_H)S}. \tag{7}$$

Better bank types receive cheaper and more debt funding: $\frac{dD_O^*}{dS} < 0$ and $\frac{d(1-k_O^*)}{dS} > 0$.

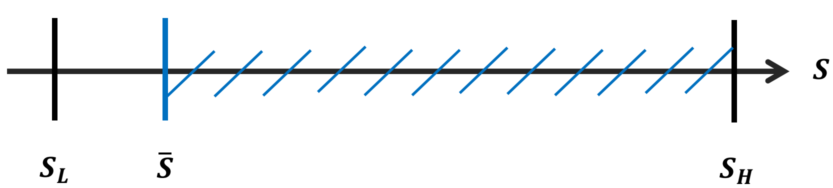

Figure 1: Observable types: parameter restriction (6) induces partial intermediation, whereby high-quality banks receive debt funding, invest, and exert effort. Low-quality banks, by contrast, receive no funding because the leverage required for banker participation exceeds the leverage allowed by investors. The cutoff $\bar{S}$ solves $RoE(\bar{S}) \equiv z$.

Capital regulation with observable types is risk sensitive and achieves first best.

Safer banks – better banker types with higher scrap value of investment – receive cheaper funding from investors. Ceteris paribus, safer banks keep more when investment is successful, resulting in better incentives to exert effort. Therefore, safer banks are allowed by the regulator to reduce the amount of their own funds invested and lever up with borrowed funds. Hence, a safer bank's return on equity is higher (and ensures banker participation for a larger range of the opportunity cost of capital).

5 Unobserved types

We turn to an uninformative signal (U) and maintain a simple debt contract and a minimum capital requirement imposed by the regulator. The requirement no longer depends on the signal – a leverage ratio – so either all banker types or none participate (pooling).[6] If intermediation occurs, risk-insensitive debt is priced according to the average type, $\mu \equiv \int S\, dG(S)$. Since all banker types default upon failure of investment, heterogeneous types do not affect individual banker participation.

Consider the effort choice conditional on banker participation. The banker exerts effort whenever it has sufficient skin in the game, $k \geq k_U(D_U) \equiv 1 - \frac{R-b}{D_U}$. If the regulator imposed a lower requirement, $k < k_U$, the banker would exert no effort and the NPV is negative, so investors cannot break even. With unobserved types, the return on equity, $RoE \equiv a_H \frac{b}{k}$, decreases in equity, $\frac{d\pi_U}{dk} < 0$. If the regulator wishes to ensure the participation of the banker, it sets $k^* = k_U$. Higher ratios have no benefit in terms of incentives but make participation of the banker harder. Given effort choice, the face value of debt, D_U, allows investors to break even given expected

[6] We abstract from potential signaling by high banker types.

banker types:

$$D_U(k_U) = \frac{1 - \frac{(1-a_H)\mu}{1-k_U}}{a_H}. \tag{8}$$

Participation of the banker again requires the return on equity form intermediation to exceed the banker's opportunity cost of capital, or a lower bound on average types,

$$\mu \geq \overline{\mu} \equiv \frac{(z-1)a_H b}{z(1-a_H)} + \frac{1-a_H R}{1-a_H}. \tag{9}$$

Proposition 2 summarizes.

Proposition 2. ***Unobservable types.*** *A leverage ratio achieves efficient effort, $a^* = a_H$, but an inefficient level of intermediation. For low average quality, $\mu < \overline{\mu}$, intermediation breaks down, while intermediation is excessive for $\mu \geq \overline{\mu}$ and all types receive funding, $1 - k_U^* = (1-a_H)\mu + a_H(R-b)$, at face value $D_U^* = \frac{R-b}{a_H(R-b)+(1-a_H)\mu}$.*

When the regulator has no superior information about the banker's type, a minimum capital requirement can only take the form of a leverage ratio. Such a minimum requirement still provides participating banker types with incentives and achieves the efficient level of effort. However, the level of intermediation is inefficiently low or high, depending on the average quality of investment.

6 Noisy information about types

We turn to the case of a noisy signal about investment quality. To simplify the analysis, we consider a bivariate distribution, $S \in \{S_L, S_H\}$, where $\Pr\{S = S_H\} \equiv p \in (0,1)$ is the probability of high quality (high scrap value or low risk). The banker privately observes his type and the regulator receives a signal $x \in \{H, L\}$. Let q_x

denote the probability of a correct signal conditional on the type, e.g., $q_H = \Pr\{x = H | S = S_H\}$. We study symmetric detection probabilities, $q_H = q_L \equiv q \in (\frac{1}{2}, 1)$.

Proposition 1 states the first-best allocation for observable types and implies a separation, whereby only high-quality bankers (with scrap value S_H) participate, attract risky debt, and invest. In this section, we study whether such a separation can be implemented by a regulator with noisy information about investment quality and characterize the resulting menu of minimum capital requirements.

Let p_x be the posterior of investors about the share of high-quality banks determined by Bayesian updating. In a separating equilibrium, investor posteriors are $p_H = p_L = 1$. Since only high-quality types participate, the relevant scrap value is S_H irrespective of the signal. Thus, the funding cost depends on the signal only via capital requirements:

$$D_x(k_x) \equiv \frac{1 - \frac{(1-a_H)S_H}{1-k_x}}{a_H}, \tag{10}$$

where we supposed banker effort $a = a_H$. It requires the incentive compatibility constraints associated with exerting effort to hold irrespective of the regulator's signal,

$$b \leq R - D_x(1 - k_x), \tag{11}$$

which simplifies to $k_x \geq \underline{k} \equiv 1 - a_H(R - b) - (1 - a_H)S_H$ for both signals.

As for banker participation where only high-quality types operate, the participation constraint for type S_H must hold, while that for S_L must fail to hold:

$$\begin{aligned} z &\leq z\big[q(1-k_H) + (1-q)(1-k_L)\big] + a_H\big[R - q(1-k_H)D_H - (1-q)(1-k_L)D_L\big], \\ z &> z\big[q(1-k_L) + (1-q)(1-k_H)\big] + a_H\big[R - q(1-k_L)D_L - (1-q)(1-k_H)D_H\big], \end{aligned}$$

where both constraints have an identical structure: the left-hand side is the impatient banker's outside option of consuming his unit endowment at $t = 0$. The first term on the right-hand side is expected consumption at $t = 0$ when raising debt $1 - k_x$. The second term is expected consumption at $t = 1$: the banker receives zero upon partial default or the proceeds from investment net of debt repayment upon success of investment (with probability a_H). The expectation is taken over the signals received by the regulator and the associated capital requirements and funding costs.

Rewriting yields a natural interpretation: the expected opportunity cost of capital for a high banker type is below the expected net present value of investment:

$$(z - 1)[qk_H + (1 - q)k_L] \leq a_H R + (1 - a_H)S_H - 1 \equiv NPV. \tag{12}$$

The respective participation constraints, PC_H and PC_L, are[7]

$$k_H \leq \bar{k}_H(k_L) \equiv \frac{\alpha}{q} - \frac{1-q}{q}k_L, \qquad k_H \geq \underline{k}_H(k_L) \equiv \frac{\alpha}{1-q} - \frac{q}{1-q}k_L, \tag{13}$$

where $\alpha \equiv \frac{NPV}{z-1} > \underline{k}$.[8] Both participation constraints intersect at $k_L = k_H = \alpha$. Finally, the feasibility constraint is $k_x \leq 1$ for both x. Figure 2 shows all constraints and the set of feasible minimum capital requirements (k_H, k_L) consistent with separation.

We turn to the objective function. In a separating equilibrium, a high-quality type participate (with probability p). Utilitarian welfare W is the sum of expected payments to the banker and investors. Up to a constant related to the mass of investors, welfare equals the payoffs when the banker consumed all of his endowment at $t = 0$ and the present value of investment minus the opportunity cost of capital

[7] We henceforth use the weak inequality for PC_L. This approach is without loss of generality if we add some amount $\epsilon > 0$ to the minimum requirement of a banker with a low signal and consider the limit $\epsilon \to 0$ to ensure non-participation of low banker types.

[8] This inequality holds if and only if $z < RoE(S_H)$, which we assumed in condition (6).

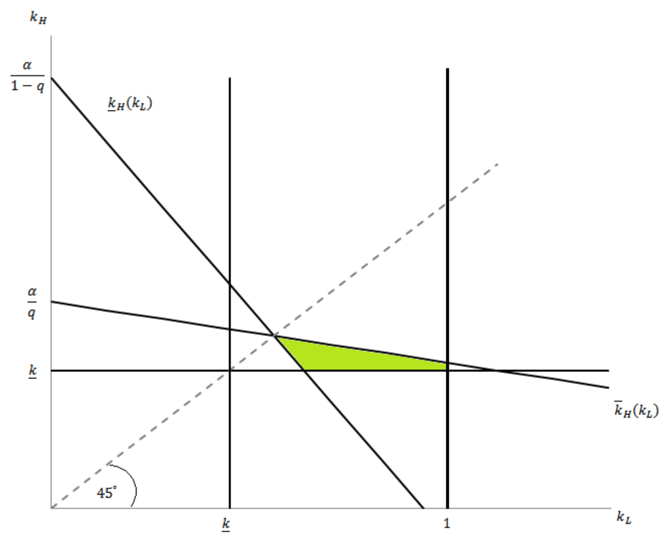

Figure 2: Constraints in separating equilibrium. The green-colored area is the set of feasible capital requirements for a slack feasibility constraint, $k_L \leq 1$.

required for incentives, so the iso-welfare curves have a slope of $-\frac{1-q}{q}$:

$$W \sim z + p\Big(PV - (z-1)\big[qk_H + (1-q)k_L\big]\Big). \tag{14}$$

The degree of risk sensitivity of bank capital requirements is $\rho \equiv k_L - k_H$, the difference in requirements after a low signal and a high signal. (High types are more likely to produce a high signal.) We state a main result on risk sensitivity.

Proposition 3. ***Noisy signal about types and risk sensitivity of capital requirements.*** *There exists a unique detection probability* $q^* \equiv \frac{z\alpha - a_H b}{1+NPV-a_H b} < 1$:

(a) if $q \geq q^*$*, the capital requirements are* $k_H^* = \underline{k}$ *and* $k_L^* = \underline{k} + \frac{\alpha - \underline{k}}{q} \leq 1$ *(slack feasibility constraint) and the degree of risk sensitivity is* $\rho^* = \frac{\alpha-\underline{k}}{q} > 0$*;*

(b) if $q < q^*$*, the capital requirements are* $k_H^* = \frac{\alpha - q}{1-q}$ *and* $k_L^* = 1$ *(binding feasibility*

constraint) and the degree of risk sensitivity is $\rho^* = \frac{1-\alpha}{1-q} > 0$.

The threshold detection probability is interior, $q^* > \frac{1}{2}$, *if the opportunity cost of capital is low enough,* $z < \bar{z} \equiv \frac{1+a_H b+NPV}{1+a_H b-NPV}$.

Proof. See Appendix A. ■

For a slack feasibility constraint, the equilibrium is given by the intersection of IC_H, the incentive compatibility constraint after a high signal, and PC_L, the participation constraint of a low-quality banker (Figure 3). The capital requirement after a high signal is independent of the detection probability, while the requirement after a low signal decreases as detection becomes better: $\frac{dk_H^*}{dq} = 0 > -\frac{\alpha-k}{q^2} = \frac{dk_L^*}{dq}$. Intuitively, better detection allows the regulator to reduce the requirement imposed to ensure the non-entry of low-quality bankers. Better detection improves welfare because a high-quality banker misidentified as low-quality has to hold less capital.

For a binding feasibility constraint, $k_L^* = 1$, the equilibrium is at its intersection with PC_L (Figure 4). To ensure the non-entry of low-quality bankers, raising the requirements after a low signal has reached its limit, so the requirement after a high signal has to raise, moving along PC_L. With better detection, this requirement can be relaxed, $\frac{dk_H^*}{dq} = -\frac{1-\alpha}{(1-q)^2} < 0 = \frac{dk_L^*}{dq}$, because of condition (6).

We turn to the comparative statics of the risk sensitivity of capital requirements.

Proposition 4. ***Comparative statics and non-monotonicities.*** *The optimal risk sensitivity of bank capital regulation* $\rho^* = k_L^* - k_H^*$

(a) is non-monotone in the detection probability q: $\frac{d\rho^*}{dq} < 0$ *if* $q > q^*$ *and* $\frac{d\rho^*}{dq} > 0$ *if* $q < q^*$;

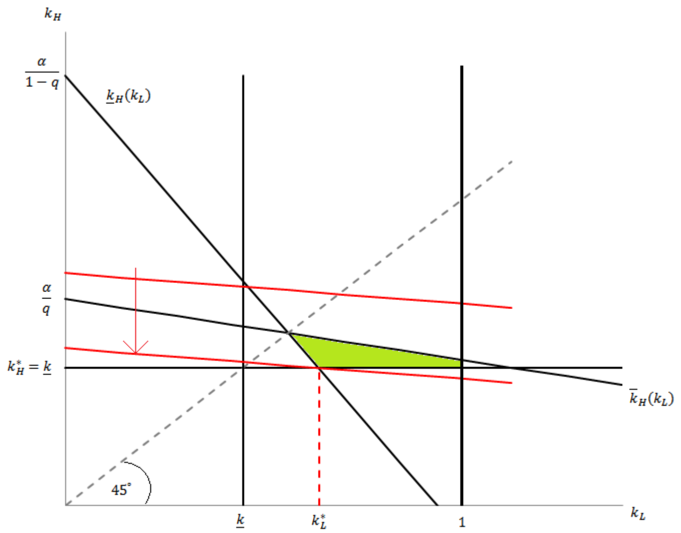

Figure 3: Separating equilibrium for a slack feasibility constraint. The red lines are iso-welfare curves.

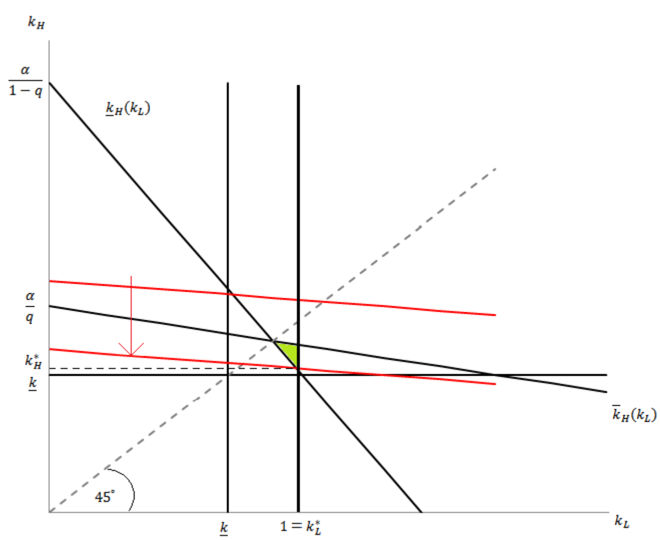

Figure 4: Separating equilibrium for a binding feasibility constraint.

(b) is non-monotone in investment characteristics $y \in \{R, a_H, S_H\}$*:* $\frac{d\rho^*}{dy} > 0$ *if* $q > q^*$ *and* $\frac{d\rho^*}{dy} < 0$ *if* $q < q^*$*;*

(c) increases in the opportunity cost of capital z: $\frac{d\rho^*}{dz} > 0$.

Proof. See Appendix B. ■

For a slack feasibility constraint, $q \geq q^*$, the equilibrium is given by the intersection of IC_H and PC_L, and we study how parameter changes affect these constraints. A higher opportunity cost of capital z makes it easier for the regulator to ensure non-participation of low types and relaxes PC_L, $\frac{dk_L^*}{dz} < 0$. Since IC_H and k_H^* are unaffected, risk sensitivity of capital requirements decreases. Better investment characteristics either reduce the cost of funding, e.g., $\frac{dD_x}{dS_H} < 0$, or directly increase what the banker keeps when investment is successful (e.g., higher R). As a result, the banker's incentives improve, allowing for a more lenient capital requirement after a high signal, $\frac{dk_H^*}{dy} < 0$. At the same time, better investment characteristics tighten PC_L because both better characteristics and a lower capital requirement after a high signal increase the incentives of a low-quality banker to participate. Hence, tighter capital requirement after a low signal is required, $\frac{dk_L^*}{dy} > 0$, which increases risk sensitivity. To recap, a better detection probability allows for a lower capital requirement after a low signal without affecting the requirement after a high signal, reducing risk sensitivity.

For a binding feasibility constraint, $q < q^*$, equilibrium is given by the intersection of PC_L and $k_L^* = 1$. Since IC_H is slack, our emphasis is on the impact of parameter changes on PC_L. Better investment characteristics again tighten the non-participation constraint of low types and require a higher capital requirement after a high signal: $\frac{dk_H^*}{dy} > 0$. Such deterrence reduces risk sensitivity of capital regulation. A better detection probability allows for a lower capital requirement after a high signal without affecting the requirement after a low signal, increasing risk sensitivity.

7 Discussion

7.1 Partial deposit insurance

Having focused on the private allocation with uninsured deposits, we next consider a deep-pocketed deposit insurance fund. If deposit insurance were complete, the pricing of deposits (and all equilibrium allocations) would be insensitive to investment quality. Instead, we focus on the more interesting case of partial deposit insurance, whereby the deposit insurance fund pays $\phi \in (0, 1 - S_H)$ upon partial default of the banker. Consistent with evidence that suggests that deposit insurance may not be actuarially fairly priced, we abstract from any deposit insurance premium for simplicity.

Proposition 5. ***Partial deposit insurance.***

(a) With observed types, higher deposit insurance coverage reduces the capital requirement, $\frac{dk_O^*}{d\phi} < 0$*, but does not affect its risk sensitivity,* $\frac{d^2 k_O^*}{dS d\phi} = 0$.

(b) With observed types, higher deposit insurance coverage reduces the average quality of investment above which all types receive funding, $\frac{d\bar{\mu}_\phi}{d\phi} < 0$.

(c) With noisy information, higher deposit insurance coverage has an asymmetric effect, raising risk sensitivity for $q > q^*$ *and reducing it for* $q < q^*$*. It also increases the threshold detection probability,* $\frac{dq^*}{d\phi} > 0$.

Proof. See Appendix C. ■

Partial deposit insurance effectively subsidizes the banker and reduces the funding cost for a given capital level. For a revealing signal, the banker keeps more when investment succeeds, providing better incentives to exert effort, so the regulator can reduce the minimum capital requirement. For a noisy signal, the capital requirement after a high signal decreases for the same reason. As a result, it is more attractive for a low-quality banker to enter with partial deposit insurance, so the minimum requirement after a low signal increases. Taking both of these changes together, risk sensitivity increases in deposit insurance for a slack feasibility constraint. Once this constraint binds, however, the non-entry of low-quality bankers is ensured via higher minimum capital requirements after a high signal, reducing the degree of risk sensitivity.

7.2 Discussion of timing and banker participation

Our analysis has focused on the timeline presented in Figure 1. Its implicit assumption is that the banker is committed to being active and cannot reverse this decision upon learning the signal communicated by the bank regulator. While we find it natural that the regulator only receives signals if the banker is active, and the empirically relevant case appears to be that banks are active over long periods of time, the purpose of this discussion is to argue for this assumption. We do so by offering an alternative sequence of events that also yields all of our results without banker commitment.

Consider a sequence in which the banker chooses whether to participate before information is revealed and, critically, debt is issued in the absence of information about the quality of investment. Then, all bankers face the same funding cost and the same level of capital. In the next step, the regulator disseminates the information x, based on which the banker renegotiates the level and pricing of debt with his creditors. If the information is favorable, the banker increases his leverage by issuing more debt, which allows the banker to consume more at $t = 0$. If the information is unfavorable, in contrast, debtors ask the banker to reduce leverage and increase his skin in the game. This demand by creditors is credible when debt is demandable and can be withdrawn upon the arrival of the regulator's information. As a result, only a high-quality banker has incentives to participate in the first place.

8 Conclusion

We provided a simple model to study the optimal risk sensitivity of bank capital regulation. Its key ingredients are a moral hazard problem in the bank's risk choice that necessitates costly bank capital for incentives, heterogeneous quality of bank investment, and a rationale for regulation based on superior information of the regulator.

When the regulator fully observes bank quality, risk-sensitive minimum capital requirements achieve the first-best allocation (in risk levels and intermediation volume). This form of regulation can be interpreted as using risk weights for assets with different losses given default. When the regulator does not observe bank quality, a risk-insensitive leverage ratio still provides incentives for the banker to control risk but leads to a two-sided inefficiency on the extensive margin: excessive intermediation for high average quality and a breakdown of intermediation for low average quality. With an informative but noisy signal, in turn, the regulator can achieve the efficient volume of intermediation via a menu of risk-sensitive minimum capital requirements. The degree of risk sensitivity of bank capital regulation is non-monotone in the precision of the signal and in investment characteristics. More precise regulatory information about bank risk can therefore reduce the degree of risk sensitivity.

Our analysis can be extended along several directions. First, we have so far allowed for two possible banker types in the case of the noisy signal. It would be interesting to study the case of more types to examine jointly the extensive margin (low types still do not participate) and the extensive margin (high types obtain cheaper funding and face lower capital requirements than medium types). Second, the role of the regulator is, effectively, to disseminate information about the banker's type to investors, for example, by direct information release, a stress test, or a strongly monotonic menu of capital requirements. However, there may also be other regulatory measures, such as closure of the bank. Third, our emphasis here is on a microprudential regulator by focusing on an individual bank without any systemic risk considerations. A macroprudential dimension could be added in an extension with multiple bankers and market-determined scrap values.

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A Proof of Proposition 3

Maximizing welfare subject to the set of constraints is a simple linear maximization problem. For a slack feasibility constraint, the equilibrium is given by the intersection of PC_L and IC_H, which yields the minimum capital requirements stated in point (a). For the feasibility constraint to indeed be slack, we require $k_L^* \leq 1$, which yields $q \geq q^*$. For a binding feasibility constraint, $k_L^* = 1$, k_H^* is obtained from a binding PC_L, resulting in the expression in point (b). The measure of risk sensitivity, $\rho = k_L^* - k_H^*$, follows for both cases. Finally, the stated expression for $\bar{z}$ arises from $q^* = \frac{1}{2}$.

B Proof of Proposition 4

We separate two cases. If $q > q^*$, then the comparative statics of the level of risk sensitivity of bank capital regulation, $\rho^* = \frac{\frac{z}{z-1}NPV - a_H b}{q}$, are

$$\begin{aligned} \frac{d\rho^*}{dq} &= -\frac{\alpha - \underline{k}}{q^2} < 0, \qquad \frac{d\rho^*}{dz} = -\frac{NPV}{q(z-1)^2} < 0, \qquad \frac{d\rho^*}{dR} = \frac{a_H z}{q(z-1)} > 0, \\ \frac{d\rho^*}{dS_H} &= \frac{(1-a_H)z}{q(z-1)} > 0, \qquad \frac{d\rho^*}{da_H} = \frac{1}{q}\left(\frac{z(R-S_H)}{z-1} - b\right) > \frac{R - S_H - b}{q} > 0. \end{aligned}$$

If $q < q^*$, then the comparative statics of $\rho^* = \frac{1-(z-1)NPV}{1-q}$ are

$$\begin{aligned} \frac{d\rho^*}{dq} &= \frac{1-\alpha}{(1-q)^2} > 0, \qquad \frac{d\rho^*}{dz} = -\frac{NPV}{1-q} < 0, \qquad \frac{d\rho^*}{dR} = -\frac{a_H(z-1)}{1-q} < 0, \\ \frac{d\rho^*}{dS_H} &= -\frac{(1-a_H)(z-1)}{1-q} < 0, \qquad \frac{d\rho^*}{da_H} = -\frac{(z-1)(R-S_H)}{1-q} < 0. \end{aligned}$$

C Proof of Proposition 5

For observed types, the capital requirement with deposit insurance is $k_O^* = 1 - a_H(R - b) - (1 - a_H)(S + \phi)$. The stated results follow immediately. For observed types, the leverage ratio is independent of investment quality by definition. Since the threshold of average types is $\bar{\mu}_\phi \equiv \bar{\mu} - \phi$, it decreases in deposit insurance coverage, ϕ.

Finally, for noisy information, the threshold of the detection probability with deposit insurance, q_ϕ^*, takes the same functional form as q^* but NPV is replaced by $NPV_\phi \equiv NPV + (1 - a_H)\phi$. Since $\frac{dq_\phi^*}{d\phi}$ is proportional to $\frac{z - a_H b}{z - 1}$, the derivative is positive. Welfare is again the sum of expected payoffs to investors and the banker. The results on the risk sensitivity follow directly from $\frac{d\alpha}{d\phi} > 0$ and $\frac{dk}{d\phi} < 0$.