Seeking Safety*

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August 23, 2018

Abstract

The scale of safe assets suggests a structural demand for a safe wealth share beyond transaction and liquidity roles. We study how investors achieve a reference wealth level by combining self-insurance and contingent liquidation of investment. Intermediaries improve upon autarky, insuring investors with poor self-insurance and limiting liquidation. However, delegation creates a conflict in states with residual risk. Demandable debt ensures safety-seeking investors can withdraw to implement a safe outcome, so private safety provision is fragile. Public debt crowds out private credit supply and investment, while deposit insurance crowds them in by reducing liquidation in residual risk states.

*We thank Jason Allen, Denis Gromb, Bengt Holmström, Charles Kahn, Simas Kucinskas, Rafael Repullo, Tanju Yorulmazer and audiences at Bonn, BU, CEPR Conference on Modelling Credit Cycles, Chicago Financial Institutions Conference 2017, Columbia, EFA 2016 (Oslo), FTG Summer School 2017, Frankfurt School, Goethe, HBS, HEC Paris, Mannheim, MIT, NY Fed, NYU, Queen's, Toronto, and Yale. These views and all remaining errors are ours. Duncan Whyte provided excellent research assistance.

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1 Introduction

Recent evidence suggests an inelastic demand for safe (i.e., nominally riskless) assets as a stable share of wealth (Gorton et al., 2012). Historical evidence points to segmented pricing and a distinct safety premium that responds to public debt supply and affects bank funding as well as the degree of maturity transformation (Krishnamurthy and Vissing-Jorgensen, 2012, 2015). This evidence has led to a focus on the role of banks as the main private providers of safety (Stein, 2012; Dang et al., 2017).[1]

A segmented market for safe assets can largely be explained by liquidity and transaction needs (Diamond and Dybvig, 1983; Gorton and Pennacchi, 1990), which predict a correlation of safe-asset demand with aggregate transaction volumes, such as GDP. Yet U.S. safe assets have risen dramatically as a share of GDP, while as a share of total wealth they have been remarkably stable (see Figure 1).[2] Along with its role in transactions and for liquidity, the historical demand for safe assets appears to reflect a structural portfolio choice by investors.

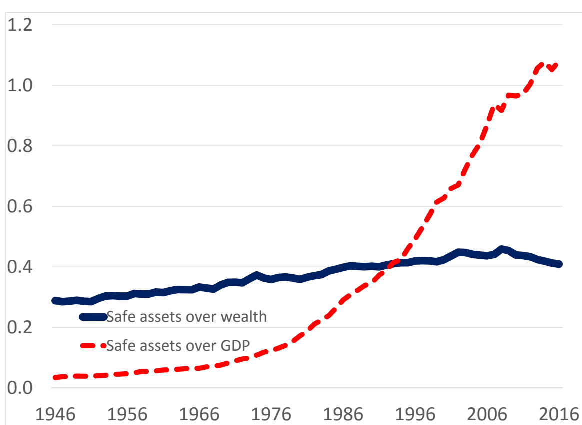

Figure 1: Safe assets as a fraction of wealth and GDP.

[1] Gorton (2017) and Caballero et al. (2017) review the safe-asset literature.

[2] Estimates of safe assets follow Gorton et al. (2012) but we exclude long-term private debt to match the model definition of safety.

We offer an interpretation with a model of structural preference for safety. Investors have a reference level of wealth in all states, similar to Stone-Geary preferences in macroeconomics and habit formation in asset pricing.[3] Investors can ensure their safety through a portfolio of self-insurance and contingent liquidation of real investment. Some self-insurance is achieved by direct control over personal assets (such as human capital) that ensures a higher minimum return in all states. Direct control protects returns in non-verifiable contingencies (Grossman and Hart, 1986). Personal returns differ across investors, reflecting personal skills or circumstances (e.g., exposure to theft or expropriation), and cannot be safely transferred contractually to others (Hart and Moore, 1994). Productive investment yields a higher expected return and its downside risk can be controlled by contingent liquidation.

In autarky, all investors choose some self-insurance and invest the rest to improve their average return. Investors with good personal returns choose to bear more investment risk, avoiding interim liquidation when the expected value of investment is positive (committed investment). Investors with poor self-insurance options instead prefer to liquidate upon a chance of loss. In contrast, the efficient benchmark maximizes expected output subject to providing safety for all investors. Avoiding self-insurance by low-return investors boosts aggregate investment and expected output, while their safety is ensured by reallocating the proceeds from contingent liquidation.

Our main result is that competitive private intermediaries implement the efficient allocation. They carve out safe and risky claims from contractible investment payoffs, issuing safe debt and a sufficient amount of equity. In the intermediation equilibrium, investors with high self-insurance returns still take care of their own safety, while others invest in a debt claim backed by sufficient bank equity. Any

[3] Habit preferences can explain a high volatility of asset returns (Campbell and Cochrane, 1999) and account for macroeconomic dynamics (King and Rebelo, 1993; Fuhrer, 2000).

residual endowment is invested in committed form, either directly or via bank equity, and equity is priced to match the expected return on direct committed investment.

For the senior debt claim to be safe, two conditions are necessary. The intermediary needs adequate loss-absorption capacity, a minimum safety capital requirement. Investors in junior claims take losses when investment is liquidated but gain a large levered payoff in high states. However, seniority in itself is insufficient to ensure safety, as insurance naturally creates a risk conflict between senior and junior claims in the residual risk state. When interim information signals a positive expected value but also a risk of loss, junior claimants prefer continuation (committed investment), while safety-seeking debt holders prefer liquidation. Since the interim state is non-verifiable, intermediaries cannot commit to a contingent liquidation plan or a shift in control rights (Aghion and Bolton, 1992; Dewatripont and Tirole, 1994).

A demandable senior claim resolves this conflict and implements the efficient outcome. The option to withdraw upon demand empowers safety-seeking investors to force partial liquidation and allows banks to credibly promise safety.[4]

Our theory has rich implications for the volume and pricing of safe debt. Higher profitability, lower opacity, and lower liquidation cost boost private supply and lead to higher safe rates (a lower safety premium) and higher volumes of intermediation and investment. Greater demand for safety, via a first-order stochastic dominance shift in the distribution of self-insurance returns, increases the volume of bank debt and investment. Under perfect competition, private safety supply adjusts to shifts in net demand at a constant safe rate, until the private insurance capacity becomes scarce. Then, adding loss-absorbing bank equity requires attracting investors with

[4] Calomiris and Gorton (1991) offer evidence that demandability ensured a safe return in U.S. banking history even before deposit insurance. Kacperczyk et al. (2017) show European banks can issue debt at a safety premium only at short-term maturity.

lower self-insurance returns, so the safe rate drops.[5] Under imperfect competition on deposits, the volume of bank funding and the safe rate always respond to shifts in net safety demand. Both implications for the safe rate's response to changes in safety demand are consistent with evidence presented in Krishnamurthy and Vissing-Jorgensen (2012).

Since private safety provision relies on withdrawals, its expansion leads to greater output volatility relative to autarky, even under scale-invariant default risk. This fragility raises the issue of the public provision of financial safety. While a government cannot create additional safety by taxing investment returns, it can reallocate it by taxing personal returns because of its superior enforcement power relative to private contracting.[6] We study public safety provision via public debt issuance and deposit insurance. Both forms directly induce a larger scale of self-insurance by high-return investors in anticipation of taxation to balance the government's budget.

Public provision of safety can complement or substitute its private provision. Government debt issued to fund a public good crowds out the private provision of safety in the form of safe bank debt, as it historically did (Krishnamurthy and Vissing-Jorgensen, 2015). Its effect on investment depends on whether the value of public investment compensates for the crowding out of private investment. In contrast, deposit insurance can complement private intermediation. It reduces the interim rate of liquidation in risky states and induces a higher safe rate and higher volumes of safe debt (crowding in). Deposit insurance is effective at boosting investment when interim asset opacity, measured by the chance of residual risk, and safety-seeking withdrawals from banks are likely.

[5] Scarce safety capacity may be caused by a sharp fall in personal income or depressed liquidation values, or when some assets are no longer deemed safe, as in Gennaioli et al. (2013).

[6] We abstract from any direct provision of safety, such as unemployment or health insurance. As this form of government intervention is redistributive, our analysis does not assess its welfare implication but simply describes its effect on investment and stability.

The model is stylized but its results are robust to milder assumptions. Self-insurance returns could be risky as long as direct control offers a minimum return above the liquidation value of investment. Self-insurance returns may also be partially contractible as in Holmström and Tirole (1998), provided their pledgeable amount is below the minimum self-insurance return. A general information and return structure implies the same interim states, with a conflict whenever the signal reveals a positive continuation value but also a chance of loss on the senior claim. Finally, a market solution can prevent costly liquidation if the private sector could create its own safe liquidity on better terms than by liquidating investment. Since liquidating personal assets or selling forward personal income is quite costly, an arbitrage strategy requires additional self-insurance and is unprofitable for infrequent residual risk.

Literature. Recent work has described safety demand as infinite risk aversion for a subset of the population (Gennaioli et al., 2013; Caballero and Farhi, 2018) or as an argument in the utility function (Stein, 2012; Krishnamurthy and Vissing-Jorgensen, 2012). Even a linear utility gain from safe assets can lead to extreme fragility for banks (Stein, 2012) as well as shadow banks (Hanson et al., 2015) in the presence of systemic risk. Our contribution to this effort is a structural preference for safety common to all investors and a general treatment of safety options for investors before financial claims are introduced. Banks funded with demandable debt emerge to offer an efficient private insurance solution that improves upon autarky.

One motive for demandable debt is that it provides liquidity. In Diamond and Dybvig (1983), it implements the efficient insurance across investors subject to liquidity shocks. Demandable debt also facilitates transactions (Stein, 2012; Krishnamurthy and Vissing-Jorgensen, 2015), avoids adverse selection in secondary markets (Gorton and Pennacchi, 1990), and discourages information production (Dang et al., 2017).

To establish safety preferences as an independent rationale, we abstract from liquidity and transactions motives, asymmetric information, a monitoring or screening role of intermediaries (Diamond, 1984; Holmström and Tirole, 1997), and idiosyncratic risk that favors pooling (Diamond, 1984; Diamond and Dybvig, 1983).

In our paper and in earlier work, demandability emerges as a solution to an agency conflict. Calomiris and Kahn (1991) show that a run commits the banker to not absconding with funds in bad states, thus preserving asset value. In Diamond and Rajan (2001), demandability is a threat that induces the banker to use relationship-specific skills to maximize value and to not renegotiate its own debt. In contrast, the conflict in our setup is over future risk choices, and it arises only in residual risk states when continuing investment maximizes expected value but endangers the safety of senior debt.

A demand for safety as an independent driver of credit supply has rich implications. Krishnamurthy and Vissing-Jorgensen (2012) document how bank credit volume historically responds to changes in government debt supply, creating lending pressure independently from real funding needs. Credit supply shocks appear to boost demand above productivity and add to volatility (Krishnamurthy and Vissing-Jorgensen, 2015; Mian et al., 2017). Safety-seeking capital inflows add to domestic risk concentration and financial fragility (Caballero and Krishnamurthy, 2008).

2 Model

There are three dates, $t = 0, 1, 2$, and a single good. Investors of unit mass are endowed with one unit at $t = 0$. They are risk-neutral once they consume a reference level S at either $t = 1$ or $t = 2$ but suffer a large disutility below this level:

$$u(c_1, c_2) = \begin{cases} c_1 + c_2 & \text{if } c_1 + c_2 \geq S \\ -\infty & \text{if } c_1 + c_2 < S, \end{cases} \tag{1}$$

where c_t is consumption at date t and investors do not prefer early consumption at $t = 1$. These preferences imply *safety seeking*, that is, investors wish to achieve an income of at least S in all states to guarantee the reference level of total consumption.

Investors can choose between two investment technologies at $t = 0$. Each investor has an individual self-insurance option with a safe return r at $t = 2$ that is heterogeneous across investors and distributed according to $F(r)$ over the support $[r_L, r_H]$. The self-insurance return is an investor's type. These returns are observable but non-contractible, since direct control makes self-insurance inaccessible to others.[7]

Productive investment is available to all investors. Its perfectly correlated and contractible return at $t = 2$ is R with probability $\gamma \in (0, 1)$ or 0.[8] Interim liquidation of investment yields $\alpha \in (0, R)$, so it is efficient when the final return is 0.

At $t = 1$, a non-verifiable signal occurs with probability $\delta \in (0, 1)$, resolving all uncertainty over the investment return at $t = 2$ (so $1-\delta$ is a measure of asset opacity). There are three interim states summarized in Figure 2. The return is certainly R in the high state H, while it is certainly 0 in the low state L. Without a signal, there is residual risk (state RR). The signal occurs independently of the investment return.

Continuation in state RR yields a higher expected return than liquidation, $\gamma R > \alpha$, so an investor who has achieved safety prefers a committed investment that is liquidated in state L only. Uncommitted investment is also liquidated in state RR.

[7] Self-insurance assets are illiquid at $t = 1$, which we relax in section 4.3.

[8] We consider a continuous return and information structure in section 4.4.

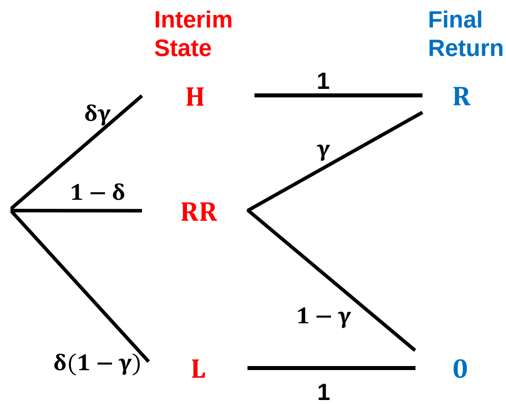

Figure 2: Payoffs and information structure of investment.

Self-insurance returns exceed the liquidation value, while the expected return on committed investment PV exceeds self-insurance returns. Finally, investors cannot achieve safety on their own simply by investing because the liquidation value is low:

$$PV \equiv \gamma R + (1-\gamma)\delta\alpha > r_H > r_L \geq S > \alpha. \tag{2}$$

Autarky. We denote by x the choice of self-insurance, so the residual $1-x$ is invested. It is never optimal to self-insure more than $\overline{x} \equiv \frac{S}{r}$, as this amount ensures safety, and investment has a higher expected return. It is also never optimal to self-insure less than $\underline{x} \equiv \frac{S-\alpha}{r-\alpha} < \overline{x}$, as even full liquidation of investment would not achieve safety in states L and RR. Each investor maximizes expected autarky output Y^A subject to achieving safety (the reference level of total consumption) in all states:

$$x^A \equiv \arg\max_{x\in[\underline{x},\overline{x}]} Y^A(x;r) = PV - x[PV-r] - (1-\delta)\left(\frac{\gamma R}{\alpha} - 1\right)(S - rx), \tag{3}$$

where expected output contains three terms: the present value of full committed investment minus the opportunity cost of self-insurance and any expected loss from liquidation in state RR. When the interim state is L (H), investors fully liquidate (do not liquidate). In state RR, investors liquidate the minimum fraction needed to achieve safety, $\ell(x) \equiv \frac{S-rx}{\alpha(1-x)} \in [0,1]$, where $\ell(\overline{x}) = 0$ and $\ell(\underline{x}) = 1$ for all r.

Since all expected returns are linear in x, the optimal portfolio choice in autarky is a corner solution. It is defined by a threshold return on self-insurance such that the marginal investor is indifferent between achieving safety by more self-insurance and committed investment or by less self-insurance and uncommitted investment:

$$r^A \equiv \frac{PV}{1 + (1-\delta)\left(\frac{\gamma R}{\alpha} - 1\right)}. \tag{4}$$

We assume this threshold is interior to its support, $r^A \in (r_L, r_H)$. This condition can be expressed in terms of bounds of the probability of revelation, $0 < \underset{\sim}{\delta} < \delta < \widetilde{\delta} < 1$.

Proposition 1 ***Autarky.*** *Investor types $r < r^A$ self-insure the amount $\underline{x}$ and fully liquidate in states RR and L. By contrast, investor types $r > r^A$ self-insure the amount $\overline{x}$ and fully liquidate in state L only. Aggregate self-insurance in autarky is $X^A = \int_{r_L}^{r^A} \underline{x}(r)dF(r) + \int_{r^A}^{r_H} \overline{x}(r)dF(r)$ and investment is $I^A \equiv 1 - X^A$.*

Efficient allocation. Our benchmark is the allocation chosen by a social planner. This allocation is constrained efficient as the proceeds from self-insurance are non-contractible. As a result, investors cannot self-insure on behalf of others, nor can the proceeds from self-insurance be redistributed by the planner. However, the planner can redistribute the proceeds from investment and thus improve upon autarky.

Appendix A derives and solves the planner's problem. With safety preferences, maximizing utilitarian welfare is maximizing aggregate expected output conditional on safety for all investors. A planner requires high-return investors to achieve their own safety via self-insurance and to invest all residual endowment. The planner commits to allocating contingent liquidation to low-return investors for their safety. This allocation avoids unproductive self-insurance, as only investors whose return is above some threshold r^E self-insure, $x^E(r) = \frac{S}{r}\mathbf{1}_{\{r \geq r^E\}}$, where $\mathbf{1}$ is the indicator function. Aggregate self-insurance is $X(r^E) \equiv \int_{r^E} \frac{S}{r} dF(r)$ and investment is $I(r^E) \equiv 1 - X(r^E)$.

The efficient threshold r^E maximizes expected output in (5) subject to an aggregate safety capacity constraint in (6):

$$\max_{r^E} Y(r^E) = PV - \int_{r^E}^{r_H} (PV - r)\frac{S}{r} dF(r) - (1-\delta)\left(\frac{\gamma R}{\alpha} - 1\right) SF(r^E) \quad (5)$$

$$\text{s.t.} \quad S\ F(r^E) \leq \alpha\ I(r^E). \quad (6)$$

The safety capacity constraint ensures that the liquidation value of total investment suffices for the safety for all low-return investors, $r \leq r^E$. We first study a slack constraint and subsequently consider parameter values that imply scarce safety capacity.

Proposition 2 ***Efficient allocation.*** *If safety capacity is abundant, $S\ F(r^A) \leq \alpha\ I(r^A)$, the unique efficient allocation is defined by a threshold $r^E = r^A$ above which investors self-insure an amount $x^E(r) = \frac{S}{r}$ and invest all the residual endowment. Safety for low-return investors is ensured by liquidating a fraction of investment, $\ell(r^E) = \frac{SF(r^E)}{\alpha I(r^E)} \leq 1$, in the residual risk state. Aggregate self-insurance is lower than in autarky, $X^E < X^A$, and investment and expected output are higher, $I^E > I^A$ and $Y^E > Y^A$.*

Proof See Appendix A.

Figure 3 shows self-insurance volumes. High-return investors ensure their own safety in both cases, while low-return investors self-insure less than in autarky.

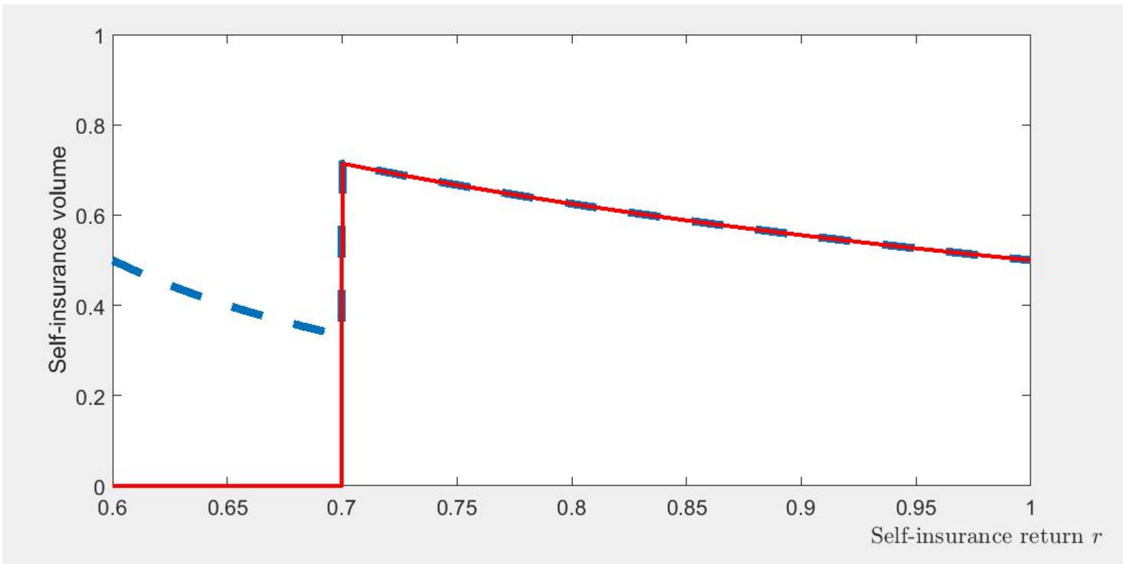

Figure 3: Self-insurance under autarky x^A (dashed, blue) and the efficient allocation x^E (solid, red). Parameters: $\alpha = 0.4$, $S = 0.5$, $r_L = 0.6$, $r_H = 1$, $\gamma = 0.5 = \delta$, $R = 4$.

3 Intermediation with demandable debt

This section shows how competitive intermediaries implement the efficient allocation. Any implementation needs to consider the participation constraints of investors. Intermediation implies a delegation of control over investment, whereby intermediary equity holders decide on interim liquidation. We will see how investor participation shapes the contractual form required for intermediaries to attract funding.

Table 1 shows the timeline. At $t = 0$, intermediaries attract funding in competitive markets. Next, investors can self-insure, and both investors and intermediaries invest. At $t = 1$, the interim state is realized and withdrawals occur (if a claim is demandable). Direct investors and bank equity holders choose whether to liquidate some investment and interim consumption occurs. At $t = 2$, all returns are realized, claims are paid out, and final consumption occurs.

$t = 0$	$t = 1$	$t = 2$
1. Intermediaries issue claims 2. Risky investment and self-insurance	1. Interim state realized (2. Withdrawals) 3. Liquidation 4. Consumption	1. Maturity of risky and safe technologies 2. Intermediaries pay claims 3. Consumption

Table 1: Timeline of events.

To offer a safe claim, intermediaries can carve out the senior portion (debt) of the verifiable investment return, backed by an adequate amount of risk-absorbing junior claims (equity). Let e be equity and d be debt with face value r^*, the safe rate. When all funding is invested, $I = d+e$, the safety of debt requires that the liquidation value of investment suffices to repay all debt claims, $\alpha I \geq r^* d$. This minimum equity level, $\underline{e} \equiv \frac{r^*-\alpha}{\alpha} d$, represents a market-imposed capital ratio required to attract funding from safety-seeking investors. That is, seniority ensures the safety of debt in state L, when debt and equity holders agree to liquidate investment. Equity receives a large payoff in state H, when debt and equity holders agree to continue investment.

However, seniority per se cannot achieve safety. A conflict between debt and equity holders arises in state RR, since continuation of investment has a higher expected return than liquidation but may produce losses. Equity holders value their payoff as part of their risky portfolio so they prefer continuation, while safety-seeking investors holding debt prefer to liquidate. Since the signal is not verifiable, there can be no contractual solution by a state-contingent liquidation rule or allocation of control, nor can there be trading of claims as there is no interim liquidity. Hence, safety-seeking investors would not invest in any long-term debt claim on the intermediary.

The solution is for intermediaries to offer demandable debt. The option to withdraw upon demand at $t = 1$ forces partial liquidation of investment and implements a safe payoff in the residual risk state. Debt is demandable at face value at $t = 1$ because there is no need for a liquidity premium. Proposition 3 summarizes.

Proposition 3 ***Portfolio choice of investors.*** *An intermediary can attract safety-seeking funding by offering demandable debt backed by enough loss-absorbing equity,* $\underline{e}$*. Low-return investors* $(r < r^*)$ *choose an amount of demandable debt,* $\frac{S}{r^*}$*, that ensures safety, while high-return investors* $(r \geq r^*)$ *self-insure. The residual endowment is invested in equity provided it earns a return of at least* PV*, otherwise it is invested directly.*

All investors are offered the same rate under perfect competition. Investors with $r \leq r^*$ (low-return investors) prefer demandable debt over self-insurance, so the aggregate volume is obtained by adding individual demand for safe debt, $\frac{S}{r^*}$, over the range of investors relying on the intermediary for safety, $d(r^*) = \frac{SF(r^*)}{r^*}$. Investors with $r > r^*$ (high-return investors) self-insure, so the aggregate volume is $X = \int_{r^*}^{r_H} \frac{S}{r} dF(r)$.

Taking the safe rate r^* as given, intermediaries choose debt and equity to maximize expected equity value V, that is, the value of investment net of debt payments and the expected liquidation losses under residual risk. All returns are linear, so intermediary size does not matter. As shown in Appendix B, each intermediary solves:

$$\max_{d,e} V(d,e) \equiv PV \left[e + d \left(1 - \frac{r^*}{r^A} \right) \right] \quad \text{s.t.} \tag{7}$$

$$V \geq PV\, e, \quad e \geq \underline{e}, \tag{8}$$

where the participation constraints in (8) are that equity holders require at least the expected return on direct committed investment, and debt holders require intermediaries to be sufficiently capitalized, so bank debt is indeed safe.

Proposition 4 ***Private provision of safety.*** *Competitive intermediaries attain efficiency by issuing safe demandable debt, $d^* = \frac{SF(r^E)}{r^E}$, at face value, $r^* = r^E$, backed by equity, $e^* \geq \underline{e}$, priced to match the return, PV. In the residual risk state, low-return investors withdraw safe debt, forcing liquidation of a fraction, $\ell(r^*) = \frac{r^* d^*}{\alpha(d^*+e^*)}$, of investment. Aggregate investment is $I^* = I(r^*) = 1 - \int_{r^*}^{r_H} \frac{S}{r} dF(r) = I^E = 1 - X^*$.*

Proof See Appendix B.

The capital structure of the intermediary efficiently transfers resources across investors. In state L, resources are transferred from high- to low-return investors via seniority. The same net transfer occurs in state RR via demandability, ensuring partial liquidation of investment. The combination of seniority and demandability ensures that low-return investors can achieve safety in all states via a claim on the intermediary. In state H, the equity claim receives a high-levered payoff, so high-return investors are compensated for losses in other states. Because of perfect competition, equity holders earn the opportunity cost, namely the return on committed investment, and low-return investors capture the entire surplus gained over autarky.

Intermediary equity needs to be at least $\underline{e}$ to make debt safe. Any additional investment is made either directly or through the intermediary via more equity, since investors who have achieved safety are indifferent. At maximum leverage, $\lambda^* \equiv \frac{d^*}{\underline{e}^*} = \frac{\alpha}{r^*-\alpha}$, investment is fully liquidated in state RR to ensure safety, $\ell^* = 1$. For lower leverage, all investment above what is required to repay debt withdrawals is continued, so the equilibrium allocation is invariant to intermediary leverage below λ^*.

Changes in the safe rate, r^*, can have an ambiguous effect on the supply of safety-seeking funding, $d(r^*)$. At the intensive margin, low-return investors need less demandable debt as its return r^* rises, while at the extensive margin some investors who used to self-insure switch to safe debt. To ensure that the supply of safety-seeking funding increases in the safe rate, we maintain throughout the following regulatory condition on the distribution of self-insurance returns, $r\, f(r) > F(r)$.

Proposition 5 *The **comparative statics** for a slack safety constraint are:*

(a) Better investment characteristics (higher R, γ, δ, α) increase the safe rate r^*, the volume of safe debt d^*, and investment I^*.

(b) Safer investment: a mean-preserving compression (lower return R or success probability γ and higher liquidation value α for unchanged PV) increases the safe rate r^*, volume of safe debt d^*, and investment I^*.

(c) A downward shift in the distribution of self-insurance returns $F(r)$, according to first-order stochastic dominance, increases the volume of safe debt d^* and investment I^*. The safe rate r^* and maximum leverage λ^* are unaffected.

(d) Maximum leverage λ^* decreases in R, δ, and γ.

Proof See Appendix C.

The first two comparative statics relate to the supply of safety. Better investment characteristics increase the opportunity cost of self-insurance. Thus, the return on safe debt r^* increases as more investors rely on the intermediary for safety, boosting total investment at the cost of self-insurance. A mean-preserving reduction in investment risk improves the safe component of investment (higher liquidation value), making uncommitted investment more attractive. As a result, both the return on and volume of safe debt increase and so does investment.

The next comparative static relates to the demand for safety. A first-order stochastic dominance downward shift in the distribution of self-insurance returns implies a greater mass of low-return investors. The effect is more demand for safe debt, which requires more intermediary equity and implies more investment.

Interestingly, maximum intermediary leverage only depends on liquidation value and the endogenous return on safe debt. It increases when the safe rate falls (e.g., for lower γ, δ, and R), as less equity is needed to insure the promised payment. In contrast, a lower liquidation value has an ambiguous effect. It reduces the equilibrium return on safe debt, but more equity is required to make debt safe.

As long as there is enough risk-absorption capacity and perfect competition, the efficient threshold for safety production and the pricing of safe debt r^* is invariant to the scale of intermediation. Intermediaries accommodate any change in safety demand via greater safe debt issuance backed by more loss-absorbing equity. However, once the safety capacity is constrained or when competition is imperfect, changes in safety demand affect the safe rate, as we show next.

When safety needs exceed the insurance capacity at the unconstrained rate, even full investment in bank equity fails to offer enough risk-absorption capacity. This scenario may arise under a high reference level relative to endowment (high S), or when many investors have poor self-insurance options ($F(r^A)$ is high). In the case of scarce safety capacity, more investors must self-insure (extensive margin), so the equilibrium safe rate is lower, $r^* = r^{SC} = r^E < r^A$.

Proposition 6 ***Scarce safety capacity.*** *If $S\ F(r^A) > \alpha\ I(r^A)$, then the return on safe debt is below its unconstrained efficient level, $r^{SC} < r^A$, and investment is below its unconstrained level, $I(r^{SC}) < I(r^A)$. To ensure safety, investment is fully liquidated in the residual risk state, $\ell(r^{SC}) = 1$. The safe rate, r^{SC}, increases in the liquidation value α. When $G(r)$ first-order stochastically dominates $F(r)$, then the safe rate is higher, $r_G^{SC} > r_F^{SC}$, and maximum intermediary leverage lower, $\lambda_G^* < \lambda_F^*$.*

Proof See Appendix D.

Several changes may cause a shift to a safety-capacity-constrained equilibrium. When liquidation values are low, the safety capacity constraint becomes more binding and depresses the safe rate. Similarly, a downward shift in the distribution of self-insurance returns (first-order stochastic dominance) reduces self-insurance capacity and leads to a lower safe rate. In our setup, in both cases maximum leverage λ^* decreases, as less equity is required to repay cheaper safe debt.

For imperfect competition, we suppose intermediaries have market power on the pricing of safe debt (e.g., Drechsler et al., 2017). For simplicity, we consider a monopolist intermediary that has to offer the same rate to all debt holders. Then, the monopolist intermediary trades off a lower debt volume against a higher margin.

A lower safe rate, $r^M \leq r^*$, requires lower risk-absorbing equity, $e \geq \underline{e} = \frac{r^M - \alpha}{\alpha} d$, and attracts an amount, $d^M = \frac{SF(r^M)}{r^M}$, of debt. As a result, safety-seeking investors keep some surplus relative to autarky. Given an investment, $I^M = d^M + e$, the intermediary needs to liquidate a fraction, $\ell^M = \frac{SF(r^M)}{\alpha I^M} \leq 1$, in the residual risk state, where we consider a slack aggregate safety constraint. The expected equity value is:

$$\Pi(r^M, e) \equiv PVe + PV\frac{SF(r^M)}{r^M}\left(1 - \frac{r^M}{r^A}\right). \tag{9}$$

Proposition 7 ***Imperfect Competition.*** *A monopolistic intermediary sets the safe rate below the efficient level, $r^M < r^A$, where the former is implicitly defined by:*

$$\frac{f(r^M)}{F(r^M)} = \frac{r^A}{r^M(r^A - r^M)}. \tag{10}$$

When $F(r)$ is first-order stochastically dominated by $G(r)$, the safe rate and safe debt volume are lower, $r^M_F < r^M_G$ and $d^M_F < d^M_G$, and maximum leverage is higher, $\lambda^*_F > \lambda^*_G$.

Proof See Appendix E.

More demand for safety according to a first-order stochastic dominance shift reduces the safe rate of the monopolistic intermediary, with similar results for an oligopoly. This pricing result implies the rankings for debt issuance and leverage.

4 Extensions

Having analyzed the private provision of safety, we turn to its public provision via public debt issuance or deposit insurance. We also study private arbitrage opportunities with liquid self-insurance returns and continuous investment returns. Throughout, we study the benchmark case of perfect competition and abundant safety capacity.

4.1 Public debt issuance

At $t = 0$, a government provides a public good, valued g, by issuing an amount of public debt $G > 0$ to be repaid at $t = 2$. For public debt to be safe, it must be backed by taxation in all states. Specifically, we consider a lump-sum tax T on all investors that allows the government to break even.[9] We use subscript G to denote quantities with public debt issuance, and r_G for the endogenous return on public debt.

Proposition 8 ***Crowding out.*** *If $G < \bar{G} \equiv \frac{SF(r^A)}{r^A(1-F(r^A))}$, public debt earns the safe rate, $r^G = r^* = r^A$, and is held by low-return investors. Its issuance crowds out the private provision of safety, $d_G < d^*$, and investment, $I_G < I^*$.*

Proof See Appendix F.

Since public and private debt are safety substitutes for low-return investors, their returns are equalized as long as public debt issuance does not capture the entire market ($G < \bar{G}$). The safe rate on bank debt remains constant, since interim liquidation of investment in the residual risk state is unaffected by public debt issuance. Since public debt issuance requires resources at $t = 0$, investment is crowded out. The implication of crowding out of private safety provision and investment replicates the result of Krishnamurthy and Vissing-Jorgensen (2015) in a context with safety preferences and is consistent with evidence documented in their paper.

4.2 Deposit insurance

Consider a deposit insurance fund that insures a fraction, $\phi \in (0, 1)$, of debt. The intermediary has enough funds when the investment return is high, and it is sufficiently capitalized to repay senior claims when it liquidates investment. Thus deposit insurance is relevant only in the residual risk state followed by a low investment return. We use subscript DI to denote quantities with deposit insurance.

[9]Taxing liquidation proceeds at $t = 1$ simply redistributes scarce safe assets, while taxing proceeds from risky investment at $t = 2$ may not produce any revenue. For public debt to be safe, the taxation of self-insurance returns at $t = 2$ is required, so we suppose that a government's statutory power allows their taxation, even though these proceeds cannot be transferred via contracts.

Proposition 9 ***Crowding in.*** *Marginal deposit insurance increases the safe rate,* $\left.\frac{dr_{DI}}{d\phi}\right|_{\phi\to 0} > 0$*, and increases the private provision of safety,* $\left.\frac{dd_{DI}}{d\phi}\right|_{\phi\to 0} > 0$*. Moreover, if the distribution* $f(r)$ *satisfies the decreasing reverse hazard rate property, then there exists a unique* $\bar{\bar{\delta}}$ *such that private investment increases,* $\left.\frac{dI_{DI}}{d\phi}\right|_{\phi\to 0} > 0$*, for all* $\delta \leq \bar{\bar{\delta}}$*.*

Proof See Appendix G.

Deposit insurance ensures a safe payment at $t = 2$, so debt holder incentives to withdraw are reduced. Thus, intermediaries have to liquidate less in the residual risk state, increasing their expected asset value. As a result of competitive pricing of debt, the safe rate increases and intermediaries issue more safe debt. We conclude that, for deposit insurance, the private and public provision of safety are complements.

Two opposing forces affect the impact of deposit insurance on investment. On the extensive margin, deposit insurance increases the safe rate and avoids some liquidation of investment in the residual risk state, which increases the ex-ante incentives for investment. On the intensive margin, by contrast, deposit insurance has to be funded and forces larger volumes of self-insurance in all states by those investors who self-insure. The net impact on investment depends on the probability of the residual risk state. If it is likely enough—that is, investment is sufficiently opaque, $\delta \leq \bar{\bar{\delta}}$—then crowding in extends from the private provision of safety to private investment.

4.3 Private arbitrage

In a liquidity insurance setup, withdrawals that force liquidation are undesirable (Diamond and Dybvig, 1983), while liquidation of investment in the residual risk state is an essential part of private safety provision. Nonetheless, for any investor who has achieved safety, liquidation in the residual risk state is wasteful because the expected value of continuation is larger. Would a private solution emerge whereby some investors choose to become arbitrageurs to benefit from preventing occasional liquidation?

Any arbitrage strategy requires safe resources at $t = 1$ to avoid liquidating investment. Therefore, we relax here the assumption that self-insurance is fully illiquid at $t = 1$. Selling personal self-insurance assets on short notice requires a steep discount (as in the case of one's personal residence) while selling forward future labor income may be difficult. Thus we assume that early liquidation of self-insurance assets yields a fraction $\beta > 0$ of its final return r.

Consider the payoff of arbitrage. With probability δ, no arbitrage opportunity arises and the investor consumes r at $t = 2$. With probability $1 - \delta$, the residual risk state arises and self-insurance is liquidated to yield βr. The arbitrageur buys the safe debt claim from safety-seeking investors and negotiates with the intermediary to avoid liquidation. The maximum gain from this strategy arises when the arbitrageur has all the bargaining power with no negotiating costs, with a payoff equal to the full return $\frac{R}{\alpha}$ with probability γ or zero otherwise. The expected return from arbitrage is $\delta r + (1-\delta)\beta r\gamma\frac{R}{\alpha}$. The highest arbitrage return possible can be achieved by investors with the best self-insurance option. Thus, private arbitrage is unprofitable if its gain is below its opportunity cost at $t = 0$, the present value of committed investment.

Proposition 10 ***No private arbitrage.*** *If $\beta < \tilde{\beta} \equiv \frac{\frac{PV}{r_H} - \delta}{1-\delta}\frac{\alpha}{\gamma R}$, private arbitrage cannot prevent liquidation of investment in the residual risk state.*

An arbitrageur cannot increase its expected profits by leverage. Investors who have already achieved safety have the same opportunity cost, PV; and safety-seeking investors do not invest in the arbitrage strategy since it sometimes produces losses. In sum, there is no arbitrage capital when runs are rare. Since liquidity in a residual risk state is limited, tradable claims also could not avoid liquidation.

4.4 Generalizing the return and signal structure

Demandability resolves a potential conflict at the interim date when new information arrives. Our result also holds under a more general, continuous information structure and a continuous return distribution. We assume next that the signal always occurs but is imprecise. Let the investment return, $R \geq 0$, follow a continuous distribution with cumulative distribution function, $J(R)$, and a positive present value at $t = 0$, so $\int_0^\infty R dJ(R) > r_H$. Also suppose that the posterior distribution after any signal at $t = 1$ precisely reveals the lowest possible value, R_L. The expected return conditional on this signal is $\mu(R_L) \equiv \int_{R_L}^\infty R dJ(R)$.

There are again three interim states. First, when the signal is sufficiently high, continuation has a higher expected value, $\mu(R_L) \geq \alpha$, and its minimum value ensures safety of debt, $R_L I \geq rd$. This case corresponds to state H in the baseline model. If the signal suggests a risk of loss and liquidation has a higher value, $\mu(R_L) < \alpha$, there is consensus on liquidation (as in state L). When the signal takes an intermediate value, there is a conflict between safety-seeking and return-seeking investors, $\alpha \leq \mu(R_L)$ and $R_L I < rd$, respectively. While continuation of investment still has a higher expected value than liquidation, R_L is not high enough to always ensure safety (as in state RR). This generalization highlights the role of an asset's minimum return in a safety context.

5 Conclusion

We offer a theory of financial intermediation and demandable debt based on investor preferences for safety, an approach consistent with recent evidence on the scale of safe assets and on their segmented pricing. Our simple setup analyzes how individuals may achieve their reference wealth even before the introduction of financial assets thanks to direct control over personal assets and real investment. Financial intermediaries funded by demandable debt can improve upon autarky by providing more efficient insurance, boosting investment and expected output. Specifically, by issuing demandable senior claims backed by adequate risk-bearing capital, banks can commit to a safe payoff. Although intermediation produces a conflict between debt and equity holders in residual risk states, the option to withdraw upon demand completes the contract and directly implements a safe payoff via partial liquidation of investment.

The private production of safe assets satisfies a safety demand in excess of public debt, but only at the cost of fragility. This is because it requires precautionary liquidation of investment in states when its expected value is positive yet continuation is too risky for safety-seeking investors. While the fragility associated with demandable debt is well appreciated in the literature, our contribution is to offer a structural model of safety demand, rather than in reduced form. We allow for a more precise interpretation of the causes of risk intolerance in a general framework where agents with the same preferences face different circumstances, choosing to either demand or supply safety.

In a safety setup, public debt naturally competes with private safe-asset production. Public debt expansion reduces intermediation (crowding out), while deposit insurance can support it (crowding in). The model implications are consistent with the recent historical evidence on crowding out, safety premia, and fragility (Krishnamurthy and Vissing-Jorgensen, 2012). Significantly, they offer some foundation to interpret credit supply drivers of economic cycles. Next to deregulation (Borio et al., 2011), safety demand is emerging as a novel component of credit cycles that leads to instability (Jorda et al., 2011; Krishnamurthy and Vissing-Jorgensen, 2015). Shifts in demand for safety may be due to habit changes, global imbalances, or demographics, and may lead to excess credit, consistent with recent evidence (Mian et al., 2017).

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A Proof of Proposition 2

We formally state and solve the planner's problem. An investor's type is its self-insurance return, $r \in [r_L, r_H]$, with probability density function $f(r)$ and cumulative distribution $F(r)$. Critically, the return on self-insurance is non-contractible, so the proceeds of self-insurance cannot be credibly promised to another investor or taxed and redistributed by the planner. Thus, redistribution across investors requires contractible investment and contingent liquidation at $t = 1$, as will become clear below.

The state of nature s_t is $s_1 \in \{H, RR, L\}$ and $s_2 \in \{H, L\}$. The contractible return on investment at $t = 2$ is $R(s_2) = R\,\mathbf{1}_{\{s_2=H\}}$, with non-contingent liquidation value α. The history of states σ_t is $\sigma_1 = s_1$ and $\sigma_2 = \{(H, H), (RR, H), (RR, L), (L, L)\}$. The probabilities of a history, $\pi(\sigma_t)$, are $\pi(H) = \gamma\delta = \pi((H, H))$, $\pi(L) = (1 - \gamma)\delta = \pi((L, L))$, $\pi(RR) = 1 - \delta$, $\pi((RR, H)) = \gamma(1 - \delta)$, and $\pi((RR, L)) = (1 - \gamma)(1 - \delta)$.

The choice variables are (1) self-insurance by investors of type r for their own safety purposes, $x(r) \in [0, 1]$; (2) the proportion of state-contingent interim liquidation of investment, $\ell(s_1) \in [0, 1]$; (3) the allocation of proceeds from investment at $t = 2$ contingent on type and the history of states, $\chi(r, \sigma_2) \geq 0$; and (4) consumption levels contingent on type and the history of states, $\{c_1(r, s_1), c_2(r, \sigma_2)\}$, with $c_t(\cdot) \geq 0$. The aggregate volumes are $X \equiv \int_{r_L}^{r_H} x(r)dF(r)$ for self-insurance and $I \equiv 1 - X$ for investment. The objective is to maximize the expected consumption of investors:

$$\max\ W \equiv \int_{r_L}^{r_H} \sum_{\sigma_2} \pi(\sigma_2)\left[c_1(r, \sigma_2[1]) + c_2(r, \sigma_2)\right] dF(r), \tag{11}$$

where $\sigma_2[1]$ is the first element of σ_2, for example RR in the history (RR, L). There are several constraints. A resource constraint at $t = 1$ states that aggregate consumption at $t = 1$ comes from liquidation of investment in each state, where we ignore weak inequalities because the objective function is strongly monotone in consumption:

$$\int_{r_L}^{r_H} c_1(r, s_1) dF(r) = \ell(s_1)\alpha I, \quad \forall s_1. \tag{12}$$

The proceeds from liquidation can be freely redistributed between investors, so only an aggregate constraint is relevant. A resource constraint at $t = 2$ states that consumption for each type and in each state comprises the proceeds from self-insurance and allocated proceeds from investment, where the allocation shares add up to one:

$$\begin{aligned} c_2(r, \sigma_2) &= rx(r) + \chi(r, \sigma_2)\left[1 - \ell(\sigma_2[1])\right] R(\sigma_2)\, I, \quad \forall (r, \sigma_2), & (13) \\ 1 &= \int_{r_L}^{r_H} \chi(r, \sigma_2) dF(r), \quad \forall \sigma_2. & (14) \end{aligned}$$

A safety constraint states that each investor achieves the reference consumption level:

$$c_1(r, \sigma_2[1]) + c_2(r, \sigma_2) \geq S, \quad \forall (r, \sigma_2). \tag{15}$$

Finally, a participation constraint of each investor states that each investor achieves at least its autarky level of expected consumption:

$$\sum_{\sigma_2} \pi(\sigma_2)\left[c_1(r, \sigma_2[1]) + c_2(r, \sigma_2)\right] \geq Y^A(r), \quad \forall r, \tag{16}$$

where $Y^A(r) \equiv Y^A(x^A(r), r)$ is the expected autarky output under the optimal autarky portfolio choice, $x^A(r)$. Thus, the aggregate expected output in autarky is

$$\begin{aligned} Y^A &\equiv \int_{r_L}^{r_H} Y^A(r) dF(r) & (17) \\ &= S + PV\left[1 - \frac{\alpha}{r^A}\int_{r_L}^{r^A} \frac{r - S}{r - \alpha} dF(r) - \int_{r_L}^{r^A} \frac{S - \alpha}{r - \alpha} dF(r) - \int_{r^A}^{r_H} \frac{S}{r} dF(r)\right]. \end{aligned}$$

We turn to solving for the efficient allocation. Ignore the participation constraints; we show below how they are satisfied. Integrating the resource constraint at $t = 2$ over investors, we obtain for aggregate consumption:

$$\int_{r_L}^{r_H} c_2(r, \sigma_2) dF(r) = \int_{r_L}^{r_H} r x(r) dF(r) + (1 - \ell(\sigma_2[1])) R(\sigma_2) I, \quad \forall \sigma_2, \tag{18}$$

since the allocation shares add up to one. Critically, this approach requires us to restrict attention to $x \in [0, \frac{S}{r}]$, since self-insurance cannot be done on behalf of others and it is never optimal to self-insure more than what already achieves safety in all states, given that the opportunity cost of self-insurance is $PV > r$.

In history (H, H), it is optimal not to liquidate, $\ell^*(H) = 0 = c_1^*(r, H)$, because $R > \alpha$ and the reference consumption level can be achieved at either date. The aggregate resource constraint at $t = 2$ implies that $\int_{r_L}^{r_H} c_2(r, (H, H)) dF(r) = \int_{r_L}^{r_H} r x(r) dF(r) + RI$. In history (L, L), it is optimal to liquidate fully, $\ell^*(L) = 1$, since $\alpha > 0$. The aggregate resource constraints imply that $\int_{r_L}^{r_H} c_1(r, L) dF(r) = \alpha I$ and $\int_{r_L}^{r_H} c_2(r, (L, L)) dF(r) = \int_{r_L}^{r_H} r x(r) dF(r)$. Consider state $s_1 = RR$. Let the liquidation proportion be $\ell \equiv \ell(RR) \in [0, 1]$, so the resource constraint at $t = 1$ implies $\int_{r_L}^{r_H} c_1(r, RR) dF(r) = \ell \alpha I$. The resource constraint at $t = 2$ depends on which return on investment is realized: $\int_{r_L}^{r_H} c_2(r, (RR, L)) dF(r) = \int_{r_L}^{r_H} r x(r) dF(r)$ or $\int_{r_L}^{r_H} c_2(r, (RR, H)) dF(r) = \int_{r_L}^{r_H} r x(r) dF(r) + RI$.

Since the proceeds from investment can be rearranged across investors at either date, it suffices to satisfy the safety constraint in the aggregate:

$$\int_{r_L}^{r_H} \left[c_1(r, \sigma_2[1]) + c_2(r, \sigma_2) \right] dF(r) \geq S, \quad \forall \sigma_2. \tag{19}$$

We study the safety constraint for each history. For (H, H), it reduces to $R + \int (r -$

$R)x(r)dF(r) \geq S$, where we used the definition of I. This inequality holds strictly for any choice of $x(r)$, since $R > r \geq S$. For (L, L), the safety constraint reduces to $\int_{r_L}^{r_H} (r - \alpha)x(r)dF(r) \geq S - \alpha$, which can be rewritten as:

$$\alpha I \geq S - \int_{r_L}^{r_H} rx(r)dF(r), \tag{20}$$

so some self-insurance by investors is required. The history (RR, L) is more restrictive than (RR, H) since fewer resources are available at $t = 2$. It can be stated as:

$$\ell \geq \frac{S - \int_{r_L}^{r_H} rx(r)dF(r)}{\alpha I}. \tag{21}$$

Since $\ell \leq 1$, this constraint is more restrictive than that in history (L, L).

Using the above resource constraints for each history, we simplify the objective function to the following reduced optimization problem subject to safety constraints:

$$\max_{\ell \in [0,1], \{x(r) \in [0, \frac{S}{r}]\}} W = PVI + \int_{r_L}^{r^H} rx(r)dF(r) - (1 - \delta)\left(\frac{\gamma R}{\alpha} - 1\right)\ell \alpha I \tag{22}$$

$$\text{s.t.} \quad I = 1 - \int_{r_L}^{r^H} x(r)dF(r), \quad \ell \geq \frac{S - \int_{r_L}^{r_H} rx(r)dF(r)}{\alpha I}. \tag{23}$$

Since $\frac{dW}{d\ell} = -(1 - \delta)\left(\frac{\gamma R}{\alpha} - 1\right)\alpha I < 0$, $\ell^* = \frac{S - \int_{r_L}^{r_H} rx(r)dF(r)}{\alpha I}$ and $W(\ell^*) = PVI + \int_{r_L}^{r^H} rx(r)dF(r) - (1 - \delta)\left(\frac{\gamma R}{\alpha} - 1\right)\left(S - \int_{r_L}^{r_H} rx(r)dF(r)\right)$, the reduced problem is:

$$\max_{\{x(r) \in [0, \frac{S}{r}]\}} W = PV - \int_{r_L}^{r^H} (PV - r)x(r)dF(r) - (1 - \delta)\left(\frac{\gamma R}{\alpha} - 1\right)\left(S - \int_{r_L}^{r_H} rx(r)dF(r)\right)$$

$$\text{s.t.} \quad S - \int_{r_L}^{r_H} rx(r)dF(r) \leq \alpha\left(1 - \int_{r_L}^{r^H} x(r)dF(r)\right). \tag{24}$$

Note that $\frac{dW}{dx(r)} = PVf(r)\left[\frac{r}{r^A} - 1\right]$. Therefore, it is optimal to use a threshold strategy, whereby investors of high types fully self-insure, $x^E(r) = \frac{S}{r}\,\mathbf{1}_{\{r \geq r^E\}}$, for some r^E to be determined. Moreover, it follows from $\frac{dW}{dx(r)}$ that $r^E \leq r^A$, depending on whether the safety constraint is slack.

Slack safety constraint. Suppose that the safety constraint is slack. Then, the efficient threshold above which full self-insurance occurs is $r^E = r^A$. For the safety constraint to be indeed slack, we require $SF(r^A) \leq \alpha I(r^A)$, where $I(r^E) \equiv 1 - \int_{r^E}^{r_H} = \frac{S}{r} dF(r) \equiv 1 - X(r^E)$. We next characterize this inequality and when it holds. In particular, we state and then prove the following lemma.

Lemma 1 *The safety capacity constraint binds at a unique threshold, $r^{SC} \in (r_L, r_H)$. Moreover, there exists a unique level, $\bar{S} > \alpha$, such that $r^A \leq r^{SC}$ if and only if $S \leq \bar{S}$.*

Consider the implicit function $H(r) \equiv SF(r) - \alpha(1 - X(r))$. We first show that H equals zero once in its domain $[r_L, r_H]$, thus defining a threshold value r^{SC} at which the safety capacity binds: $H(r^{SC}) \equiv 0$. Uniqueness follows from strong monotonicity, since $\frac{dH(r)}{dr} = Sf(r)(1 - \frac{\alpha}{r}) > 0$. Existence follows from different signs of its bounds, since $H(r_L) = -\alpha\left(1 - \int_{r_L}^{r_H} \frac{S}{\rho} dF(\rho)\right) < 0$ and $H(r_H) = S - \alpha > 0$. Thus, a unique $r^{SC} \in (r_L, r_H)$ exists. The aggregate safety constraint can be expressed as $r^E \leq r^{SC}$.

We turn to the construction of the bound $\bar{S}$. By the implicit function theorem, $\frac{dr^{SC}}{dS} < 0$ because $\frac{dH}{dS} = F(r) + \alpha \int_{r^E}^{r_H} \frac{dF(\rho)}{\rho} > 0$. This strong monotonicity ensures the uniqueness of $\bar{S}$. To ensure $\bar{S} > \alpha$ (so $r^A < r^{SC}$ at $S \to \alpha$), it suffices to show $H(r^A) < 0$ when $S \to \alpha$. This condition always holds because $\alpha F(r^A) - \alpha\left(1 - \int_{r^A}^{r_H} \frac{\alpha}{\rho} dF(\rho)\right) < 0 \Leftrightarrow \int_{r^A}^{r_H} \left(\frac{\alpha}{\rho} - 1\right) dF(\rho) < 0$. This boundary condition ensures the existence of $\bar{S}$.

Taking these results together, we have $r^E = r^A$ when the constraint is slack.

Binding safety constraint (scarce safety capacity). If $S > \bar{S}$, then the safety constraint is violated at r^A, so the efficient threshold return has to be lower. As a result, $r^E = r^{SC} < r^A$, since $\frac{dW}{dx}\big|_{r=r^{SC}} < 0$ and the planner does not wish to increase self-insurance beyond what is required to satisfy the aggregate safety constraint.

Participation constraints. Finally, we need to show that the participation constraints bind. Since the proceeds from investment can be freely rearranged across investors (the liquidation proceeds at $t = 1$ and the return at $t = 2$), it suffices to show that the expected output under the efficient allocation, Y^E, is no smaller than the expected autarky output, Y^A. For $S \leq \bar{S}$, we have:

$$Y^E \equiv W(\ell^*, x(r) = x^E(r)) = PV + S - PV\left[\frac{SF(r^A)}{r^A} + \int_{r^A}^{r_H} \frac{S}{r} dF(r)\right]. \tag{25}$$

Comparing both expected outputs, it can be shown that $Y^E > Y^A$ whenever:

$$0 < \int_{r_L}^{r^A} \frac{(r^A - r)(\alpha - S)}{r - \alpha} dF(r), \tag{26}$$

which always holds. Efficiency pins down the liquidation and self-insurance choices but the distribution of the surplus across investors is indeterminate.

B Proof of Proposition 4

We derive the expected equity value—that is, the value of investment net of debt repayment and the cost of liquidation in state RR. The bank takes the face value of safe debt r^* as given and chooses its levels of safe debt, d, and equity, e, to invest, $I = d + e$. In state H, the bank continues investment and pays r^*d to debt holders at $t = 2$ out of its proceeds, RI. In state L, the bank pays r^*d to debt holders at $t = 1$ out of the liquidation proceeds, αI. In state RR, the bank partially liquidates a fraction, $\ell(r^*) = \frac{r^*d}{\alpha I}$, to pay debt holders. Residual investment, $(1 - \ell)I$, earns a return, R, with conditional probability, γ. Adding up, the expected equity value is:

$$\begin{aligned} V(d+e) &= \gamma\delta\left[RI - r^*d\right] + (1-\gamma)\delta\left[\alpha I - r^*d\right] + (1-\delta)\left[\overbrace{\ell\alpha I - r^*d}^{=0} + (1-\ell)\gamma RI\right] \\ &= PVe + PVd\left(1 - \frac{r^*}{r^A}\right). \end{aligned} \tag{27}$$

We construct the equilibrium. Let us ignore the participation constraints for now (see below); characterizing the demand for safety-seeking funding is straightforward given the linearity of the problem. If $r^* > r^A$, then demand is zero. Conversely, if $r^* < r^A$, then demand is unbounded. Since the supply of safety-seeking funding is positive but finite, market clearing implies that $r^* = r^E = r^A$ in any equilibrium. Turning to the participation constraints, the expected equity value in equilibrium is $V^* = V(r^* = r^A) = PVe$, so the participation constraint of return-seeking investors is just satisfied. There is also an indifference between direct and indirect investment via holding bank equity. It follows that bank profits are zero. Finally, the participation constraint of safety-seeking investors requires $e^* \geq \underline{e}$, which pins down the minimum equity holdings of banks. Beyond this minimum, our model is silent on the distribution of endowment between additional bank equity and direct investment.

We turn to the distribution of surplus. By perfect competition, safety-seeking investors receive all the surplus upon autarky. Banks and equity holders break even.

Finally, we show that the aggregate allocation is feasible. By Lemma 1, sufficiently small safety needs, $S \leq \bar{S}$, imply that $r^A \leq r^{SC}$, so the aggregate safety capacity does not bind. (We consider scarce safety capacity below.)

C Proof of Proposition 5

We study the comparative statics of the safe rate, $r^* = r^A$, and aggregate variables, such as the volume of safe debt, $d^* = \frac{SF(r^*)}{r^*}$; self-insurance, $X^* = \int_{r^*}^{r_H} \frac{S}{r} dF(r)$; investment, $I^* = 1 - X^*$; and maximum bank leverage, $\lambda \equiv \frac{d^*}{\underline{e}^*} = \frac{\alpha}{r^* - \alpha}$.

First, increases in R, α, δ, or γ increase $r^* = r^A$ (see Proposition 1). By definition of X^*, self-insurance decreases and, therefore, investment I^* increases. Second, consider a reduction in the investment return and an increase in the liquidation value that keeps PV constant. So, $dPV = 0$ implies $-dR = \frac{1-\gamma}{\gamma}\delta d\alpha > 0$. Inserting this relationship in the total derivative of r^A yields:

$$dr^A = \frac{(1-\delta)PV}{\alpha\left[1 + (1-\delta)\left(\frac{\gamma R}{\alpha} - 1\right)\right]^2}\left[\delta(1-\gamma) + \frac{\gamma R}{\alpha}\right] d\alpha > 0. \tag{28}$$

Similarly, a reduction in the success probability accompanied by an increase in the liquidation value that keeps PV constant implies $-d\gamma = \frac{(1-\gamma)\delta}{R - \delta\alpha} d\alpha > 0$, which yields:

$$dr^A = \frac{(1-\delta)PV}{\alpha\left[1 + (1-\delta)\left(\frac{\gamma R}{\alpha} - 1\right)\right]^2}\left[\frac{R\delta(1-\gamma)}{R - \delta\alpha} + \frac{\gamma R}{\alpha}\right] d\alpha > 0. \tag{29}$$

Thus, in both cases, the equilibrium return, $r^* = r^A$, and investment, $I^* = I(r^*)$, increase, while self-insurance, X^*, decreases.

Third, consider a first-order stochastic dominance deterioration in the distribution of self-insurance returns, where $G(r)$ is FOSD by $F(r)$. First, $r^* = r^A$ is unaffected by this change as long as $S \leq \bar{S}$ still holds. It follows that $d^*_G = \frac{SG(r^*)}{r^*} > \frac{SF(r^*)}{r^*} = d^*$, $X^*_F = \int_{r^*}^{r_H} \frac{S}{r} dG(r) < X^*$, and therefore $I^*_G > I^*_F$. Maximum bank leverage, $\lambda^* = \frac{\alpha}{r^{SC} - \alpha}$, is unchanged, however, since minimum bank equity increases with the level of debt. Finally, its derivative with respect to $\omega \in \{R, \gamma, \delta\}$ is

$\frac{d\lambda^*}{d\omega} = \frac{\alpha}{(r^*-\alpha)^2}\frac{dr^*}{d\omega}$.

D Proof of Proposition 6

For scarce safety capacity, $S > \bar{S}$, we have $r^E = r^{SC} < r^A$ from the binding constraint, $r^E \leq r^{SC}$, since the expected output increases at this point, $\frac{dY}{dr^E}\Big|_{r^E=r^{SC}} > 0$ (see Appendix A and Lemma 1). As shown before, the competitive banking equilibrium decentralizes the efficient allocation, so $r^* = r^{SC}$. Since the safe rate is below its unconstrained level, aggregate self-insurance increases and investment decreases.

Turning to comparative statics, we use $H(r)$ with $H(r^{SC}) \equiv 0$ from Lemma 1 and $\frac{dH}{d\alpha} = -I < 0$. Thus, $\frac{dr^{SC}}{d\alpha} > 0$ from the implicit function theorem. If $G(r)$ first-order stochastically dominates $F(r)$, then $\tilde{H}(r) \equiv SG(r) - \alpha\left(1 - \int_r^{r_H}\frac{S}{\rho}dG(\rho)\right)$ with $\tilde{H}(r_G^{SC}) \equiv 0$. $\tilde{H}$ places less weight on realizations that yield S and more weight on realizations that yield $S\frac{\alpha}{r} < S$, so $\tilde{H}(r_F^{SC}) < 0$ and $r_G^{SC} > r_F^{SC}$ from the strong monotonicity of $\tilde{H}$ in r. Since $\lambda^* = \frac{\alpha}{r^{SC}-\alpha}$, the ranking of maximum leverage follows.

E Proof of Proposition 7

The monopolist banker sets r^M, d, e to maximize expected equity value, taking the impact of the safe rate on the supply of safety-seeking funding, $d(r^M) = \frac{SF(r^M)}{r^M}$, into account. Inserting $d(r^M)$ into equation (7) yields the objective function. Total differentiation with respect to the safe rate and equalizing with zero yields the first-order condition stated in equation (10), and implies $r^M \in (0, r^A)$. If $G(r)$ dominates $F(r)$ according to the reverse hazard rate, we have $\frac{g(r)}{G(r)} > \frac{f(r)}{F(r)}$ at $r = r_F^M$, so $\frac{g(r_F^M)}{G(r_F^M)} > \frac{r^A}{r_F^M(r^A - r_F^M)}$. Thus, $\frac{d\Pi}{dr}\Big|_{G(\cdot),r=r_M^F} > 0$, which implies $r_F^M < r_G^M$. A reduction in the debt level follows from the regularity condition on $F(r)$, while this ranking of safe rates again implies the ranking for maximum bank leverage.

F Proof of Proposition 8

Since both public and private debt are substitutes for achieving safety, their returns are equalized, $r^G = r^*$. If $r^G < r^*$, then the demand for public debt is zero and its market fails to clear since $G > 0$, so its return must rise. If $r^G > r^*$, then there is no demand for private debt. We assume that the supply of government debt is too small to satisfy all private demand, $G < \bar{G}$, so that the safe rate must rise.

The portfolio choice of investors is as in the main model, except that the total safety needs increase to $S + T$. Thus, if $r > r^*$, investors self-insure an amount $\frac{S+T}{r}$, so aggregate self-insurance is $X_G = \int_{r^*}^{r_H} \frac{S+T}{r} dF(r)$. If $r < r^*$, investors hold bank debt or public debt, so total bank debt is $d_G = \frac{(S+T)F(r^*)}{r^*} - G$. Since public debt issuance does not affect the interim liquidation of investment, the competitive pricing of bank debt is unaffected, $r^* = r^A$. Using the balanced budget constraint, $T = Gr^A$, we can solve for the upper bound on government debt from $d_G > 0$ and obtain the value stated in Proposition 8. Crowding out follows: $X_G > X^*$, $I_G < I^*$, and $d_G < d^*$.

We turn to welfare Y^G. We add up the value of the public good, g, the investor income from self-insurance, private debt and public debt, $S + T$, and the equilibrium value of the bank (dispersed to investors), $PV(I_G - d_G)$, which yields:

$$Y^G = Y^E + g + Gr^A \left[1 - PV \left(\int_{r^A} \frac{dF(r)}{r} + \frac{F(r^A)}{r^A} \right) \right]. \tag{30}$$

G Proof of Proposition 9

The equilibrium lump-sum tax rate is $T = \frac{\phi S F(r_{DI})}{1-\phi F(r_{DI})}$, so the volume of debt, the private provision of safety, is $d_{DI} = \frac{SF(r_{DI})}{r_{DI}(1-\phi F(r_{DI}))}$ and self-insurance is $X_{DI} = \frac{S}{1-\phi F(r_{DI})} \int_{r_{DI}}^{r_H} \frac{dF(r)}{r}$.

Partial deposit insurance reduces interim withdrawals and liquidation of a bank, $\ell = \frac{(1-\phi)rd}{\alpha I}$. Thus, the value of a bank with debt d and equity e is:

$$V_{DI} = PV(d+e) - rd\left[\delta + \gamma(1-\delta)\left(\phi + (1-\phi)\frac{R}{\alpha}\right)\right] + \gamma(1-\delta)\frac{\phi(1-\phi)d^2}{\alpha(d+e)}. \quad (31)$$

Since $\frac{dV_{DI}}{de} < PV$, it follows that $e^* = \underline{e}$ and $I^* = \frac{rd}{\alpha}$. For an interior solution of the debt level (given by the demand for debt stated above), we require $\frac{dV_{DI}}{dd} = 0$. Evaluating at e^* yields the safe rate as a function of partial deposit insurance:

$$r_{DI}(\phi) = \frac{PV - \gamma(1-\delta)\phi(1-\phi)\alpha}{\frac{PV}{r^A} - \gamma(1-\delta)\phi\left[\frac{R}{\alpha} - 1 - 2(1-\phi)\right]}. \quad (32)$$

It is easy to see that $r_{DI}(0) = r^A$. Differentiating and evaluating at $\phi = 0$ yields the impact of marginal deposit insurance on the safe rate:

$$\left.\frac{dr_{DI}}{d\phi}\right|_{\phi=0} = \frac{\gamma(1-\delta)\left[\gamma\delta(R-\alpha) + \frac{R}{\alpha}PV\right]}{PV} > 0. \quad (33)$$

As a result of this and $\frac{\partial d}{\partial \phi} = \frac{SF^2}{r(1-\phi F)^2} > 0$, a sufficient condition for an increase in the private provision of safety after marginal deposit insurance is $\frac{dd}{dr_{DI}} \geq 0$, which can be verified to hold strictly.

Finally, the marginal impact on investment is $\left.\frac{dI_{DI}}{d\phi}\right|_{\phi=0} = 1 - \left.\frac{dX_{DI}}{d\phi}\right|_{\phi=0}$, where:

$$\left.\frac{dX_{DI}}{d\phi}\right|_{\phi=0} = \frac{SF(r^A)}{(1-F(r^A))^2}\int_{r^A}^{r_H}\frac{dF(r)}{r} - \frac{Sf(r^A)}{r^A(1-F(r^A))}\left.\frac{dr_{DI}}{d\phi}\right|_{\phi=0}. \quad (34)$$

Since $\int_{r^A}^{r_H} \frac{dF(r)}{r} < \frac{1-F(r^A)}{r^A}$, a sufficient condition for $\left.\frac{dI_{DI}}{d\phi}\right|_{\phi=0} > 0$ is $\frac{F(r^A)}{f(r^A)} \leq \left.\frac{dr_{DI}}{d\phi}\right|_{\phi=0}$. If $f(r)$ has a decreasing reverse hazard rate, the left-hand side of (34) decreases in δ, while its right-hand side increases in δ. The existence of a unique $\bar{\delta}$ follows.