A Wake-Up Call Theory of Contagion

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Abstract

We offer a theory of contagion based on the information choice of investors after observing a financial crisis elsewhere. We study global coordination games of regime change in two regions with an unobserved common macro shock as the only link between regions. A crisis in the first region is a wake-up call to investors in the second region. It induces them to reassess the regional fundamental and acquire information about the macro shock. Contagion can even occur after investors learn that regions are unrelated (zero macro shock). Our results rationalize empirical evidence about contagious bank runs and currency crises after wake-up calls. We also derive new implications and discuss how these can be tested.

Keywords: wake-up call, information choice, financial crises, contagion, bank run, global games, regime change, fundamental re-assessment.

JEL Classification: G01, G21.

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1 Introduction

Understanding the causes of financial contagion is an important question in banking and international finance. For example, Forbes (2012) distinguishes four mutually non-exclusive channels of contagion: trade, banks, portfolio investors, and wake-up calls. According to the wake-up call hypothesis, a popular explanation for contagion put forward by Goldstein (1998), a financial crisis in region 1 is a wake-up call to investors in region 2 that induces them to re-assess and inquire about the fundamentals of region 2. Such a re-appraisal of risk can lead to a contagious spread of a crisis to region 2. In this paper, we offer a theory of contagion based on a re-assessment of local fundamentals and information acquisition after a wake-up call.

The empirical literature documents support for wake-up call contagion across markets and over time. Studying equity markets during the global financial crisis of 2007–09, Bekaert et al. (2014) identify wake-up calls as the key driver of contagion. Analyzing eurozone sovereign bond markets, Giordano et al. (2013) find evidence for contagion based on the wake-up call of the Greek crisis of 2009–10. For bond markets during the Asian crisis in 1997, Basu (2002) finds evidence for contagion based on the re-assessment of risks in some countries. Karas et al. (2013) find a wake-up call effect during the Russian banking panic of 2004, where deposit flows remained sensitive to bank capital, regardless of the introduction of deposit insurance. For the Panic of 1893, Ramirez and Zandbergen (2013) document contagion based on the wake-up call of newspaper reports about distant bank runs, which led to elevated deposit withdrawals in Montana. Despite this body of empirical evidence, there has been little theoretical work on the wake-up call hypothesis.

We offer a wake-up call theory of contagion based on a re-assessment of local fundamentals and information acquisition after observing a crisis elsewhere. We define contagion as an increase in the probability of a crisis in region 2 after a crisis in region 1, relative to the case of no crisis in region 1. Specifically, wake-up call contagion is the increase in the probability of a crisis in region 2 after a crisis in region 1 that prevails even if investors learn after the crisis in region 1 that regional fundamentals are uncorrelated. In our model, the wake-up call contagion effect arises from the endogenous choice of investors to acquire information about the exposure of region 2 to a common macro shock only after a wake-up call. Since tail events are never fully unexpected, the unfavorable news of a crisis elsewhere induces investors to acquire information about the risk of exposure (to a macro shock) that is otherwise considered unlikely.

While empirical work often measures the contribution of different contagion channels, our theoretical approach aims to isolate the wake-up call component in the transmission of financial crises. Therefore, we abstract from both common investors and balance sheet links. Building on global games (Carlsson and van Damme 1993), we study how information acquisition shapes the fundamental re-assessment and derive a set of new testable implications consistent with existing empirical evidence.

We develop a global coordination game of regime change with incomplete information about the fundamental (Morris and Shin 2003). In contrast to the standard game, our model has two regions that move sequentially and whose only link is the exposure to an initially unobserved common macro shock. A financial crisis occurs in a given region if sufficiently many investors act against the regime (attack a currency peg, withdraw funds from a bank, or refuse to roll over debt). Investors in region 1 decide whether to act, which determines the outcome in region 1. Afterwards, investors in region 2 observe the endogenous public signal of whether a regime change occurred in region 1 and update their beliefs about the macro shock. Subsequently, investors decide whether to learn the macro shock at a cost and next whether to act.

If crises are rare events and the macro shock is negatively skewed, investors in region 2 have a higher incentive to acquire information after the wake-up call of observing a crisis in region 1 (Lemma 2). Intuitively, the negative skewness creates an asymmetry that makes it more valuable to acquire information after the rare event of observing a crisis. For an intermediate range of information costs, investors learn the macro shock if and only if a crisis occurred in region 1 (Proposition 1). This differential information choice is at the core of the fundamental re-assessment and shapes contagion. It arises since investors face an elevated risk of a strongly negative macro shock after a wake-up call, while the negative macro shock is less likely after no crisis. The value of information is higher after a crisis in region 1, since investors in region 2 benefit more from understanding whether regional fundamentals are linked. Specifically, an investor's benefit of tailoring its attack rule to the realized macro shock is higher after a crisis in region 1 than after no crisis.

We show that contagion can occur even if all investors learn that the macro shock is zero and regional fundamentals are unrelated (Proposition 2). That is, the probability of a crisis in region 2 after a crisis in region 1 and learning that region 2 has no exposure to region 1 is higher than the probability of a crisis in region 2 after no crisis in region 1. This result consists of two parts: endogenous information choice of investors and contagion. Endogenous information is critical for the contagion result that, in turn, is driven by Bayesian updating about the macro shock. Observing a crisis in region 1 and learning about a zero macro shock means that the crisis is unrelated to the fundamental in region 2. In contrast, absent a crisis in region 1, investors in region 2 choose not to acquire information and form a more optimistic belief about the macro shock. As a result, the probability of a crisis in region 2 is lower after no crisis in region 1.

A key assumption for the result on the differential information choice of investors is the negatively skewed distribution of the macro shock. There is an extensive empirical literature on the negative skewness of important macroeconomic variables, including GDP growth, individual stock returns, and the aggregate stock market. The negative skewness plays a prominent role in the context of financial liberalization in developing countries, but also in the literature on asymmetric business cycles in advanced economies.[1] To the extent that the negative skewness of the macro shock applies, the wake-up call theory of contagion may be informative for a range of economic applications. For currency crises, speculators observe a currency attack and are uncertain about the magnitude of trade or financial links or institutional similarity.[2] For rollover risk and bank

[1]Campbell and Hentschel (1992) and Bae et al. (2007) study the sources of the negative skewness of stock returns. An extensive empirical and theoretical literature on asymmetric business cycles studies the occurrence of sharp recessions and slow booms (for example, Neftçi 1984). Theoretical explanations for the negative skewness of output and total factor productivity include Acemoglu and Scott (1997), Veldkamp (2005), and Jovanovic (2006). For developing countries, Rancière et al. (2003) find that financially liberalized countries exhibit a negatively skewed growth of both GDP and credit. Rancière et al. (2008) argue that a negative skewness captures systemic risk and is therefore a systemic component.

[2]See Morris and Shin (1998) and Corsetti et al. (2004) for a one-regional global game that builds on the earlier works of Krugman (1979), Flood and Garber (1984), and Obstfeld (1986). For trade links, financial links, and institutional similarities see Glick and Rose (1999), Van Rijckeghem and Weder (2001, 2003), and Dasgupta et al. (2011).

runs, wholesale investors observe a run elsewhere and are uncertain about interbank exposures.[3] For sovereign debt crises, bond holders observe a sovereign default elsewhere and are uncertain about the macroeconomic links, the commitment of the international lender of last resort, or the resources of multilateral bail-out funds.[4] For political regime change, activists observe a revolution, for example during the Arab spring, and are uncertain about the impact on their government's ability to stay in power.[5]

We derive four additional implications and discuss how these can be tested. First, information acquisition can amplify volatility, when measured as the increase in the dispersion of probabilities of a crisis in region 2 conditional on the realized macro shock. Second, the extent of information acquisition about the exposure to aggregate or market-wide shocks is higher after observing a financial crisis elsewhere than after observing no crisis. This prediction is consistent with empirical evidence in Vlastakis and Markellos (2012). In a study on the U.S. stock market, they document a positive association between the demand for information on the market level, proxied by internet search intensity (Da et al. 2011) and may be interpreted as information acquisition about the common component of stock returns, with measures of volatility.

Third, the extent of information acquisition about the exposure to aggregate or market-wide shocks is higher for more negatively skewed fundamentals. This prediction can be tested by separating segments with varying degrees of skewness or downside risk. Fourth, the extent of information acquisition about the exposure to aggregate or market-wide shocks increases if public disclosure of information on the idiosyncratic level is improved, which can be interpreted as an increase in market disclosure standards, the precision of information provided by rating agencies, or as an increase in the transparency of bank stress tests. Flannery et al. (2015) evaluate the information of the Federal Reserve stress tests. Consistent with our prediction, they find no evidence that stress test disclosures have reduced the production of information by analysts, with analyst coverage and forecast accuracy somewhat increasing.

[3]See Rochet and Vives (2004) and Goldstein and Pauzner (2005) for one-region games building on Diamond and Dybvig (1983).

[4]See Corsetti et al. (2006). See Drazen (1999) for membership contagion.

[5]For a one-regional global game of political regime change with endogenous information manipulation or dissemination, see Edmond (2013) and Shadmehr and Bernhardt (2015), respectively.

Other theories of contagion have been proposed. Regarding balance sheet links, see Allen and Gale (2000) and Dasgupta (2004) for interbank links and Kiyotaki and Moore (2002) for balance-sheet contagion. For a common discount factor channel, see Ammer and Mei (1996) and Kodres and Pritsker (2002). Regarding a common investor base, see Goldstein and Pauzner (2004) and Cole et al. (2016) for wealth effects, Pavlova and Rigobon (2008) for portfolio constraints, Taketa (2004) and Oh (2013) for learning about other investors. In terms of ex-post exposures, see Basu (1998) for a common risk factor, and Acharya and Yorulmazer (2008), Manz (2010) and Allen et al. (2012) for asset commonality among banks and information contagion. To distinguish our theory of contagion after wake-up calls more clearly from the existing literature, we nest a version of the standard information contagion channel as a special case.[6]

Calvo and Mendoza (2000) and Mondria and Quintana-Domeque (2013) also have endogenous information. Contagion arises in Calvo and Mendoza since globalization shifts the incentives of risk-averse investors from costly information acquisition to imitation and herding. In Mondria and Quintana-Domeque, the contagion mechanism is based on the reallocation of limited attention by risk-averse investors, where a higher relative attention allocated to one market induces a higher price volatility in another market. In contrast, we highlight a complementary channel where a wake-up call induces information acquisition about a macro shock and contagion without common investors, risk-aversion, or information processing constraints. While alternative modeling approaches are available,[7] we regard the global games approach with endogenous information as an effective tool to address the issue that also allows us to derive novel testable implications consistent with existing empirical evidence.

Our modeling approach is closest to the literature on information choice in global coordination games initiated by Hellwig and Veldkamp (2009). They show that the information choices of investors inherit the strategic motive of an underlying beauty contest, which can result in multiple equilibria. Our game of regime change with complementarity in actions also yields strategic complementarity in information choices. While multiple equilibria exist, a sufficiently negatively

[6]See Chen (1999) for a model with information contagion and uninformed junior claimants, Li and Ma (2016) for or a model on uninformed asset buyers and Chen and Suen (2013) for a model of information contagion in the context of model uncertainty.

[7]An alternative could be to build on behavioral aspects (e.g., Caplin and Leahy (1994)), or the rational inattention literature.

skewed macro shock ensures a uniqueness for an intermediate range of information costs. In contrast to the acquisition of publicly available information, Szkup and Trevino (2015) examine private information acquisition in global coordination games of regime change.

The paper proceeds as follows. We describe the model in section 2. We solve for its equilibrium in section 3 and describe our contagion results. Section 4 offers testable implications. Section 5 discusses some extensions and the robustness of our results. Finally, section 6 concludes. All proofs are in the Appendix.

2 Model

We study global coordination games of regime change played sequentially in two regions $t = 1,2$. Each region has a different unit continuum of risk-neutral investors $i \in [0,1]$. Investors in region $t = 1$ move first, followed by investors in region $t = 2$.

Attack decision. In each region, investors simultaneously decide whether to attack the regime, $a_{it} = 1$, or not, $a_{it} = 0$. The outcome of the attack depends on the aggregate attack size, $A_t \equiv \int_0^1 a_{it}\, di$, and a regional fundamental $\Theta_t \in \mathbb{R}$ that measures the strength of the regime. A regime change occurs if enough investors attack, $A_t > \Theta_t$. Following Vives (2005), an attacking investor in region t receives a benefit $b_t > 0$ if a regime change occurs and otherwise incurs a loss $\ell_t > 0$, where $\gamma_t \equiv \frac{\ell_t}{b_t+\ell_t} \in (0,1)$ is the relative cost of failure:

$$u(a_{it} = 1, A_t, \Theta_t) = b_t \;\; \mathbf{1}_{\{A_t > \Theta_t\}} - \ell_t \;\; \mathbf{1}_{\{A_t \leq \Theta_t\}}. \tag{1}$$

The payoff from not attacking is normalized to zero, so the relative payoff from attacking increases in the attack size A_t (global strategic complementarity in attack decisions) and decreases in the fundamental Θ_t.

A regime change is a currency attack, bank run, or debt crisis. The fundamental is interpreted as the ability of a monetary authority to defend its currency (Morris and Shin 1998; Corsetti et al. 2004), the measure of investment profitability (Rochet and Vives 2004; Goldstein and Pauzner

2005; Corsetti et al. 2006), or the resources or willingness of a debtor to repay. Investors are interpreted as currency speculators, as retail or wholesale bank creditors who withdraw funds, or as debt holders who refuse to roll over.

Macro shock. Each regional fundamental Θ_t comprises a regional component θ_t and a common component m. This common macro shock is the only link between regions:

$$\Theta_t = \theta_t + m, \tag{2}$$

where each θ_t follows an independent normal distribution with mean $\mu_t \equiv \mu \in (-\infty, \infty)$ and precision $\alpha_t \equiv \alpha \in (0, \infty)$ that is independent of the macro shock. Depending on its realization, the macro shock induces a positive correlation between regional fundamentals Θ_1 and Θ_2. Specifically, region 2 is exposed to region 1 if $m \neq 0$. The macro shock takes one of three values:

$$m = \begin{cases} \Delta & \quad p \\ -s\Delta \quad \text{w.p.} & \quad q \\ 0 & \quad 1-p-q, \end{cases} \tag{3}$$

where $p \in [0,1]$, $q \in [0, 1-p]$, $\Delta > 0$, and $s > 0$. We impose $q = \frac{p}{s}$ to ensure an unbiased macro shock. Its variance is affected by Δ and its skewness by s, where $s > 1$ corresponds to negative skewness. The macro shock is initially unobserved, motivated by our applications to financial crises. For currency attacks or sovereign debt crises, this uncertainty about the macro shock reflects the unknown relevance of certain institutional similarities or of real or financial linkages across debtors. For bank runs, it reflects the uncertainty about bank portfolios and interbank exposures.

Incomplete information. Following Carlsson and van Damme (1993), there is incomplete information about the fundamental. Each investor receives a noisy private signal x_{it} before deciding whether to attack (Morris and Shin 2003):

$$x_{it} \equiv \Theta_t + \varepsilon_{it}. \tag{4}$$

Idiosyncratic noise ε_{it} is identically and independently normally distributed across investors with zero mean and precision $\beta \in (0, \infty)$. Each noise term is independent of the macro shock and the regional component.

Information acquisition. An information stage precedes the coordination stage in region 2, as summarized in the timeline in Table 1. First, investors in region 2 observe whether there is a crisis in region 1. Second, investors in region 2 can acquire costly information about the macro shock.[8] Investors simultaneously decide whether to purchase a perfectly revealing signal about the macro shock at cost $c > 0$. In terms of wholesale investors or currency speculators, costly information acquisition could be the access to Bloomberg and Datastream terminals or hiring analysts who assess publicly available data.

Date 1	**Date 2**
1. Macro shock m and regional component θ_1 realized but unobserved	1. Regional component θ_2 realized but unobserved
	Information stage in region 2
Coordination stage in region 1	
	2. Investors choose whether to acquire information about macro shock m at cost $c > 0$
2. Investors receive private information x_{i1} and choose whether to attack the regime, $a_{i1} \in \{0, 1\}$	
	Coordination stage in region 2
3. Payoffs to investors in region 1	
	3. Investors receive private information x_{i2} and choose whether to attack the regime, $a_{i2} \in \{0, 1\}$
4. Outcome of regime publicly observed	
	4. Payoffs to investors in region 2

Table 1: Timeline of events.

[8] We abstract from information acquisition in region 1 without loss of generality (see also section 5).

3 Equilibrium

3.1 Region 1

We first consider the equilibrium in region 1. A Bayesian equilibrium is an attack decision a_{i1} for each investor i and an aggregate attack size A_1 that satisfy both individual optimality for all investors, $a^*_{i1} = \arg\max_{a_{i1}\in\{0,1\}} \mathbb{E}[u(a_{i1}, A_1, \Theta_1)|x_{i1}]$, and aggregation, $A^*_1 = \int_0^1 a^*_{i1} di$. Let $n_1 \in [0,1]$ be the proportion of investors in region 1 informed about the macro shock. If all investors are informed, $n_1 = 1$, the analysis is standard (see, e.g., Morris and Shin (2003), Morris and Shin (2004)). If some investors are uninformed, $n_1 < 1$, the analysis is non-standard and requires the use of mixture distributions. In the main text, we focus on the case of uninformed investors, $n_1 = 0$, but derive the general case, $0 < n_1 < 1$, in Appendix A.6.

Lemma 1 ***Equilibrium in region 1.*** *Suppose $n_1 = 0$. If private information is sufficiently precise, there exists a unique monotone Bayesian equilibrium. Each investor attacks when the private signal is below a signal threshold, $x_{i1} < x^*_1$. A crisis occurs when the fundamental is below a fundamental threshold, $\Theta_1 < \Theta^*_1$.*

Proof See Appendix A.1.

Lemma 1 extends the analysis in standard global games models (for example, Morris and Shin 2003) to the case where the posterior of investors follows a mixture distribution over different macro states, comprising conditional normal distributions. The equilibrium is characterized by an indifference condition from individual optimality and by a critical mass condition which states that the proportion of attacking investors A^*_1 equals the fundamental threshold Θ^*_1. The equilibrium conditions can be reduced to one equation in one unknown. Using the results of Milgrom (1981) and Vives (2005), the best-response function of individual investors are strictly increasing in the thresholds used by other investors (Appendix A.1.1). The common requirement of sufficiently precise private information suffices for uniqueness in monotone equilibrium in the case of mixture distributions.

3.2 Region 2

We next consider region 2. Let $n_2 \in [0,1]$ be the proportion of investors in region 2 who acquire information about the macro shock m and let $d_i \in \{I,U\}$ be the information choice of investor i, with corresponding attack rules of informed and uninformed investors given by $a_I \equiv a_{i2}(d_i = I)$ and $a_U \equiv a_{i2}(d_i = U)$.

Definition 1 *A pure-strategy monotone perfect Bayesian equilibrium in region 2 comprises an information choice for each investor, $d_i^* \in \{I,U\}$, an aggregate proportion of informed investors, $n_2^* \in [0,1]$, an attack rule for informed and uninformed investors, $a_I^*(m,\cdot)$ and $a_U^*(\cdot)$, and an aggregate attack size, A_2^*, such that:*

1. *At the information stage, investors optimally choose d_i.*
2. *The proportion of informed investors is consistent with individual information choices, $n_2^* = \int_0^1 d_i^*\, di$.*
3. *At the coordination stage, attack rules are optimal, where uninformed investors use $a_U^*(\cdot)$ and informed investors use $a_I^*(m,\cdot)$ for each macro shock.*
4. *The aggregate attack size is consistent with attack rules for each macro shock:*

$$A_2^* \quad = \quad n_2^* \int_0^1 a_I^*(m,\cdot)di + (1-n_2^*) \int_0^1 a_U^*(\cdot)di. \tag{5}$$

To derive analytical results, we maintain the following assumption throughout.

Assumption 1 *Private information is precise, $\beta > \underline{\beta}$, public information imprecise, $\alpha < \overline{\alpha}$, a zero macro shock is unlikely, $1-p-q < \eta$, and crisis are rare with $\mu > \underline{\mu}$ and a negatively skewed macro shock, $s > \underline{s}$.*

Assumption 1 states sufficient conditions for the main result on wake-up call contagion, where the bounds are described in the proofs. The rareness of crises is key, since it implies a strong fundamental re-assessment after the wake-up call of a crisis in region 1. A sufficiently high relative precision of private information is common in the global games literature. The sufficiently low probability of a zero macro shock simplifies the analysis, as it allows us to focus on the favorable and unfavorable macro states that are at the core of the re-assessment. We stress that these conditions are sufficient, but not necessary, and help with tractability and exposition. The numerical examples below show that our results also obtain under less restrictive conditions. We discuss robustness of our results further in the discussion section.

We proceed by constructing the equilibrium in region 2. Investors in region 2 observe whether a crisis occurred in region 1, and use Bayes' rule to re-assess the fundamental of region 2, specifically the macro shock m. Since only a proportion of investors may choose to acquire information, we allow for heterogeneous priors. There are three distinct fundamental thresholds – one for each realized macro shock – and thus three critical mass conditions. Similarly, there are four indifference conditions – one for uninformed investors and one for informed investors for each macro shock realization. The system of equations is derived in Appendix A.2. If some investors are informed, we denote the fundamental thresholds in region 2 as $\Theta_I^*(m)$. If all investors are uninformed, we denote the fundamental thresholds in region 2 as Θ_U^*.

Proposition 1 ***Equilibrium in region 2.*** *For intermediate information costs, $c \in (\underline{c}, \overline{c})$, there exists a unique monotone perfect Bayesian equilibrium. At the information stage, investors acquire information only after a wake-up call, $n_2^* = \mathbf{1}_{\{\Theta_1 < \Theta_1^*\}}$. At the coordination stage, investors use threshold strategies:*

1. *After no crisis in region 1, investors choose to be uninformed and attack whenever their private signal is sufficiently low, $x_{i2} < x_U^*$, and a crisis occurs whenever the fundamental is sufficiently low, $\Theta_2 < \Theta_U^*$.*

2. *After a crisis in region 1, investors choose to be informed and attack whenever their private signal is sufficiently low relative to a macro-shock-specific threshold $x_{i2} < x_I^*(m)$, and a crisis occurs whenever the fundamental is sufficiently low relative to a macro-shock-specific threshold, $\Theta_2 < \Theta_I^*(m)$.*

Proof See Appendix A.2.5 for a proof and Appendix A.2 for a derivation of the equilibrium conditions, as well as the required results on information acquisition discussed below.

The equilibrium is in dominant actions at the information stage. Irrespective of the information choices of other investors, each investor acquires information *only* after the wake-up call of a crisis in region 1. This occurs whenever the fundamental in region 1 is below its threshold, $\Theta_1 < \Theta_1^*$. When investors in region 2 choose to be uninformed, they use the same attack threshold, x_U^*, and there is one fundamental threshold, Θ_U^*, where both thresholds are independent of the macro shock. In contrast, when investors choose to be informed, they tailor their attack rule to the macro state, $x_I^*(m)$, and there is one fundamental threshold for each state, $\Theta_I^*(m)$. We proceed by building intuition for the differential information choice in Proposition 1.

First, we examine the value of information and trace out how the incentives of investors to acquire information about the macro shock are affected by the wake-up call and the information choices of other investors. Let $f \in \{0,1\}$ indicate whether a crisis occurred in region 1. After a wake-up call, $f = 1$, investors learn that the fundamental in region 1 was low, $\Theta_1 < \Theta_1^*$. Conversely for $f = 0$, the fundamental was high, $\Theta_1 \geq \Theta_1^*$. Using Bayesian updating, Lemma 3 in Appendix A.2.1 states that a less (more) favorable realization of the macro shock is more likely after a crisis (no crisis).

The resulting re-assessment determines the incentives of investors to acquire information, whereby the value of information is higher after a wake-up call. Since crises are rare events, there is a strong Bayesian updating channel after a wake-up call. The negatively skewed macro shock generates an asymmetry, which assures that it is more valuable to acquire information about the realized macro shock after observing a crisis in region 1. The probability of a negative macro shock is small without a crisis in region 1, but it is substantially higher after a crisis. Hence, investors in region 2 have a high benefit from learning about the macro shock and tailoring their attack decision. This is the key effect behind the differential information choice in Proposition 1.[9]

Although we restrict ourselves to analyzing the differential incentives to acquire information in region 2, allowing for information acquisition in region 1 as well would not alter the insight. More specifically, there exists an intermediate range of information costs such that information acquisition never takes place in region 1, while it only takes place in region 2 after the wake-up

[9]The differential information reverses under the implausible assumption of frequent crises and a sufficiently positively skewed macro shock.

call of a crisis in region 1.

Next, we discuss the value of information and how it affects the incentives to acquire information. The value of information is defined as the difference between the expected utility of an informed investor, EU_I, and an uninformed investor, EU_U, and is derived in Appendix A.2.3. It depends on both the proportion of informed investors and on whether a crisis occurred in region 1:[10]

$$v(n_2, f) \equiv EU_I - EU_U. \tag{6}$$

Informed investors observe whether a crisis occurred and take into account the possible realizations of m, since these affect the signal thresholds, $x_I^*(m)$. By contrast, uninformed investors cannot tailor their attack strategy and must use the *same* signal thresholds x_U^* for all realized macro shocks. As a result, the signal thresholds of informed and uninformed investors differ and $v(n_2, f) > 0$.

Information about the macro shock allows an investor to tailor her behavior and reduce two types of errors. First, when an investor attacks the regime although no crisis occurs, she incurs a loss (type-I error). Second, when an investor does not attack although a crisis occurs, she could have earned a benefit (type-II error). The value of information is governed by the relationship between these two types of errors. The marginal benefit of increasing $x_I^*(-s\Delta)$ above x_U^* is positive because the type-II error is relatively more costly than the type-I error. By contrast, the marginal benefit of decreasing $x_I^*(\Delta)$ below x_U^* is positive because the type-I error is more costly. In sum, informed investors attack more aggressively upon learning the low macro shock realization, $m = -s\Delta$, and less aggressively upon learning the high realization, $m = \Delta$.

Next, we turn to the strategic aspect of information acquisition. The signal thresholds of informed and uninformed investors depend on the proportion of informed investors. We find that the difference in signal thresholds increases in the proportion of informed investors, as derived in Lemma 4 in Appendix A.2.2. The divergence of signal thresholds with an increasing proportion of informed investors induces a strategic complementarity in information choice, $\frac{dv(n_2,f)}{dn_2} \geq 0$, as

[10] To evaluate the incentives of investors to acquire information, we study the optimal attack behavior for any given proportion of informed investors and allow for some investors to be informed while others are uninformed, resulting in heterogeneous priors about the macro shock that follow a mixture distribution. In another global game with mixture distributions, Chen et al. (2012) develop a theory of rumors during political regime change. However, they abstract from both contagion and information choice.

derived in Lemma 5 in Appendix A.2.3. Intuitively, the individual attack decision of an informed investor is more strongly adjusted the larger the proportion of informed investors, which in turn increases the value of information. In the words of Hellwig and Veldkamp (2009), investors want to know what others know in order to do what others do.

Figure 1 shows the attack threshold of informed and uninformed investors. First, informed investors attack more (less) aggressively after observing a negative (positive) macro shock. Second, conditional on observing a wake-up call, both types of investors attack more aggressively. Comparing the left and right panel, all signal thresholds are higher after a wake-up call for each macro state and for all $n_2 \in [0,1)$. When all investors are informed, $n_2 = 1$, signal thresholds coincide irrespective of whether a crisis occurred in region 1, since region 1's outcome does not contain any information beyond the macro shock.

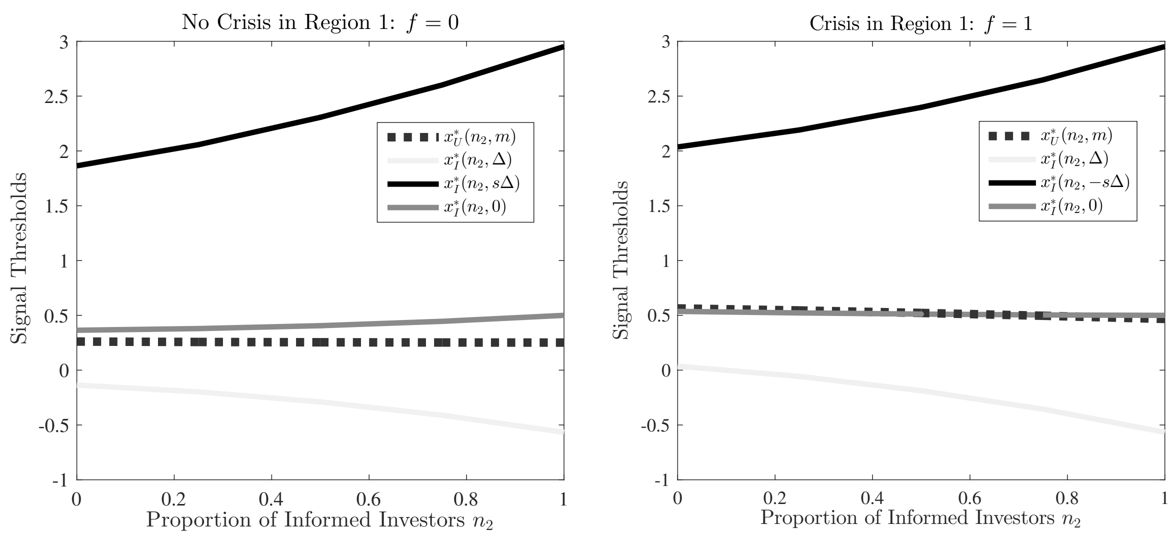

Figure 1: Signal thresholds of informed and uninformed investors, x_I^* and x_U^*, as a function of the proportion of informed investors, n_2, after no crisis (left panel) and a crisis (right panel). The signal thresholds are monotonic in the proportion of informed investors. Both informed and uninformed investors attack more aggressively after a crisis in region 1: the signal thresholds are higher in the right panel than in the left panel for each macro state and for any $n_2 \in [0,1)$. Parameter values are $\alpha = \beta = 1$, $\mu = 1/2$, $p = 1/4$, $\Delta = 1/2$, $s = 3$, $\gamma_1 = \gamma_2 = 1/2$.

Lemma 2 ***Wake-up call and the value of information.*** *The value of information is higher after a crisis in region* 1 *independent of the proportion of informed investors:*

$$v(1,1) > v(0,1) > v(1,0) > v(0,0), \tag{7}$$

and the differential value of information increases if the macro shock is more negatively skewed:

$$\frac{d[v(1,1)-v(1,0)]}{ds} > 0. \tag{8}$$

Proof See Appendix A.2.4.

Lemma 2 ranks the value of information that affects the information choices of investors. The first and third inequality in (7) represent the strategic complementarity in information choices. The second inequality is due to the negatively skewed macro shock. For a sufficiently negatively skewed macro shock, as guaranteed by Assumption 1, we have $v(0,1) > v(1,0)$. As a result, there exists an intermediate range of information costs $c \in (\underline{c}, \overline{c})$ with $\underline{c} \equiv v(1,0)$ and $\overline{c} \equiv v(0,1)$ such that all investors choose to acquire information if and only if a crisis occurs in region 1 (the wake-up call).

Figure 2 shows the value of information, which depends positively on the proportion of informed investors (strategic complementarity) and the occurrence of a crisis in region 1. A sufficiently high negative skewness of the macro shock leads to a strong Bayesian updating channel that ensures a unique equilibrium for intermediate values of information costs despite strategic complementarity in information choices. This intermediate region depicted by the shaded area expands strongly when the skewness parameter s increases.

3.3 Contagion

Having established a unique equilibrium for intermediate information costs, we turn to the question of contagion after a wake-up call. Contagion is defined as the increase in the likelihood of a crisis in region 2 after a crisis in region 1, compared to no crisis in region 1.

Our main result is that contagion occurs even if investors learn that region 2 is not exposed to region 1 (zero macro shock). This result isolates the wake-up call component of contagion. It builds on the equilibrium information choices in Proposition 1 and holds under the sufficient conditions of Assumption 1.

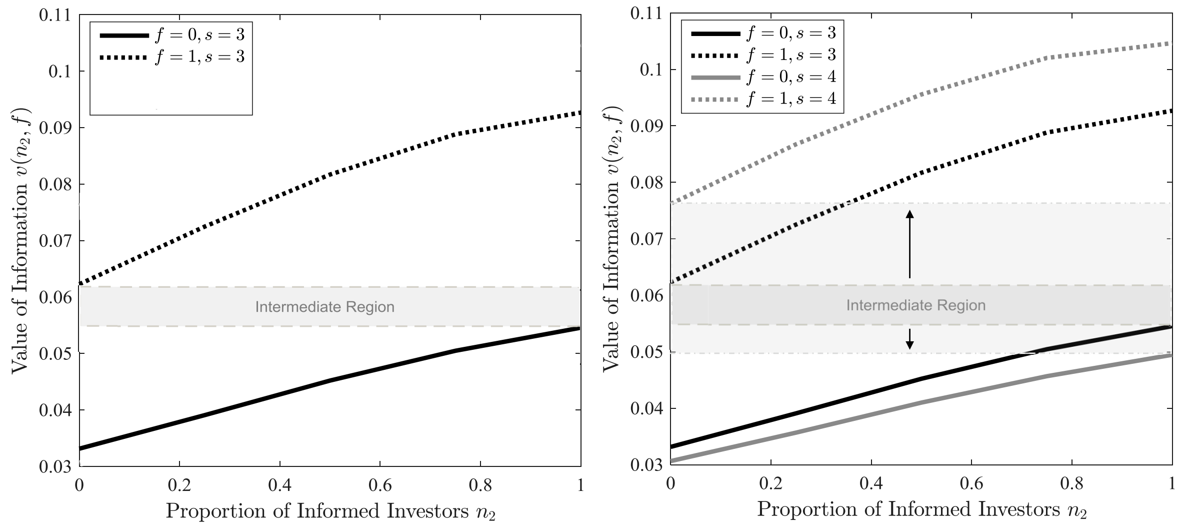

Figure 2: Dependence of the value of information v on the proportion of informed investors n_2 and the occurrence of a wake-up call f. The value of information increases in the proportion of informed investors and it is uniformly higher after a wake-up call of a crisis in region 1 for a sufficiently skewed macro shock. The shaded area depicts the intermediate region where $v(0,1) > v(1,0)$. It expands if s increases. Parameter values are $\alpha = \beta = 1$, $\mu = 1/2$, $p = 1/4$, $\Delta = 1/2$, $s \in \{3,4\}$, $\gamma_1 = \gamma_2 = 1/2$.

Proposition 2 ***Wake-up call contagion.*** *Suppose that $c \in (\underline{c}, \overline{c})$. A financial crisis in region* 2 *is more likely after a crisis in region* 1 *when all investors acquire information and learn that the macro shock is zero, than after no crisis in region* 1*, when all investors choose not to acquire information:*

$$\Pr\{\Theta_2 < \Theta_I^*(m) | m = 0\} > \Pr\{\Theta_2 < \Theta_U^*\}. \tag{9}$$

Proof See Appendix A.3.

This contagion result rests on the unique equilibrium for intermediate information costs. The left-hand side of inequality (9) is the probability of a crisis in region 2 after a crisis in region 1, a wake-up call, that induces investors *to acquire information* and when they learn that the macro shock is zero. The right-hand side is the probability of a crisis in region 2 after no crisis in region 1, that induces investors *not to acquire information*. Hence, the conditional probability allows for any realization of the unobserved macro shock.

We find that a crisis in region 2 is more likely after a crisis in region 1 than after no crisis in region 1 *even if* all investors acquire information and learn that the macro shock is zero. Learning that the macro shock is zero implies that the crisis in region 1 is unrelated to the fundamental in region 2. In contrast, no crisis in region 1 implies a more favorable view about the fundamental in region 2 due to the unobserved macro shock. Hence, the decreased crisis probability after observing no crisis in region 1 is a key driver of the result. This effect tends to lower the right-hand side of inequality (9).

While Bayesian updating is mechanical, the result of wake-up call contagion arises endogenously. For intermediate information costs, investors choose to acquire information only after the wake-up call. In other words, the comparison of scenarios in equation (9) hinges on the differential information choice. Critically, our mechanism of wake-up call contagion based on endogenous information is distinct from the information contagion literature with ex-post correlated fundamentals (see section 5). It allows for contagion even when the fundamentals of the two regions are uncorrelated ex-post, but potentially correlated ex-ante.

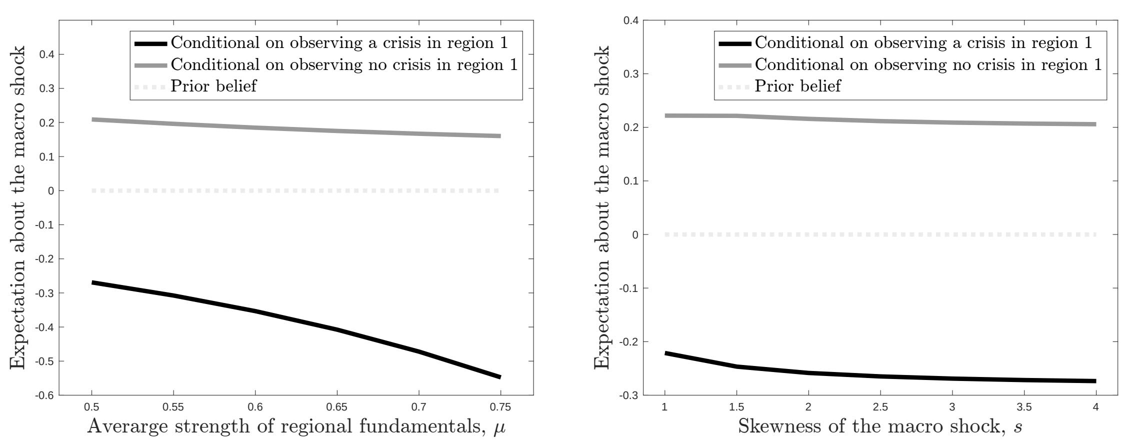

Figure 3: The two panels illustrate the updating of the belief about the mean of the macro shock for different strengths of regional fundamentals (left panel) and for different skewness parameters (right panel). The parameter values are as in Figure 1.

The Bayesian updating channel is about the re-assessment of the macro shock. The left panel of Figure 3 shows this re-assessment for different values of the average strength of regional fundamentals, μ. The dotted line reflects the prior about the macro shock, which has zero mean, $E[m] = 0$. After *observing a crisis* in region 1, the conditional expectation about the macro shock is negative, $E[m|f = 1] < 0$ (dark solid line). This result is due to an upward revision of the probability of a negative macro shock after bad news, $\Pr\{m = -s\Delta|f = 1\} > q$. After *observing no crisis* in region 1, by contrast, the expectation about the macro shock is positive (grey solid line), since the probability of a negative macro shock is revised down. The re-assessment is stronger, that is the difference between $E[m|f = 1]$ and $E[m|f = 0]$ is higher, when the average strength of regional fundamentals is higher and crises are rare. The right panel shows the re-assessment of the mean of the macro shock for different values of skewness, s. The main insights are unaltered and an increase in s increases the magnitude of the re-assessment for the same reasons. However, this panel does show that the Bayesian updating results arise even if $s = 1$ (no skewness).

We wish to highlight that our contagion results do not hinge upon a common investor base or balance sheet links across regions. Therefore, Proposition 2 isolates the wake-up call component of contagion by showing that contagion occurs even if investors learn that the macro shock is zero. The assumption of a negatively skewed macro shock is crucial for the underlying information choice in equation (9) driven by the strong fundamental reassessment. However, the assumption is inessential for the Bayesian updating channel that gives rise to the inequality. Hence, the wake-up call effect and the associated novel testable implications hinge on the possibility of rare but strongly negative shocks to fundamentals.

Figure 4 illustrates the magnitude of the wake-up call contagion effect. We compare the crisis probability by plotting both sides of inequality (9). In the numerical example, the wake-up call contagion effect is significant and its magnitude can exceed 10 percentage points. The bounds on the precision of private and public signals are not more stringent than the standard conditions used to assure uniqueness of equilibria in global games models (Morris and Shin 2003; Svensson 2006). Notably, the result of wake-up call contagion even prevails when the probability of a zero macro shock is rather high (such as $1 - p - q = 2/3$) and when crises are relatively frequent (such as $\mu = 1/2$), illustrating the robustness of the key results.

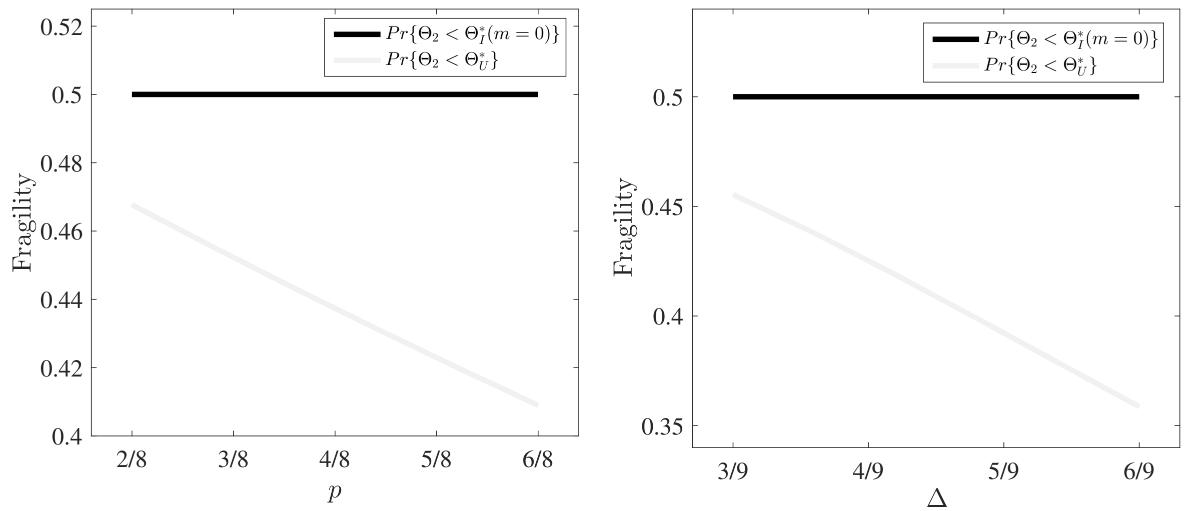

Figure 4: The magnitude of the wake-up call contagion effect in isolation. The left panel shows the increase in wake-up call contagion under the baseline if p increases from $1/4$ to $3/4$. The right panel shows the increase in wake-up call contagion if $p = 3/4$ and Δ increases from $1/3$ to $2/3$. The other parameter values are as before, $\alpha = \beta = 1$, $\mu = 1/2$, $s = 3$, $\gamma_1 = \gamma_2 = 1/2$.

The magnitude of wake-up call contagion is governed by the strength of the Bayesian updating channel. Absent a crisis in region 1, uninformed investors place a positive probability $\Pr\{m = \Delta | f = 0\} > p$ on a positive realization of the macro shock. Given that an increase in p is associated with a more favorable view about fundamentals after not observing a crisis in region 1, the difference in likelihoods of a crisis in region 2, $\Pr\{\Theta_2 < \Theta_I^* | m = 0, f = 1\} - \Pr\{\Theta_2 < \Theta_U^* | f = 0\}$, is positive and increasing in p (left panel). The magnitude of the wake-up call effect also increases in Δ (right panel), the level of the positive macro shock.

This contagion result is further strengthened when regions are indeed related ex-post, that is when the macro shock takes a negative value.

Corollary 1 ***Negative macro shock.*** *The result of Proposition 2 is strengthened if all investors choose to acquire information after a crisis in region* 1 *and learn that the macro shock is negative:*

$$\Pr\{\Theta_2 < \Theta_I^*(m) | m = -s\Delta\} > \Pr\{\Theta_2 < \Theta_I^*(m) | m = 0\}. \tag{10}$$

Proof The proof parallels that of Proposition 2 and is therefore omitted.

The left-hand (right-hand) side of inequality (10) is the probability of a crisis in region 2 after a crisis in region 1, a wake-up call, that induces investors to acquire information and when they learn that the macro shock is negative (zero). On the left-hand side, beliefs are less favorable and the crisis is more likely.

4 Testable Implications

Our theory of contagion after a wake-up call offers three sets of novel testable implications described in this section. We discuss how these implications have already been tested or can be tested in future work.

4.1 Wake-up call contagion

Since the wake-up call contagion result relies on endogenous information acquisition, we describe novel predictions related to information acquisition.

Prediction 1: The extent of information acquisition about the exposure to aggregate or market-wide shocks is positively associated with an increase in volatility.

The first prediction is on financial fragility and highlights how information acquisition can amplify volatility, when measured as the increase in the dispersion of probabilities of a crisis in region 2 conditional on the macro shock realization (as may be observed by the empiricist). To see this, recall that, the occurrence of information acquisition about the exposure to aggregate or market-wide shocks hinges on observing the wake-up call of a crisis elsewhere and on the cost of information. Moreover, the acquisition of information about the macro shock induces investors to tailor their attack strategy to the observed macro shock. Hence, the total contagion effect after a wake-up call is stronger when investors learn that the macro shock realization is negative, $m = -s\Delta$ (see also Corollary 1). Conversely, the total contagion effect after a wake-up call is weaker when investors learn that the macro shock realization is positive, $m = \Delta$.

Taken together, the dispersion in the crisis probabilities conditional on the macro state increases as a result of the information acquisition after observing the wake-up call of a crisis elsewhere. That is, a crisis elsewhere amplifies volatility through endogenous information acquisition. As a result, this wake-up call mechanism can contribute to explaining the spread of volatility in episodes of emerging market turmoil in a way that is complementary to other contagion channels.

4.2 Information acquisition after a wake-up call

The subsequent predictions focus on the differential incentives to acquire information about the macro shock.

Prediction 2: The extent of information acquisition about the exposure to aggregate or market-wide shocks is higher after observing a financial crisis elsewhere than after observing no crisis.

The second prediction stems from equation (7) in Lemma 2 and is consistent with empirical evidence. In a study on the U.S. stock market, Vlastakis and Markellos (2012) find evidence for a positive association between the demand for market information, proxied by internet search intensity (Da et al. 2011), with measures of volatility. The demand for information at the market level may be interpreted as information acquisition about the common component of stock returns. In the context of our model, the market information refers to information about the aggregate component m and measures of volatility can be associated with a crisis occurring in region 1. Furthermore, Vlastakis and Markellos document that the demand for market information (information about m) is important relative to the demand for idiosyncratic information (information about θ_2). For idiosyncratic information, they find only mixed results in direction and strength. Taken together, the empirical results support our focus on information acquisition about a macro shock.

Macroeconomic news are known to play an important role in asset markets (e.g., Andersen et al. (2007)). Macro factors, or global factors in the case of sovereign debt (Longstaff et al. 2011), can explain the majority of credit risk. This suggests that information acquisition about aggregate, or market-wide, shocks plays an important role also in bond markets. Regarding sovereign debt, our theory predicts a larger extend of information acquisition about common macro risks after another country with similar characteristics enters a period of financial distress. Provided the availability of proxies for information acquisition like the internet search intensity for macro news that relate to potential common risk factors, this prediction may be tested.

The second prediction may also testable in the corporate debt market. Consider a firm with publicly traded debt to be rolled over by investors. A crisis elsewhere refers to a spike in the credit risk of other firms in the same industry sector that may be associated with a substantial ratings downgrade or earnings warning. It is well know that institutional lenders like banks seek to insure against industry-specific risks when confronted with a significant exposure via portfolio trading or the loan book. Our theory predicts a high sensitivity of debt holders to negative news that may convey information about changes in industry-specific factors (such as demand factors, new trends, or innovations that affect all firms in that industry), as well as an increase in the incentive to acquire information about potential industry shocks.

Our theory suggests that information acquisition occurs for an intermediate information cost. An empiricist can separate industries according to whether information acquisition is cheap or expensive. There are several potential proxies an empiricist may use for the information cost. Some industry sectors comprise mostly smaller firms that are not publicly listed, which makes it more difficult and costly for analysts to gauge relevant changes in industry-specific factors. Similarly, the growth prospects of high-tech industry sectors are more difficult to analyze than, for instance, utilities. Moreover, less homogenous industry sectors also suggest to be more difficult to evaluate. Based on our theory, the corporate debt from industry sectors with lower information costs are more likely to exhibit an increase in the extend of information acquisition by investors after observing a firm in the industry that suffers a ratings downgrade or earnings warning.

Finally, the second prediction may also be tested in the market for bank commercial paper, which is rolled over frequently. Apart from a downgrade or an earnings warning, a crisis elsewhere could also be a downward revision of another bank's asset quality by the supervisor. Again, an empiricist would need to separate circumstances under which information about the exposure of other banks is easy to acquire from those where it may be difficult. To this end, the opaqueness and complexity of banks play an important role, which may be affected by the extend of securitization activities, cross-border linkages and the organizational structure (Flannery et al. 2013; Cetorelli and Goldberg 2014; Goldberg 2016).

Next, we turn to another testable implication.

Prediction 3: The extent of information acquisition about the exposure to aggregate or market-wide shocks increases if the aggregate component of fundamentals is more negatively skewed.

The third prediction relates to the skewness of the macro shock. Empirically, skewness is difficult to measure, especially in bond markets, due to its time-varying nature. There is an extensive empirical literature on downside risk in stock markets (e.g., Ang et al. (2006)), as well as bond markets for sovereign debt and corporate debt, including emerging markets and bank debt. Here downside risk often manifests itself in the form of systemic risk. The prediction could be tested provided the availability of proxies for information acquisition. More specifically, the observed cross-sectional variation in downside risk across different bond market segments, especially for emerging markets (e.g., Erb et al. (2000)), may be associated with the extend of information acquisition about market-wide shocks by investors in a given market segment relative to other market segments. Similarly, the time-variation of downside risk may be associated with a variation in the extent of information acquisition about market-wide shocks.

Prediction 3 is also testable in equity markets, for instance, by extending Vlastakis and Markellos (2012). An empiricist has to separate market segments with varying degrees of skewness, or downside risk of the return distribution. When recasting the result from equation (8) in Lemma 2 in the context of an equity market study, our model suggests that the demand for market segment specific information in a high volatility environment should be higher, the more downside risk on the market specific level.

4.3 Transparency and information acquisition

Next, we state a testable implication on how transparency affects the incentives to acquire information. We study how greater transparency, measured as a higher α, affects the value of information about the macro shock. As before, the result holds under the sufficient conditions of Assumption 1.

Proposition 3 ***Transparency.*** *Greater transparency increases the incentives to acquire information:*

$$\frac{dv(1,f)}{d\alpha} > 0, \quad f \in \{0,1\}. \tag{11}$$

Proof See Appendix A.4.

With greater transparency, higher incentives to acquire information arise from the larger benefit of tailoring the signal thresholds to the realized macro shock. Intuitively, an increase in transparency is associated with less aggressive attacks against the regime if the prior about the fundamentals is strong, which occurs if investors observe $m = \Delta$. Greater transparency is associated with more aggressive attacks against the regime if the prior about the fundamentals is weak, which occurs if investors observe $m = -s\Delta$. As a result, signal thresholds diverge, which is associated with an increase in the value of information.

We state an empirical implication based on the result in Proposition 3.

Prediction 4: The extent of information acquisition about the exposure to aggregate or market-wide shocks increases if public disclosure of information on the idiosyncratic level is improved.

Depending on the application, an increase in α can, for instance, be interpreted as an increase in market disclosure standards, the precision of information provided by rating agencies or as an increase in the transparency of bank stress tests.

In the context of the debate about bank stress tests, higher transparency can be seen as a commitment of the banking regulator to disclose more details of bank-specific information. Testing Prediction 4 requires to study episodes of changes in the conduct, design and publicity of bank stress tests. After an increase in public disclosure, our result implies more information acquisition about variables common across banks, such as the impact of regulation on bank profitability, the soundness of bank business models, and the impact of central bank policy. When evaluating the information of the Federal Reserve stress tests, findings of the emerging empirical literature are consistent with Prediction 4. Flannery et al. (2015) find no evidence that stress test disclosures have reduced the production of information by analysts, with analyst coverage and forecast accuracy somewhat increasing.

5 Discussion

In this section, we differentiate our result of contagion after a wake-up call from the literature on information contagion reviewed in the introduction. Next, we relate the result on transparency to the existing literature. Finally, we discuss the robustness of our results.

5.1 Information contagion

There exists a literature on information contagion. Manz (2010) establishes information contagion due to ex-post correlated fundamentals in a global games framework. Acharya and Yorulmazer (2008) show that the funding cost of one bank increases after bad news about another bank when the banks' loan portfolio returns have a common factor. To avoid information contagion ex post, banks herd their investment ex ante. Allen et al. (2012) compare the impact of information contagion on systemic risk across asset structures. Adverse news about the solvency of the banking system leads to runs on multiple banks.

We can nest a version of the standard information contagion channel as a special case in our modeling framework with endogenous information, where investors are uncertain about the common component of regional fundamentals and update their beliefs about the exposure after observing a crisis elsewhere. The information contagion result is stated below and purely relies on Bayesian updating of uninformed investors. That is, when the information cost is high, $c > v(1,1) \equiv \tilde{c} > \bar{c}$, information acquisition never occurs.

Proposition 4 ***Information contagion.*** *If $c > \tilde{c}$, then there exists a unique monotone perfect Bayesian equilibrium in region 2 in which no investor acquires information, $n_2^* = 0$. In this case, a crisis in region 2 is more likely after a crisis in region 1 than after no crisis in region 1:*

$$\Pr\{\Theta_2 < \Theta_U^* | f = 1\} > \Pr\{\Theta_2 < \Theta_U^* | f = 0\}. \tag{12}$$

Proof See Appendix A.5.

Proposition 4 compares the probability of a crisis in region 2 conditional on whether a crisis occurred in region 1. For a sufficiently high information cost, investors in region 2 choose not to acquire information irrespective of the occurrence of a crisis in region 1. In this case, a crisis in region 1 is unfavorable news about the fundamental in region 1. Since the macro shock is a common component of both regional fundamentals, this crisis is also unfavorable news about the fundamental in region 2. As a result, the re-assessment of the local fundamental Θ_2 via Bayesian updating increases the probability of a crisis in region 2.

Different to the result on wake-up contagion, the result of information contagion only rests on Bayesian updating about the macro shock but not on the endogenous (differential) information choice of investors, which allows us to isolate the wake-up call component of contagion and to derive novel testable implications.

5.2 Transparency

We measure greater transparency as an increase in α (e.g., Morris and Shin (2002)). Given Assumption 1, Proposition 3 states that greater transparency induces more information acquisition in a coordination game of regime change. In other words, there is a complementary relationship between public disclosure and information acquisition. This result contrasts with some of the existing literature that has analyzed the impact of transparency on information acquisition in coordination games. In the context of a beauty contests with private information acquisition, Colombo et al. (2014) find a crowding-out effect of public information. The incentives to acquire more precise private information decrease in the precision of public information.

In contrast, Szkup and Trevino (2015) study continuous information choice subject to a convex information cost that is homogeneous across investors. They analyze efficiency when information choices are complements or substitutes, and the trade-off between public and private information, focusing on the precision of public information. Ahnert and Kakhbod (2017) study binary private information choice subject to heterogeneous information costs, finding that greater disclosure sometimes increases fragility. In contrast, we study the acquisition of publicly available information in a regime change game. Finally, there is an earlier literature studying the effect of transparency on the incidence of a regime change with exogenous information (Morris and Shin 1998; Heinemann and Illing 2002; Bannier and Heinemann 2005).

5.3 Robustness

Our analytical results are derived under the conditions of Assumption 1. Given that the conditions might seem restrictive, it is worth noting that the conditions are sufficient but not necessary for our results. Most importantly, the benchmark parameter values used for the numerical analysis provided in the figures illustrate that wake-up call contagion also holds for a high probability of the zero macro shock, suggesting that the bound η is merely relevant for analytical tractability. Also the bounds on the precision of private and public signals are not more stringent than the standard sufficient conditions for equilibrium uniqueness in global games models (Morris and Shin 2003; Svensson 2006).

To simplify the exposition, we abstract from information acquisition in region 1. This allows us to focus on how the wake-up call of a crisis in region 1 affects the incentives to acquire information in region 2 and may therefore result in contagion. Allowing for information acquisition in region 1 does not affect our main insights. For some intermediate region of information costs, there is a unique equilibrium with no information acquisition in region 1 and information acquisition in region 2 only after a crisis in region 1.

Next, we discuss two extensions and an alternative modeling approach. First, an important channel of our paper is how a wake-up call affects the incentives of investors in region 2 to acquire information about the macro shock. An additional channel of interest could be private information acquisition with convex costs (Szkup and Trevino 2015), whereby investors improve the precision of their private information at a cost after the wake-up call. It can be shown that the effect of wake-up call contagion is even larger when private information acquisition is also allowed.

Second, one could consider an alternative model setup, where learning is not about the realization of the macro shock but about whether region 2 is exposed to the macro shock itself. In this setup, two macro shock realizations suffice, so $1-p-q=0$, where the scenario of no exposure to the macro shock is equivalent to $m = 0$. As before, both observing a crisis in region 1 and learning about an exposure to the macro shock suggest that the fundamentals in region 2 are likely to be affected by a negative macro shock that also contributed to the crisis in region 1. Conversely, learning about no exposure to the macro shock after observing a crisis in region 1 is favorable information for the local fundamentals in region 2. No crisis in region 1 would still imply a more favorable view about the fundamental in region 2 when an exposure to the macro shock has positive weight due to the Bayesian updating channel. Hence, the wake-up call component of contagion can be isolated in the same way in this alternative model setup, and the incentives to acquire information about the macro shock in such an alternative setup are similar to the present version.

Third, we have so far considered the case of a perfect signal about the macro shock. A generalization to noisy signals is possible but would not affect the key insights of the paper. Related to this extension, a plausible modification of our model is to allow for communication between investors about the macro shock realization. With noisy signals about m and limits to communication (that is, each investor can only communicate with few peers), there is still a benefit for individual investors from acquiring information about the macro shock (that is, no free-riding on the information of others). Hence, our results should be preserved for limited communication when learning about m is imperfect.

6 Conclusion

We offer a theory of contagion to explain how wake-up calls may transmit financial crises. We study global coordination games of regime change that are often applied to currency attacks, bank runs, and debt crises. There are two regions whose only link is an initially unobserved and negatively skewed macro shock. A crisis in region 1 is a wake-up call for investors in region 2 and induces them to re-assess the local fundamental in region 2. Since crises are rare events and the macro shock is negatively skewed, investors have an incentive to acquire information only after a wake-up call. The probability of a crisis in region 2 is higher after crisis in region 1 than after no crisis, even if investors learn that the macro shock is zero and, hence, that there is no exposure to the crisis in region 1. In short, we isolate the wake-up call component of contagion without ex-post exposure to the crisis region, common lender effects, or balance sheet links.

A distinctive feature of our theory is that it combines information acquisition with a Bayesian updating channel. The optimal information choices of investors are driven by wake-up calls and shape the fundamental re-assessment, giving rise to novel implications. Information choices are strategic complements but a unique equilibrium obtains for intermediate information cost levels. We describe how the incentives to acquire information about the macro shock depend on distributional characteristics of the macro shock and on disclosure. Based on these results, we derive a set of testable implications about the information choices of investors. We argue that several of these are consistent with existing empirical evidence and discuss how the new implications can be tested.

While the focus of this paper is on how information acquisition about common shocks can transmit financial crises, implications for the ex-ante level of welfare are an interesting area left for future work.

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A Appendix: Proofs

A.1 Equilibrium in region 1

To simplify the exposition, we focus on the case of uninformed investors, $n_1 = 0$. The equilibrium analysis for the general case $n_1 \in [0,1]$ can be found in Appendix A.6. We first discuss Bayesian updating of uninformed investors receiving a private signal x_{i1} about the regional fundamental and derive the equilibrium conditions in section A.1.1. Thereafter, we present the proof of Lemma 1 in section A.1.2.

A.1.1 Deriving the equilibrium in region 1 for the case $n_1 = 0$

Bayesian updating. Uninformed investors use Bayes' rule to form a belief about the macro shock, where $\hat{p} \equiv \Pr\{m = \Delta | x_{i1}\}$, and $\hat{q} \equiv \Pr\{m = -s\Delta | x_{i1}\}$:

$$\hat{p} = p \Pr\{x_{i1} | m = \Delta\} \Gamma_1^{-1} \qquad \hat{q} = q \Pr\{x_{i1} | m = -s\Delta\} \Gamma_1^{-1}, \tag{13}$$

where $\Gamma_1 = p \Pr\{x_{i1} | m = \Delta\} + q \Pr\{x_{i1} | m = -s\Delta\} + (1 - p - q) \Pr\{x_{i1} | m = 0\}$ and:

$$\Pr\{x_{i1} | m\} = \frac{1}{\sqrt{\mathrm{Var}[x_{i1} | m]}} \phi\left(\frac{x_{i1} - \mathbb{E}[x_{i1} | m]}{\sqrt{\mathrm{Var}[x_{i1} | m]}}\right) = \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^{-\frac{1}{2}} \phi\left(\frac{x_{i1} - (\mu + m)}{\sqrt{\frac{1}{\alpha} + \frac{1}{\beta}}}\right).$$

Using $p = qs$, we obtain $\frac{d\hat{p}}{dx_{i1}} > 0$, $\quad \frac{d\hat{q}}{dx_{i1}} < 0$, and $\frac{d(1-\hat{p}-\hat{q})}{dx_{i1}} \left[x_{i1} - \mu + \frac{1-s}{2}\Delta\right] \leq 0$, with strict inequality if $x_{i1} \neq \mu + \frac{1-s}{2}\Delta$. An investor in region 1 places more weight on the probability of a positive (negative) macro shock after a higher (lower) private signal. The relationship between the posterior probability of a zero macro shock and the private signal, x_{i1}, is non-monotone. It increases if $x_{i1} > \underline{x_1}(s, \Delta) \equiv \mu + \frac{1-s}{2}\Delta$. The bound is below μ if the macro shock is negatively skewed ($s > 1$).

Equilibrium conditions. For the case of $n_1 = 0$, we derive a system of equations that comprises the critical mass and indifference condition for region 1. The critical mass condition states that the proportion of attacking investors $A_1^*(m)$ equals the fundamental threshold $\Theta_1^*(m)$ for each realized

$m \in \{-s\Delta, 0, \Delta\}$:

$$\Theta_1^*(m) = \Phi\big(\sqrt{\beta}[x_1^* - \Theta_1^*(m)]\big). \tag{14}$$

Given the invariant attack rule, the fundamental thresholds are equal, $\Theta_1 \equiv \Theta_1^*(m)\ \forall m$. The indifference condition states that an uninformed investor with threshold signal $x_{i1} = x_1^*$ is indifferent whether to attack:

$$\hat{p}^*\Psi(\Theta_1^*, x_1^*, \Delta) + \hat{q}^*\Psi(\Theta_1^*, x_1^*, -s\Delta) + (1 - \hat{p}^* - \hat{q}^*)\Psi(\Theta_1^*, x_1^*, 0) \equiv J(\Theta_1^*, x_1^*) = \gamma_1. \tag{15}$$

where $\hat{p}^* = \hat{p}(x_1^*)$, $\hat{q}^* = \hat{q}(x_1^*)$ and $\Psi(\Theta_1^*, x_1^*, m) \equiv \Phi\big(\Theta_1^*\sqrt{\alpha+\beta} - \frac{\alpha(\mu+m)+\beta x_1^*}{\sqrt{\alpha+\beta}}\big)$. Solving equation (14) for x_1^* and plugging into equation (15), we arrive at one equation in one unknown.

Monotone equilibria. Using the results of Milgrom (1981) and Vives (2005), we can show that the best-response function of an individual investor strictly increases in the threshold used by other investors. Using Proposition 1 of Milgrom (1981), we conclude that $\Pr\{\Theta_1 \leq \Theta_1^* | x_{i1}\}$ monotonically decreases in x_{i1}. Hence, $\frac{d\Pr\{\Theta_1 \leq \Theta_1^* | \hat{x}_1\}}{d\Theta_1^*} > 0$. Equation (15) then implies:

$$0 \leq \frac{d\hat{\Theta}_1(\hat{x}_1)}{d\hat{x}_1} \leq \left(1 + \sqrt{2\pi\beta^{-1}}\right)^{-1}. \tag{16}$$

Thus, our focus on monotone equilibria is valid. Equation (16) is used to determine conditions sufficient for a unique monotone Bayesian equilibrium in Lemma 1.

A.1.2 Proof of Lemma 1

The proof consists of two steps. First, we show that $J(\Theta_1, x_1) \equiv J(\Theta_1) \to 1 > \gamma_1$ as $\Theta_1 \to 0$, and $J(\Theta_1) \to 0 < \gamma_1$ as $\Theta_1 \to 1$. Second, we show that $\frac{dJ(\Theta_1)}{d\Theta_1} < 0$ for some sufficiently high but finite values of β, such that J strictly decreases in Θ_1. We denote this lower bound as $\underline{\beta}_1$. Therefore, if Θ_1^* exists, it is unique. Notably, this argument implicitly defines the lower and upper dominance regions of the game. However, as Θ_1 can be any real number, the limit used here is one-sided.

Step 1 (limiting behavior): After solving equation (14) for x_1^* and plugging into equation (15), let $\Psi(\Theta_1, x_1, m) \equiv \Psi(\Theta_1, m)$. Observe that $J(\Theta_1)$ is a weighted average of the $\Psi(\Theta_1, m)$'s evaluated at the different levels of m. As $\Theta_1 \to 0$, then $\Psi(\Theta_1, m) \to 1$ for any $m \in \{-s\Delta, 0, \Delta\}$, so $J(\Theta_1) \to 1 > \gamma_1$. Likewise, as $\Theta_1 \to 1$, then $\Psi(\Theta_1, m) \to 0$ for any $m \in \{-s\Delta, 0, \Delta\}$, so $J(\Theta_1) \to 0 < \gamma_1$.

Step 2 (strictly negative slope): The total derivative of J is:

$$\begin{aligned}
\frac{dJ(\Theta_1)}{d\Theta_1} &= \hat{p}(x_1(\Theta_1))\frac{d\Psi(\Theta_1,\Delta)}{d\Theta_1} + \hat{q}(x_1(\Theta_1))\frac{d\Psi(\Theta_1,-s\Delta)}{d\Theta_1} \\
&+ (1-\hat{p}(x_1(\Theta_1)) - \hat{q}(x_1(\Theta_1)))\frac{d\Psi(\Theta_1,0)}{d\Theta_1} \\
&+ \frac{d\hat{p}(x_1(\Theta_1))}{dx_1}\frac{dx_1(\Theta_1)}{d\Theta_1}\big[\Psi(\Theta_1,\Delta) - \Psi(\Theta_1,0)\big] \\
&+ \frac{d\hat{q}(x_1(\Theta_1))}{dx_1}\frac{dx_1(\Theta_1)}{d\Theta_1}\big[\Psi(\Theta_1,-s\Delta) - \Psi(\Theta_1,0)\big]. \qquad (17)
\end{aligned}$$

The proof proceeds by inspecting the individual terms of equation (17). For the analysis of the special case where all investors are informed, $n_1 = 1$, we can use a result from standard global games models: $\frac{d\Psi(\Theta_1,m)}{d\Theta_1} < 0$ if $\beta > \frac{\alpha^2}{2\pi}$ for all m. Thus, the first three components of the sum are negative and finite for sufficiently high but finite private noise. The sign of the two terms in square brackets in the last two summands in (17) is negative and positive, respectively: $\Psi(\Theta_1^*, \Delta) \leq \Psi(\Theta_1^*, 0)$ and $\Psi(\Theta_1^*, \Delta) \geq \Psi(\Theta_1^*, 0)$. However, the difference vanishes in the limit when $\beta \to \infty$. The last terms to consider are $\frac{d\hat{p}(x_1(\Theta_1))}{dx_1(\Theta_1)}\frac{dx_1}{d\Theta_1}$ and $\frac{d\hat{q}(x_1(\Theta_1))}{dx_1(\Theta_1)}\frac{dx_1}{d\Theta_1}$. Given the previous sufficient conditions on the relative precision of the private signal:

$$0 < \frac{dx_1}{d\Theta_1} = 1 + \left(\sqrt{\beta}\phi(\Phi^{-1}(\Theta_1))\right) < 1 + \sqrt{2\pi\alpha^{-1}}.$$

The derivative is finite for $\beta \to \infty$. Taken together with the zero limit of the first factor of the third and forth term, this terms vanish in the limit. As a result, by continuity, there must exist a finite level of precision $\beta > \underline{\beta}_1 \in (0, \infty)$ such that $\frac{dJ(\Theta_1)}{d\Theta_1} < 0$ for all $\beta > \underline{\beta}_1$. This concludes the proof of Lemma 1.

A.2 Equilibrium in region 2

To study the equilibrium in region 2, we first analyze the coordination stage in section A.2.1. The main results are on Bayesian updating and on the existence of unique attack rules are summarized in Lemma 3 and Corollary 2, respectively. Next, we analyze the information stage in sections A.2.2 and A.2.3. The main results are summarized in Lemma 4, which describes how fundamental and signal thresholds depend on the proportion of informed investors and in Lemma 5, which establishes a strategic complementarity in information choices. Finally, we prove Lemma 2 in section A.2.4 and Proposition 1 in section A.2.5.

A.2.1 Coordination stage in region 2

The optimal behavior of investors in region 2 at the coordination stage can be described by extending the results from region 1. Let $f \in \{0,1\}$ indicate whether a crisis occurred in region 1, where $f=1$ corresponds to a crisis and $f=0$ corresponds to no crisis. Investors use the information about region 1 to update their prior about their beliefs about the distribution of the macro shock, using Bayes' rule:

$$p' \equiv \Pr\{m=\Delta|f\} \quad = \quad p\Pr\{f|m=\Delta\}\,\Gamma_2^{-1} \tag{18}$$

$$q' \equiv \Pr\{m=-s\Delta|f\} \quad = \quad q\Pr\{f|m=-s\Delta\}\,\Gamma_2^{-1}, \tag{19}$$

with $\Pr\{f=1|m\} = \Pr\{\Theta_1 < \Theta_1^*|m\}$ and $\Gamma_2 \equiv p\Pr\{f|m=\Delta\} + q\Pr\{f|m=-s\Delta\} + (1-p-q)\Pr\{f|m=0\}$.

Lemma 3 states the evolution of the beliefs about the macro shock.

Lemma 3 ***Beliefs about the macro shock.*** *The wake-up call of a crisis in region* 1 *is associated with less favorable beliefs about the macro shock, while no crisis in region* 1 *is associated with more favorable beliefs about the macro shock:*

$$\begin{cases} p' < p, \;\; q' > q & \textit{if } f=1 \\ p' > p, \;\; q' < q & \textit{if } f=0. \end{cases}$$

Moreover, we can state that:

$$\begin{cases} \frac{p'}{1-q'} < \frac{p}{1-q}, \frac{q'}{1-p'} > \frac{q}{1-p} & if\ f=1\ and\ n_1 \in \{0,1\} \\ \frac{p'}{1-q'} > \frac{p}{1-q}, \frac{q'}{1-p'} < \frac{q}{1-p} & if\ f=0\ and\ n_1 \in \{0,1\}. \end{cases}$$

The first set of inequalities are an extension of a comparative static in Morris and Shin (2003) and Vives (2005). For the special case of $n_1 = 1$, we have $\frac{d\Theta_1^*(1,m)}{dm} < 0$. Similarly for the general case, a more favorable information about fundamentals is associated with a lower fundamental threshold. The results follow from Bayesian updating in equations (18) and (19). The second set of inequalities on the right-hand side follow from $\frac{d}{dm}\big(\Pr\{f=1|m\} - \Pr\{f=0|m\}\big) < 0$. The results are immediate for $n_1 \in \{0,1\}$ and also hold for the general case, $n_1 \in [0,1]$, if the thresholds are monotone in n_t. We show this monotonicity in Lemma 4.

Using the updated p' and q' as weights, the belief about Θ_2 prior to receiving a private signal x_{i2} follows again a mixture distribution. It is an average over the cases of negative, zero and positive macro shocks with weights depending on f:

$$\Theta_2|f \equiv p'\left[\Theta_2|m=-s\Delta\right] + q'\left[\Theta_2|m=\Delta\right] + (1-p'-q')\left[\Theta_2|m=0\right]. \tag{20}$$

For the general case of $n_2 \in [0,1]$ we have seven equations in seven unknowns. Three critical mass conditions state that the proportion of attacking investors $A_2^*(m)$ equals the fundamental threshold $\Theta_2^*(m)$ for each realized $m \in \{-s\Delta, 0, \Delta\}$:

$$\Theta_2^*(m) = n_2\Phi\big(\sqrt{\beta}[x_I^*(m) - \Theta_1^*(m)]\big) + (1-n_2)\Phi\big(\sqrt{\beta}[x_U^* - \Theta_1^*(m)]\big), \tag{21}$$

where the short-hands are $\Theta_2^*(m) \equiv \Theta_2^*(n_2,m)$, $x_I^*(m) \equiv x_{2I}^*(n_2,m)$, and $x_U^* \equiv x_{2U}^*(n_2)$ for the fundamental threshold and the signal thresholds of informed and uninformed investors, respectively.

The first indifference condition states for each $n_2 \in [0,1]$ that an uninformed investor with threshold signal $x_{i2} = x_U^*$ is indifferent whether to attack:

$$\underbrace{\begin{aligned}&\hat{p}^{*}\Psi(\Theta_2^*(\Delta),x_U^*,\Delta)+\hat{q}^{*}\Psi(\Theta_2^*(-s\Delta),x_U^*,-s\Delta)\\&\qquad\qquad+(1-\hat{p}^{*}-\hat{q}^{*})\Psi(\Theta_2^*(0),x_U^*,0)\end{aligned}}_{\equiv J(n_2,\Theta_2^*(\Delta),\Theta_2^*(-s\Delta),\Theta_2^*(0),x_U^*)} = \gamma_2 \tag{22}$$

where $\hat{p}'^{*}=\hat{p}'(x_U^*)$ and $\hat{q}'^{*}=\hat{q}'(x_U^*)$ solve equation (13) after replacing p and q with p' and q'. Moreover, $\Psi(\Theta_2^*(m),x_d^*,m)\equiv\Phi\big(\Theta_2^*\sqrt{\alpha+\beta}-\frac{\alpha(\mu+m)+\beta x_d^*}{\sqrt{\alpha+\beta}}\big)$ for $d\in\{I,U\}$ and $m\in\{-s\Delta,0,\Delta\}$.

Three additional indifference conditions, one for each realized macro shock, state that an informed investor is indifferent between attacking or not upon receiving the threshold signal $x_{i2}=x_I^*(m)$:

$$\Psi\left(\Theta_2^*(n_2,m),x_I^*(m),m\right)=\gamma_2\ \ \forall\, m\in\{-s\Delta,0,\Delta\}. \tag{23}$$

For the special case of the equilibrium in region 1 with $n_1=0$, we had two thresholds x_1^* and Θ_1^* for each m. There, the objective was to establish aggregate behavior by inserting the critical mass condition, which states x_1^* in terms of Θ_1^*, into the indifference condition. This yields one equation implicit in Θ_1^*. We pursue a similar strategy here and express the equilibrium in terms of $\Theta_2^*(-s\Delta)$, $\Theta_2^*(0)$ and $\Theta_2^*(\Delta)$ only.

To simplify the system of equations, we can use the following insight. Since uninformed investors do not observe the macro shock realization, the signal threshold must be identical across these realizations, $x_U^*\equiv x_U^*(-s\Delta)=x_U^*(0)=x_U^*(\Delta)$. In the following steps, we derive this threshold for either realization of m by using $\Theta_2^*(m)$ and equalize both expressions. First, we use the critical mass conditions in equation (21) for $\Theta_2^*(m)$ to express x_U^* as a function of each $\Theta_2^*(m)$ and $x_I^*(m)$. Second, we use the indifference condition of informed investors for each m to obtain $x_I^*(m)$ as a function of $\Theta_2^*(m)$. Thus, $\forall m$:

$$x_U^*(m)=\Theta_2^*(m)+\Phi^{-1}\left(\frac{\Theta_2^*(m)-n_2\Phi\Big(\frac{\alpha(\Theta_2^*(m)-(\mu+m))-\sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\Big)}{1-n_2}\right)/\sqrt{\beta}. \tag{24}$$

Hence, for $m\in\{-s\Delta,0,\Delta\}$, there exists a $\underline{\beta}_2\in(0,\infty)$ such that for all $\beta>\underline{\beta}_2$: $\frac{dx_U^*(m)}{d\Theta_2^*(m)}>0$.

Since the signal threshold is the same for an uninformed investor, subtracting equation (24) evaluated at $m = 0$ from the same equation evaluated at $m = -s\Delta$ or at $m = \Delta$ must yield zero. This yields the first two pair-wise implicit relationships between $\Theta_2^*(-s\Delta)$, $\Theta_2^*(0)$ and $\Theta_2^*(\Delta)$:

$$K_1(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0)) \equiv x_U^*(0) - x_U^*(-s\Delta) = 0 \tag{25}$$

$$K_2(n_2, \Theta_2^*(0), \Theta_2^*(\Delta)) \equiv x_U^*(0) - x_U^*(\Delta) = 0. \tag{26}$$

Now, we construct the third implicit relationship between the three aggregate thresholds by inserting equation (24) evaluated at each m in $\Psi(\Theta_2^*(m), x_U^*(m), m)$, respectively, and in $\hat{p}(p')$ and $\hat{q}(q')$ as used in J:

$$L(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0), \Theta_2^*(\Delta)) \equiv J(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0), \Theta_2^*(\Delta)) = \gamma_2. \tag{27}$$

Corollary 2 establishes existence and uniqueness for a given $n_2 \in [0,1]$ under the conditions of Assumption 1 by analyzing the system of equations given by (25), (26) and (27).

Corollary 2 ***Existence of unique attack rules in region 2.*** *If private information is sufficiently precise, then for any proportion of informed investors in region 2, $n_2 \in [0,1]$, there exist unique attack rules for informed investors, $a_I^*(m,\cdot)$, and for uniformed investors, $a_U^*(\cdot)$.*

Proof Notice that the first and the second equation depend only on two thresholds, $K_1(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0)) = 0$ and $K_2(n_2, \Theta_2^*(0), \Theta_2^*(\Delta)) = 0$, while the third equation depends on all three, $L(n_2, \Theta_2^*(-s\Delta), \Theta_2^*(0), \Theta_2^*(\Delta)) = \gamma_2$. In a first step, we analyze, for a given n_2, the relationship between $\Theta_2(-s\Delta)$ and $\Theta_2(0)$, as governed by K_1. We obtain $\frac{\partial K_1}{\partial \Theta_2^*(0)} > 0$, $\frac{\partial K_1}{\partial \Theta_2(-s\Delta)} < 0$, and $\frac{\partial K_1}{\partial \Theta_2(\Delta)} = 0$. Hence, $\frac{d\Theta_2(0)}{d\Theta_2(-s\Delta)} > 0$ by the implicit function theorem. Likewise, we analyze the relationship between $\Theta_2^*(0)$ and $\Theta_2^*(\Delta)$, as governed by K_2. We obtain $\frac{\partial K_2}{\partial \Theta_2^*(0)} > 0$, $\frac{\partial K_2}{\partial \Theta_2(-s\Delta)} = 0$, and $\frac{\partial K_2}{\partial \Theta_2(\Delta)} < 0$. Hence, $\frac{d\Theta_2(0)}{d\Theta_2(\Delta)} > 0$ by the implicit function theorem. These results do not require a bound on the precision of private information.

In a second step, we analyze, for a given n_2, the relationship between all three fundamental thresholds, as governed by L. We know from our analysis of the case of informed investors that $\frac{d\Psi(\Theta_2,m)}{d\Theta_2} < 0$ for all m if $\beta > \frac{\alpha^2}{2\pi}$. Analogous to the argument in the proof of Lemma 1, there exists a sufficiently high but finite value of the private precision such that $\frac{\partial L}{\partial \Theta_2(m)} < 0$ for all m. Hence, in the limit $\frac{d\Theta_2(0)}{d\Theta_2(-s\Delta)} < 0$ for a given $\Theta_2(\Delta)$, $\frac{d\Theta_2(0)}{d\Theta_2(\Delta)} < 0$ for a given $\Theta_2(-s\Delta)$, and $\frac{d\Theta_2(\Delta)}{d\Theta_2(-s\Delta)} < 0$ for a given $\Theta_2(0)$. By continuity, there exists a finite precision of private information, $\underline{\beta}_2 \in (0,\infty)$, that guarantees the inequality if $\beta > \underline{\beta}_2$.

In a third step, we establish uniqueness conditional on existence. Thus suppose for now that an equilibrium exists. Then, due to the monotonicity and the opposite signs of the respective derivatives, we have that there is a single crossing of K_1 and L in the $(\Theta_2(-s\Delta),\Theta_2(0))$ space and a single crossing of K_2 and L in the $(\Theta_2(\Delta),\Theta_2(0))$ space, as shown in Figure 5. Observe that this is a "partial equilibrium" argument since the third threshold is taken as given. We now move to a "general equilibrium" argument. The argument builds on a second feature of the system, the opposite signs of the respective derivatives are not only a sufficient condition for a single crossings in the two panels of Figure 5, but they also imply that $\Theta_2(-s\Delta)$ and $\Theta_2(0)$ are each decreasing in $\Theta_2(\Delta)$ (left panel), where an increase in $\Theta_2(\Delta)$ shifts the L curve inwards. Likewise, $\Theta_2(\Delta)$ and $\Theta_2(0)$ are each decreasing in $\Theta_2(-s\Delta)$ (right panel). Hence, starting from a general equilibrium, any modification of $\Theta_2(\Delta)$ and $\Theta_2(-s\Delta)$ must lead to a violation of the system of equations. Hence, given $\frac{\partial L}{\partial\Theta_2(\Delta)} < 0$ and $\frac{\partial L}{\partial\Theta_2(-s\Delta)} < 0$, the combination of fundamental thresholds $(\Theta_2^*(-s\Delta),\Theta_2^*(0),\Theta_2^*(\Delta))$ that satisfies K_1 and L in the $(\Theta_2(-s\Delta),\Theta_2(0))$ space and K_2 and L in the $(\Theta_2(\Delta),\Theta_2(0))$ space is unique.

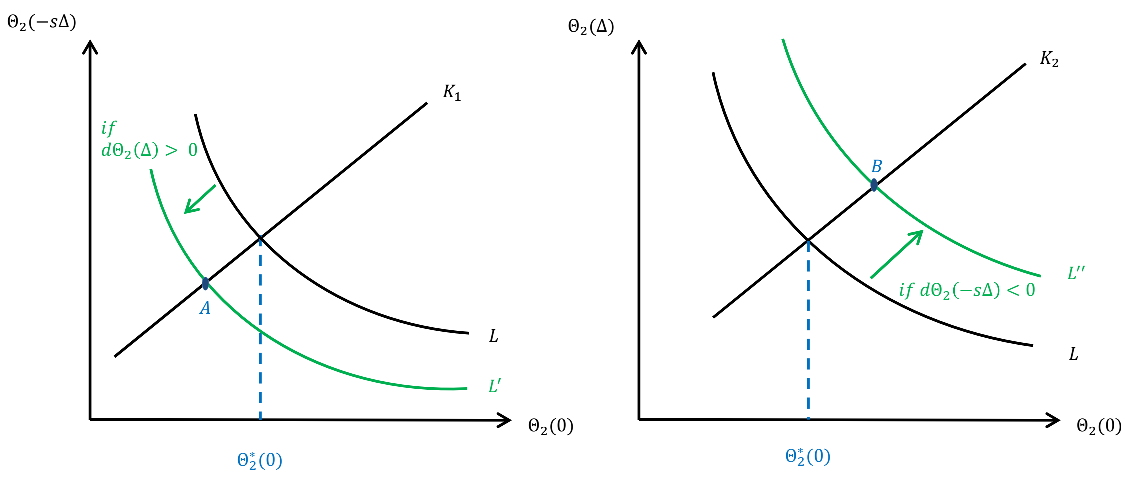

Figure 5: Single crossing.

In a fourth step, we establish the existence of a combination of fundamental thresholds. Existence can be shown by proving the following sequence of points: (i) for the highest permissible value of $\Theta_2(-s\Delta)$, the value of $\Theta_2(0)$ prescribed by K_1 is strictly larger than the value of $\Theta_2(0)$ prescribed by L; (ii) for the lowest permissible value of $\Theta_2(-s\Delta)$, the value of $\Theta_2(0)$ prescribed by K_1 is strictly smaller than the value of $\Theta_2(0)$ prescribed by L; (iii) for the highest permissible value of $\Theta_2(\Delta)$, the value of $\Theta_2(0)$ prescribed by K_2 is strictly larger than the value of $\Theta_2(0)$ prescribed by L; (iv) for the lowest permissible value of $\Theta_2(\Delta)$, the value of $\Theta_2(0)$ prescribed by K_2 is strictly smaller than the value of $\Theta_2(0)$ prescribed by L; (v) for the lowest (highest) permissible value of $\Theta_2(-s\Delta)$, also $\Theta_2(0)$ must be at its lowest (highest) permissible value from K_1 and, hence, also $\Theta_2(\Delta)$ must be at its lowest (highest) permissible value from K_2, leading to a violation of L in both the $(\Theta_2(-s\Delta),\Theta_2(0))$ space and the $(\Theta_2(\Delta),\Theta_2(0))$ space; (vi) a successive increase (decrease) in $\Theta_2(0)$ shifts L continuously inwards (outwards) in both spaces until a fixed point is reached.

Before addressing formally points (i)-(iv), we start by analyzing the following auxiliary step. For any $\Theta_2(m) \geq \Theta_2^*(1,m)$, it can be shown that:

$$\frac{\partial}{\partial n_2}\Phi^{-1}\left(\frac{\Theta_2(m) - n_2\Phi\left(\frac{\alpha(\Theta_2(m)-(\mu+m))-\sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\right)}{1-n_2}\right) \geq 0 \tag{28}$$

because $J(1,\Theta_2) \leq \gamma_2$ for any m. Note that both the previous expression and the partial derivative hold with strict inequality if $\Theta_2(m) > \Theta_2^*(1,m)$. Inspecting the inside of the inverse of the cdf, Φ^{-1}, we define the highest permissible values of $\Theta_2(m)$ that are labeled $\overline{\Theta}_2(n_2,m)$ for all m:

$$1 = \frac{\overline{\Theta}_2(n_2,m) - n_2\Phi\left(\frac{\alpha(\overline{\Theta}_2(n_2,m)-(\mu+m))-\sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_2)}{\sqrt{\beta}}\right)}{1-n_2}. \tag{29}$$

Hence, $1 \geq \overline{\Theta}_2(1,m) \geq \Theta_2^*(1,m)\ \forall m$, where the first (second) inequality binds if and only if $n_2 = 0$ ($n_2 = 1$).

We now prove points (i) and (iii). Evaluate K_1 and K_2 at the highest permissible value, $\Theta_2(0) = \overline{\Theta}_2(n_2,0)$, which yields $\overline{\Theta}_2(n_2,-s\Delta)$ and $\overline{\Theta}_2(n_2,\Delta)$, respectively. Likewise, evaluate L at the highest permissible values, $\overline{\Theta}_2(n_2,0)$ and $\overline{\Theta}_2(n_2,-s\Delta)$, which yields $\Theta_2(\Delta) < \overline{\Theta}_2(n_2,\Delta)$. Similarly, evaluate L at $\overline{\Theta}_2(n_2,0)$ and $\overline{\Theta}_2(n_2,\Delta)$, which yields $\Theta_2(-s\Delta) < \overline{\Theta}_2(n_2,-s\Delta)$. This proves points (i) and (iii).

Next, we now proceed with points (ii) and (iv). We can similarly define the lowest permissible value of $\Theta_2(m)$, which is labeled $\underline{\Theta}_2(n_2,m)$ for all m. Now, $0 \leq \underline{\Theta}_2(1,m) \leq \Theta_2^*(1,m)\ \forall m$, where the first (second) inequality binds if and only if $n_2 = 0$ ($n_2 = 1$). Evaluate K_1 and K_2 at the lowest permissible value, $\Theta_2(0) = \underline{\Theta}_2(n_2,0)$, which yields $\underline{\Theta}_2(n_2,-s\Delta)$ and $\underline{\Theta}_2(n_2,\Delta)$, respectively. Likewise, evaluate L at the lowest permissible values, $\underline{\Theta}_2(n_2,0)$ and $\underline{\Theta}_2(n_2,-s\Delta)$, which yields $\Theta_2(\Delta) > \underline{\Theta}_2(n_2,\Delta)$. Similarly, evaluate L at $\underline{\Theta}_2(n_2,0)$ and $\underline{\Theta}_2(n_2,\Delta)$, which yields $\Theta_2(-s\Delta) > \underline{\Theta}_2(n_2,-s\Delta)$. This proves points (ii) and (iv). The proof of points (v)-(vi) follows, which completes the overall proof of Corollary 2.

A.2.2 Information stage in region 2: proportion of informed investors and equilibrium thresholds

To characterize the value of information about the macro shock to investors, we describe how the fundamental and signal thresholds depend on the proportion of informed investors, as summarized below.

Lemma 4 ***Proportion of informed investors and equilibrium thresholds.*** *If Assumption 1 holds, then:*

(A) Boundedness. *The fundamental thresholds in case of informed investors bound the fundamental thresholds in case of asymmetrically informed investors:*

$$\Theta_2^*(1,\Delta) \leq \Theta_2^*(n_2,m) \leq \Theta_2^*(1,-s\Delta)\ \ \forall m \in \{-s\Delta, 0, \Delta\} \quad \forall n_2 \in [0,1]. \tag{30}$$

(B) Monotonicity in fundamental thresholds. *The fundamental threshold in the case of a negative (positive) macro shock increases (decreases) in the proportion of informed investors. Strict monotonicity is attained if and only if the fundamental thresholds are strictly bounded, that is* $\forall n_2 \in [0,1)$*:*

$$\frac{d\Theta_2^*(n_2,-s\Delta)}{dn_2} = \begin{cases} >0 \ \ if\ \Theta_2^*(n_2,-s\Delta) < \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) > \Theta_2^*(1,\Delta) \\ =0 \ \ if\ \Theta_2^*(n_2,-s\Delta) = \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) = \Theta_2^*(1,\Delta), \end{cases} \tag{31}$$

$$\frac{d\Theta_t^*(n_2,\Delta)}{dn_2} = \begin{cases} <0 \ \ if\ \Theta_2^*(n_2,-s\Delta) < \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) > \Theta_2^*(1,\Delta) \\ =0 \ \ if\ \Theta_2^*(n_2,-s\Delta) = \Theta_2^*(1,-s\Delta) \wedge \Theta_2^*(n_2,\Delta) = \Theta_2^*(1,\Delta). \end{cases} \tag{32}$$

(C) Monotonicity in signal thresholds. *As a consequence of the monotonicity in fundamental thresholds:*

$$\frac{d(x_I^*(n_2,-s\Delta)-x_I^*(n_2,\Delta)))}{dn_2} \geq 0, \ \ \forall\, n_2 \in [0,1), \tag{33}$$

where $x_I^*(n_2,-s\Delta)-x_I^*(n_2,\Delta)) > 0, \ \ \forall\, n_2 \in [0,1].$

Proof We formally prove the results of Lemma 4 in turn. Since the argument applies for both regions, we use the subscript t. A general observation is that the updated belief on the probability of a positive macro shock becomes degenerate: $\hat{p} \to p$ for $\alpha \to 0$. Results (A) and (B) are closely linked, so we start with them. It will be useful to consider a modified system of equations where either K_1 or K_2 are used alongside K_3:

$$K_3(n_t,\Theta_t^*(-s\Delta),\Theta_t^*(\Delta)) \equiv x_{tU}^*(-s\Delta)-x_{tU}^*(\Delta)=0. \tag{34}$$

Results (A) and (B). This prove has three steps.

Step 1: We show in the first step that for $1-p-q \to 0$ the fundamental thresholds $\Theta_t^*(-s\Delta)$ and $\Theta_t^*(\Delta)$ in the case of asymmetrically informed investors lie either both within these bounds or outside of them. As a consequence of $\hat{p} \to p$, condition $L(n_t,\Theta_t^*(-s\Delta),\Theta_t^*(\Delta))=0$ prescribes that, for any n_t, the thresholds $\Theta_t^*(\Delta)$ and $\Theta_t^*(-s\Delta)$ are either simultaneously within or outside of the two bounds given by the fundamental thresholds if all investors are informed, $\Theta_t^*(1,\Delta)$ and $\Theta_t^*(1,-s\Delta)$. This is proven by contradiction. First, suppose that $\Theta_t^*(\Delta) < \Theta_t^*(1,\Delta)$ and $\Theta_t^*(-s\Delta) < \Theta_t^*(1,-s\Delta)$. This leads to a violation of $L(\cdot)=0$ because $J(\cdot) > \gamma_t \,\forall\, n_t$ if $\alpha \to 0$. Second, suppose that $\Theta_t^*(\Delta) > \Theta_t^*(1,\Delta)$ and $\Theta_t^*(-s\Delta) > \Theta_t^*(1,-s\Delta)$. Again, leading to a violation because $J(\cdot) < \gamma_t \,\forall\, n_t$ if $\alpha \to 0$. By continuity, the results continue to hold provided that $1-p-q$ is sufficiently small. That is, there exists a threshold $\eta > 0$, such that the result holds provided the sufficient condition $1-p-q < \eta$.

Step 2: We now derive the derivatives of the fundamental thresholds with respect to the proportion of informed investors, $\frac{d\Theta_t^*(m)}{dn_t}$:

$$\frac{d\Theta_t^*(n,-s\Delta)}{dn_t} = \frac{\begin{vmatrix} -\frac{\partial K_{1,2}}{\partial n_t} & \frac{\partial K_{1,2}}{\partial\Theta_t(n_t,0)} & \frac{\partial K_{1,2}}{\partial\Theta_t(n_t,\Delta)} \\ -\frac{\partial K_3}{\partial n_t} & \frac{\partial K_3}{\partial\Theta_t(n_t,0)} & \frac{\partial K_3}{\partial\Theta_t(n_t,\Delta)} \\ -\frac{\partial L}{\partial n_t} & \frac{\partial L}{\partial\Theta_t(n_t,0)} & \frac{\partial L}{\partial\Theta_t(n_t,\Delta)} \end{vmatrix}}{\begin{vmatrix} \frac{\partial K_{1,2}}{\partial\Theta_t(n_t,-s\Delta)} & \frac{\partial K_{1,2}}{\partial\Theta_t(n_t,0)} & \frac{\partial K_{1,2}}{\partial\Theta_t(n_t,\Delta)} \\ \frac{\partial K_3}{\partial\Theta_t(n_t,-s\Delta)} & \frac{\partial K_3}{\partial\Theta_t(n_t,0)} & \frac{\partial K_3}{\partial\Theta_t(n_t,\Delta)} \\ \frac{\partial L}{\partial\Theta_t(n_t,-s\Delta)} & \frac{\partial L}{\partial\Theta_t(n_t,0)} & \frac{\partial L}{\partial\Theta_t(n_t,\Delta)} \end{vmatrix}} \equiv \frac{|M_1|}{|M|} \tag{35}$$

where $|M| \equiv \det(M)$. Similarly we can derive $\frac{d\Theta_t^*(n,0)}{dn_t} = \frac{|M_2|}{|M|}$ and $\frac{d\Theta_t^*(n,\Delta)}{dn_t} = \frac{|M_3|}{|M|}$.

To find $|M|$, recall from the proof of Proposition 2 that $\frac{\partial K_1}{\partial\Theta_t(0)} > 0$, $\frac{\partial K_1}{\partial\Theta_t(-s\Delta)} < 0$ and $\frac{\partial K_1}{\partial\Theta_t(\Delta)} = 0$, while $\frac{\partial K_2}{\partial\Theta_t(0)} > 0$, $\frac{\partial K_2}{\partial\Theta_t(-s\Delta)} = 0$ and $\frac{\partial K_2}{\partial\Theta_t(\Delta)} < 0$. Furthermore, $\frac{\partial K_3}{\partial\Theta_t(0)} = 0$, $\frac{\partial K_3}{\partial\Theta_t(\Delta)} < 0$ and $\frac{\partial K_3}{\partial\Theta_t(-s\Delta)} > 0$. Finally, $\frac{\partial L}{\partial\Theta_t(m)} < 0\ \forall m$ for a sufficiently high but finite value of β. As a result, $|M| > 0$ for a sufficiently high but finite value of β, irrespective of which of the two systems is used. That is, there exists a threshold $\beta > 0$, such that the result holds provided the sufficient condition $\beta > \underline{\beta}$.

The proof proceeds by analyzing $|M_1|$, $|M_2|$, and $|M_3|$. To do this, we first examine the derivatives $\frac{\partial K_1}{\partial n_t}$, $\frac{\partial K_3}{\partial n_t}$ and $\frac{\partial L}{\partial n_t}$. Thereafter, we combine the results to obtain the signs of the determinants.

$$\begin{aligned} \frac{\partial K_1}{\partial n_t} &= \frac{\partial x_{tU}(0)}{\partial n_t} - \frac{\partial x_{tU}(-s\Delta)}{\partial n_t} \qquad (36) \\ &= \sqrt{\frac{1}{\beta}}\,\frac{\Theta_t(0) - \Phi\left(\frac{\alpha(\Theta_t(0)-\mu)-\sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_t)}{\sqrt{\beta}}\right)}{(1-n_t)^2\phi(\Phi^{-1}(\ldots))} - \sqrt{\frac{1}{\beta}}\,\frac{\Theta_t(-s\Delta) - \Phi\left(\frac{\alpha(\Theta_t(-s\Delta)-(\mu-s\Delta))-\sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_t)}{\sqrt{\beta}}\right)}{(1-n_t)^2\phi(\Phi^{-1}(\ldots))} \\ \frac{\partial K_2}{\partial n_t} &= \frac{\partial x_{tU}(0)}{\partial n_t} - \frac{\partial x_{tU}(\Delta)}{\partial n_t} = \frac{\partial x_{tU}(0)}{\partial n_t} - \sqrt{\frac{1}{\beta}}\,\frac{\Theta_t(\Delta) - \Phi\left(\frac{\alpha(\Theta_t(\Delta)-(\mu+\Delta))-\sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_t)}{\sqrt{\beta}}\right)}{(1-n_t)^2\phi(\Phi^{-1}(\ldots))} \qquad (37) \\ \frac{\partial K_3}{\partial n_t} &= \frac{\partial x_{tU}(-s\Delta)}{\partial n_t} - \frac{\partial x_{tU}(\Delta)}{\partial n_t}. \qquad (38) \end{aligned}$$

To evaluate this partial derivatives, we can use the optimality condition in the case of symmetrically informed investors, $n_t = 1$. That is, $\Theta_t^*(1,m)$ is defined as the solution to $F_t(\Theta_t^*(1,m),m) = 0$, where uniqueness requires that F_t is strictly decreasing in the first argument. This implies:

$$\Theta_t(m) - \Phi\left(\frac{\alpha(\Theta_t(m) - (\mu + m)) - \sqrt{\alpha+\beta}\,\Phi^{-1}(\gamma_t)}{\sqrt{\beta}}\right) \lesseqqgtr 0 \; if \; \Theta_t(m) \lesseqqgtr \Theta_t(1,m).$$

Four cases are considered in turn. Case 1: $\Theta_t^*(1,\Delta) \leq \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,0) \leq \Theta_t^*(n_t,0) \leq \Theta_t^*(0,m) \leq \Theta_t^*(n_t,-s\Delta) \leq \Theta_t^*(1,-s\Delta))$. Case 2: $\Theta_t^*(1,\Delta) \leq \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(0,m) \leq \Theta_t^*(n_t,0) \leq \Theta_t^*(1,0) \leq \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,-s\Delta))$. Case 3: $\Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,\Delta) \leq \Theta_t^*(1,0) \leq \Theta_t^*(0,m) \leq \Theta_t^*(1,-s\Delta) \leq \Theta_t^*(n_t,-s\Delta)$. Case 4: $\Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,\Delta) \leq \Theta_t^*(0,m) \leq \Theta_t^*(1,0) \leq \Theta_t^*(1,-s\Delta) \leq \Theta_t^*(n_t,-s\Delta)$.

Case 1: Using K_1 and K_3 we obtain $\frac{\partial K_1}{\partial n} > 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} < 0 \,\forall\, n_t \in [0,1)$.

Case 2: Using K_2 and K_3 we obtain $\frac{\partial K_2}{\partial n} < 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} < 0 \,\forall\, n_t \in [0,1)$.

Case 3: Using K_1 and K_3 we obtain $\frac{\partial K_1}{\partial n} > 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} > 0 \,\forall\, n_t \in [0,1)$.

Case 4: Using K_2 and K_3 we obtain $\frac{\partial K_2}{\partial n} < 0 \,\forall\, n_t \in [0,1)$ and $\frac{\partial K_3}{\partial n} > 0 \,\forall\, n_t \in [0,1)$.

After having found the partial derivative for first two equilibrium conditions ($K_{1,2}$), we turn to the other equilibrium condition (L). Here, we can invoke the envelope theorem in order to obtain $\frac{\partial L}{\partial n} = 0$. The idea is the following. Since L represents the indifference condition of an uninformed investor, the proportion of informed investors enters only indirectly via x_{tU}^* and we can write:

$$\frac{\partial L}{\partial n} = \frac{\partial J}{\partial x_{tU}^*}\frac{\partial x_{tU}^*}{\partial n} + \overbrace{\frac{\partial J}{\partial n}}^{=0}. \tag{39}$$

Since x_{tU}^* is the optimal signal threshold of an uninformed investor, it satisfies $J(\cdot, x_{tU}^*) = \gamma_t$. Thus, we must have $\frac{\partial J}{\partial x_{tU}^*} = 0$, which corresponds to a first-order optimality condition. (This implicitly uses the result that the equilibrium is unique.)

To conclude, we have for all cases that $|M| > 0$ provided that $\beta > \underline{\beta}$. It shows that $|M_1| > 0$ for case 1 and $|M_3| < 0$ for case 2, while $|M_1| < 0$ for case 1 and $|M_3| > 0$ for case 2. Furthermore, for the probability of $m = 0$, i.e. $1 - p - q$, sufficiently small we have that $|M_1| > 0$ also for case 2 and $|M_3| < 0$ also for case 1, while $|M_1| < 0$ also for case 2 and $|M_3| > 0$ also for case 1. Hence, provided that $1-p-q<\eta$ and $\beta>\underline{\beta}$, we find $\forall n_t \in [0,1)$:

$$\frac{d\Theta_t^*(n_t,-s\Delta)}{dn_t}=\begin{cases} >0 \quad if\ \Theta_t^*(n_t,-s\Delta)<\Theta_t^*(1,-s\Delta)\wedge\Theta_t^*(n_t,\Delta)>\Theta_t^*(1,\Delta) \\ <0 \quad if\ \Theta_t^*(n_t,-s\Delta)>\Theta_t^*(1,-s\Delta)\wedge\Theta_t^*(n_t,\Delta)<\Theta_t^*(1,\Delta) \\ =0 \quad if\ \Theta_t^*(n_t,-s\Delta)=\Theta_t^*(1,-s\Delta)\wedge\Theta_t^*(n_t,\Delta)=\Theta_t^*(1,\Delta) \end{cases}$$

and $\forall n_t \in [0,1)$:

$$\frac{d\Theta_t^*(n_t,\Delta)}{dn_t}=\begin{cases} <0 \quad if\ \Theta_t^*(n_t,-s\Delta)<\Theta_t^*(1,-s\Delta)\wedge\Theta_t^*(n_t,\Delta)>\Theta_t^*(1,\Delta) \\ >0 \quad if\ \Theta_t^*(n_t,-s\Delta)>\Theta_t^*(1,-s\Delta)\wedge\Theta_t^*(n_t,\Delta)<\Theta_t^*(1,\Delta) \\ =0 \quad if\ \Theta_t^*(n_t,-s\Delta)=\Theta_t^*(1,-s\Delta)\wedge\Theta_t^*(n_t,\Delta)=\Theta_t^*(1,\Delta) \end{cases}$$

Step 3: In this final step, we combine the results from the previous two steps to show both boundedness and monotonicity. In particular, we use the result that the derivative of the fundamental threshold w.r.t. the proportion of informed investors is zero once the boundary is hit. Therefore, the thresholds in the general case of asymmetrically informed investors are always bounded, which proves Result (A). Given boundedness, in turn, the derivatives of the fundamental threshold can be clearly signed, yielding Result (B). That is, given the result from *step 1*, the second line of each derivative drops and equations (31) and (32) follow.

We prove that $\Theta_t^*(1,\Delta)\leq\Theta_t^*(\Delta),\Theta_t^*(-s\Delta)\leq\Theta_t^*(1,-s\Delta)$ for all n_t if α sufficiently small. First, $\Theta_t^*(1,\Delta)<\Theta_t^*(\Delta)=\Theta_t^*(0)=\Theta_t^*(-s\Delta)<\Theta_t^*(1,-s\Delta)$ if $n_t=0$, while $\Theta_t^*(1,\Delta)=\Theta_t^*(\Delta)$ and $\Theta_t^*(1,-s\Delta)=\Theta_t^*(-s\Delta)$ if $n_t=1$. Second, $\frac{d\Theta_t^*(\Delta)}{dn_t}\big|_{n_t=0}<0$, $\frac{d\Theta_t^*(-s\Delta)}{dn_t}\big|_{n_t=0}>0$ and $\frac{d\Theta_t^*(\Delta)}{dn_t}\big|_{n_t=1}=\frac{d\Theta_t^*(-s\Delta)}{dn_t}\big|_{n_t=1}=0$. Third, by continuity $\Theta_t^*(1,\Delta)\leq\Theta_t^*(\Delta),\Theta_t^*(-s\Delta)\leq\Theta_t^*(1,-s\Delta)$ and $\frac{d\Theta_t^*(\Delta)}{dn_t}\big|_{n_t=0}<0$, $\frac{d\Theta_t^*(-s\Delta)}{dn_t}\big|_{n_t=0}>0$ for small values of n_t. Fourth, if for any $\hat{n}_t\in(0,1]$ $\Theta_t^*(-s\Delta)\nearrow\Theta_t^*(1,-s\Delta)$ when $n_t\to\hat{n}_t$, then – for sufficiently small but positive values of α – it has to be true that $\Theta_t^*(\Delta)\searrow\Theta_t^*(1,\Delta)$ when $n_t\to\hat{n}_t$. This is because of the result in *step 1*. Fifth, given that the derivatives of the fundamental thresholds flip when both are outside of the bounds we have $\Theta_t^*(1,\Delta)=\Theta_t^*(\Delta)$ and $\Theta_t^*(1,-s\Delta)=\Theta_t^*(-s\Delta)$ for all $n_t\geq\hat{n}_t$. In conclusion, $\Theta_t^*(1,\Delta)\leq\Theta_t^*(\Delta),\Theta_t^*(-s\Delta)\leq\Theta_t^*(1,-s\Delta)$ for all $n_t\in[0,1]$ if α sufficiently small.

Result (C). From equation the indifference conditions for informed investors:

$$\frac{dx_{tI}^*(m)}{dn_t} \quad = \quad \frac{d\Theta_t^*(m)}{dn_t}\left(\frac{\beta}{\alpha+\beta}\right)^{-1}. \tag{40}$$

Therefore, by continuity, there exists a sufficiently small but positive value of α, say $\overline{\alpha}$, that implies the required inequality, taking into account the monotonicity of the fundamental thresholds. The distance between the fundamental thresholds is monotone for any $n_t > 0$, which implies $\frac{d(x_{tI}^*(n_t,-s\Delta)-x_{tI}^*(n_t,\Delta))}{dn_t} \geq 0 \ \forall \ n_t \in [0,1)$. Furthermore, $x_{tI}^*(n_t,-s\Delta) - x_{tI}^*(n_t,\Delta)) > 0 \ \forall \ n_t \in [0,1]$. This completes the proof.

A.2.3 Information stage in region 2: strategic complementarity in information choices

We next study the value of information about the macro shock. The value of information to an individual investor is defined as the difference in the expected utility between an informed and an uninformed investor before costs. These expected utilities are denoted by EU_I and EU_U, respectively. The expected utility of an informed investor writes:

$$\begin{aligned} &\mathbb{E}[u(d_i = I, n_2)] \equiv EU_I - c \\ = \quad & -c + p'\left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(n_2,\Delta)} b_t \int_{x_{i2}\leq x_I^*(n_2,\Delta)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|\Delta)d\Theta_2 \\ -\int_{\Theta_2^*(n_2,\Delta)}^{+\infty} \ell_2 \int_{x_{i2}\leq x_I^*(n_2,\Delta)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|\Delta)d\Theta_2 \end{array}\right)+ \\ & q'\left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(n_2,-s\Delta)} b_2 \int_{x_{i2}\leq x_I^*(n_2,-s\Delta)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|-s\Delta)d\Theta_2 \\ -\int_{\Theta_2^*(n_2,-s\Delta)}^{+\infty} \ell_2 \int_{x_{i2}\leq x_I^*(n_2,-s\Delta)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|-s\Delta)d\Theta_2 \end{array}\right)+ \\ & (1-p'-q')\left(\begin{array}{c} \int_{-\infty}^{\Theta_2^*(n_2,0)} b_2 \int_{x_{i2}\leq x_I^*(n_2,0)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|0)d\Theta_2 \\ -\int_{\Theta_2^*(n_2,0)}^{+\infty} \ell_2 \int_{x_{i2}\leq x_I^*(n_2,0)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|0)d\Theta_2 \end{array}\right), \end{aligned} \tag{41}$$

By contrast, the expected utility of an uninformed investor, $E[u(d_i = U, n_2)] \equiv EU_U$, is in constructed in the same way as EU_I with the difference that all signal thresholds have to be replaced by $x_U^*(n_2)$.

Let $v \equiv EU_I - EU_U$ be the value of information conditional on the proportion of informed investors and the information set in region 2:

$$v(n_2) = \quad p'\begin{pmatrix} \int_{-\infty}^{\Theta_2^*(n_2,\Delta)} b_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,\Delta)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|\Delta)d\Theta_2 \\ -\int_{\Theta_2^*(n_2,\Delta)}^{+\infty} \ell_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,\Delta)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|\Delta)d\Theta_2 \end{pmatrix}$$
$$+q'\begin{pmatrix} \int_{-\infty}^{\Theta_2^*(n_2,-s\Delta)} b_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,-s\Delta)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|-s\Delta)d\Theta_2 \\ -\int_{\Theta_2^*(n_2,-s\Delta)}^{+\infty} \ell_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,-s\Delta)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|-s\Delta)d\Theta_2 \end{pmatrix}$$
$$+(1-p'-q')\begin{pmatrix} \int_{-\infty}^{\Theta_2^*(n_2,0)} b_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,0)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|0)d\Theta_2 \\ -\int_{\Theta_2^*(n_2,0)}^{+\infty} \ell_2 \int_{x_U^*(n_2)}^{x_I^*(n_2,0)} g(x_{i2}|\Theta)dx_{i2}f(\Theta_2|0)d\theta_2 \end{pmatrix}. \tag{42}$$

The distribution of the fundamental conditional on the realized macro shock, $f(\Theta_2|m)$, is normal with mean $\mu+m$ and precision α. The distribution of the private signal conditional on the fundamental, $g(x|\Theta_2)$, is normal with mean Θ_2 and precision β.

To build intuition, suppose that $1-p-q \to 0$. Given $\Theta_2^*(1,-s\Delta) > \Theta_2^*(1,\Delta)$ we have that $x_I^*(n_2,-s\Delta) > x_U^*(n_2) > x_I^*(n_2,\Delta)$. Thus, the marginal benefit of increasing $x_I^*(n_2,-s\Delta)$ above $x_U^*(n_2)$ is:

$$p'\begin{pmatrix} b_2 \int_{-\infty}^{\Theta_2^*(n_2,-s\Delta)} g(x_U^*|\Theta_2)f(\Theta_2)d\Theta_2 \\ -\ell_2 \int_{\Theta_2^*(n_2,-s\Delta)}^{+\infty} g(x_U^*|\Theta_2)f(\Theta_2)d\Theta_2 \end{pmatrix} > 0, \tag{43}$$

while the marginal benefit of increasing $x_I^*(n_2,\Delta)$ above $x_U^*(n_2)$ is:

$$q'\begin{pmatrix} b_2 \int_{-\infty}^{\Theta_2^*(n_2,\Delta)} g(x_U^*|\Theta_2)f(\Theta_2|\Delta)d\Theta_2 \\ -\ell_2 \int_{\Theta_2^*(n_2,\Delta)}^{+\infty} g(x_U^*|\Theta_2)f(\Theta_2|\Delta)d\Theta_2 \end{pmatrix} < 0. \tag{44}$$

These expressions are best understood in terms of type-I and type-II errors. Each of the expressions in equations (43) and (44) have two components. The first component in each equation represents the marginal benefit of attacking when a crisis occurs. Equivalently, this is the marginal loss from not attacking when a crisis occurs (type-I error). The second component in each equation is negative and represents the marginal cost of attacking when no crisis occurs (type-II error).

Lemma 4 together with Corollary 2 imply the following. The marginal benefit of increasing $x_I^*(n_2,-s\Delta)$ above $x_U^*(n_2)$ is positive because the type-I error is relatively more costly than the type-II error. By contrast, the marginal benefit of decreasing $x_I^*(n_2,\Delta)$ below $x_U^*(n_2)$ is positive because the type-II error is more costly. In sum, informed investors attack more aggressively upon learning that $m = -s\Delta$ and less aggressively upon learning $m = \Delta$. The value of information is governed by the relationship between the type-I and type-II errors. When the signal threshold of informed and uninformed investors differ, the value of information is positive because the difference in signal thresholds increases in the proportion of informed investors. This gives rise to the result in Lemma 5.

Lemma 5 ***Strategic complementarity in information choices.*** *If Assumption 1 holds, the value of information increases in the proportion of informed investors:*

$$\frac{dv(n_2, f)}{dn_2} \geq 0, \tag{45}$$

with strict inequality for small values of n_2.

Proof Under the sufficient conditions of Assumption 1 we have that $\Theta_2^*(n_2, -s\Delta) > \Theta_2^*(n_2, \Delta)$ and $x_I^*(n_2, -s\Delta) > x_U^*(n_2, 0) > x_I^*(n_2, \Delta)$. We will prove that $\frac{dv(n_2, f)}{dn_2} \geq 0$ and $v(n_2, f) > 0 \ \forall\, n_2 \in (0, 1] \wedge f \in \{0, 1\}$. Suppose that $1 - q - p \to 0$, then the last term of $\mathbb{E}[u(d_i = I, n_2)]$ and $\mathbb{E}[u(d_i = U, n_2)]$ vanishes. Given that $\Theta_2^*(n_2, -s\Delta) > \Theta_2^*(n_2, \Delta)$ the first two summands of equation (42) are strictly positive and, hence, $v(n_2) > 0 \ \ \forall\, n_2 \in (0, 1]$. Furthermore, given Lemma 4, an increase in the proportion of informed investors is associated with a (weak) increase in both $\Theta_2^*(n_2, -s\Delta)$ and $x_I^*(n_2, -s\Delta)$ as well as a (weak) decrease in both $\Theta_2^*(n_2, \Delta)$ and $x_I^*(n_2, \Delta)$. For a given x_U^*, an increase in n_2 leads to a relative increase of the (positive) loss component in the first summand of equation (42) and a relative increase of the benefit component in the second summand. By continuity and monotonicity, any general equilibrium adjustment of $x_U^*(n_2)$ with n_2 cannot fully off-set the previous effects. For this reason, the left-hand side of equation (42) increases in n_2. Thus, $\frac{dv(n_2, f)}{dn_2} \geq 0$. By continuity, the results continue to hold if $1 - p - q$ is sufficiently small, that is if $1 - p - q < \eta$. This concludes the proof.

A.2.4 Proof of Lemma 2

We prove the results of inequalities (7) and (8) in turn. Given Assumption 1, the results of Lemma 5 apply. Therefore, the first and third inequality of (7) follow. The proof of the second inequality builds on equation (8) and consists of four steps.

Step 1: Suppose that $1-p-q \to 0$ and evaluate equation (42) at $n_2 = 1$. First, observe that the first term in brackets is only affected by s through $x_U^*(1)$. Second, observe that the second term in brackets is growing strictly larger in s for a given $x_U^*(1)$, as $x_I^*(1,-s\Delta)$ grows in s because of the indifference condition of informed investors. Third, observe that $x_U^*(1) \to x_I^*(1,\Delta)$ as $s \to \infty$. Given that the term in in the second bracket is finite and multiplied by $q = \frac{p}{s}$, we have that $v(1, f = 0) > v(0, f = 0) \to 0$ for $s \to \infty$, where the inequality is due to the result in Lemma 5.

Step 2: Now, suppose that $f = 1$ and observe that:

$$\frac{\partial}{\partial s}\left(\frac{q'}{p'} = s\frac{\Pr\{f=1, m=-s\Delta\}}{\Pr\{f=1|m=\Delta\}}\right) > 0.$$

Moreover:

$$q' = \frac{\frac{p}{s}\Pr\{f=1|m=-s\Delta\}}{\frac{p}{s}\Pr\{f=1|m=-s\Delta\} + p\Pr\{f=1|m=\Delta\} + (1-p-\frac{p}{s})\Pr\{f=1|m=0\}},$$

is increasing in μ, with $q' \to 1$ if $\mu \to \infty$. The results flip if $f = 0$.

Step 3: Observe that, for a given μ and $s > 1$, the event of a negative macro shock is never considered to be the most probable state of the world provided that s is sufficiently high. This is because $q' < p'$ holds for s sufficiently high: $s \geq \Pr\{f|m=-s\Delta\}(\Pr\{f|m=\Delta\})^{-1}$.

Step 4: Given the comparative statics in *step 2*, we have that $[q'|f=1] >> 0$ for large values of s and μ. From *step 3*, the first and the second summand of $v(1, f = 1)$ must be strictly positive and away from zero if $s \to \infty$, since now $x_U^* \nrightarrow x_I^*(1,\Delta)$ and $x_U^* \nrightarrow x_I^*(1,-s\Delta)$ as $s \to \infty$. By continuity, the result also holds for large, but finite, values of μ and s, as well as for sufficiently small $1-p-q$. Hence, inequality (7) follows provided that Assumption 1 holds and μ and s are sufficiently high. Hence, $v(n_2 = 0, f = 1) > v(n_2 = 1, f = 0)$.

Next, we prove inequality (8). Again suppose that $1-p-q \to 0$ and evaluate equation (42) at $n_2 = 1$. Recall that the first term in brackets is only affected by s through $x_U^*(1)$. Moreover, from equation (22) we have that $x_U^*(1)$ strictly decreases (increases) in s for $n_2 = 1$ and $f = 1$ ($f = 0$), since $\Theta_2^*(m)$ is independent of s and $\frac{\hat{q}}{\hat{p}}$ strictly increases (decreases) in s provided that s sufficiently high. Then $\frac{d\Pr\{x_{i1}|m=-s\Delta\}}{ds}$ is small since the slope goes to zero. Taken together with the results from *step 1* above, inequality (8) follows.

A.2.5 Proof of Proposition 1

The proof builds on the analysis of the coordination and information stages in region 2. Corollary 2 establishes the existence of unique attack rules in region 2. Lemma 2 establishes the existence of a nonempty intermediate range of information costs $c \in (\underline{c}, \overline{c})$ with $\underline{c} \equiv v(1,0)$ and $\overline{c} \equiv v(0,1)$, such that all investors choose to acquire information if and only if a crisis occurs in region 1. The result in Proposition 1 follows.

A.3 Proof of Proposition 2

The proof consists of four steps. First, suppose that $s \to \infty$. Not observing a crisis in region 1 implies that $\frac{q'}{q} \to 0$ as q' goes to zero faster than q. To see this, observe that $\Pr\{f = 0|m = -s\Delta\} \to 0$ if $s \to \infty$, since a Θ_2 drawn from a distribution with a highly negative mean, $\mu - s\Delta$, is increasingly unlikely to have a sufficiently high realization such that $f = 0$ occurs. At the same time, $\frac{p'}{p} \nrightarrow 0$ and $\frac{1-p'-q'}{1-p-q} \nrightarrow 0$ if $s \to \infty$ and $f = 0$.

Second, the right-hand side of inequality (9) has a fundamental threshold that is lower than the fundamental threshold on the left-hand side. To see this, we again use the comparative static result underlying Lemma 3. Observing $f = 0$ implies that the second summand of J in equation (22) goes to zero if $s \to \infty$. Hence, $\Theta_2^*(n_2 = 0, f = 0) < \Theta_2^*(n_2 = 1, f = 1, m = 0)$.

Third, given $s \to \infty$, the Θ's on the right-hand side of inequality (9) are drawn from equally favorable or, with a positive probability ($\frac{p'}{p} \nrightarrow 0$) that is away from zero, from a more favorable distribution if $f = 0$. Taken together, the likelihood of a crisis in region 2 is lower if $f = 0$ and $s \to \infty$.

Fourth, by continuity, the result can be generalized to hold for a sufficiently high, but finite, value of s, say $s > \underline{s}$. This concludes the proof.

A.4 Proof of Proposition 3

The proof builds on equation (42) and consists of three steps.

Step 1: Following Metz (2002), we analyze how the precision of public information affects the fundamental thresholds. That is, we derive the following comparative static in case of $n_2 = 1$:

$$\frac{d\Theta_2^*(n_2=1)}{d\alpha_2} = \begin{cases} <0 & if \;\; \Theta_2^* < (\mu+m) + \frac{1}{2\sqrt{\alpha_2+\beta}}\Phi^{-1}(\gamma_2) \\ \geq 0 & otherwise \end{cases} \tag{46}$$

Recall that the signal and fundamental thresholds are in a one-to-one relationship: $x_2^* = \Theta_2^* + \beta^{-\frac{1}{2}}\Phi^{-1}(\Theta_2^*)$. For sufficiently high values of μ and s (satisfied because of Assumption 1), we obtain $\frac{d\Theta_2^*(\Delta)}{d\alpha_2} < 0$, $\frac{dx_2^*(\Delta)}{d\alpha_2} < 0$, $\frac{d\Theta_2^*(-s\Delta)}{d\alpha_2} > 0$, and $\frac{dx_2^*(-s\Delta)}{d\alpha_2} > 0$.

Step 2: Suppose that $1-p-q \to 0$ and evaluate the value of information at $n_2 = 1$ in equation (42). All terms are affected by α_2 through the conditional distribution $f(\Theta_2|m)$, which becomes more concentrated as α_2 increases. Moreover, greater transparency affects the signal and fundamental thresholds.

We now look at these terms in greater detail. First, for a given $x_U^*(0)$, the loss term in the first bracket grows relatively larger in α_2 due to the reduction in $\Theta_2^*(1,\Delta)$. This effect is further strengthened by a more concentrated $f(\Theta_2|\Delta)$, since $\Theta_2^*(1,\Delta) < \mu+\Delta$ for a sufficiently high μ. Second, for a given $x_U^*(0)$, the benefit term in the second bracket grows relatively larger in α_2 due to the increase in $\Theta_2^*(1,-s\Delta)$. This effect is further strengthened by a more concentrated $f(\Theta_2|-s\Delta)$, since $\Theta_2^*(1,-s\Delta) > \mu - s\Delta$ if s is sufficiently high. As a result, for a given $x_U^*(0)$, the value of information $v(1,f)$ increases in α_2 for sufficiently high values of μ and s, as guaranteed by Assumption 1.

Step 3: By continuity and monotonicity, any general equilibrium adjustment of x_{2U}^* with α_2 cannot fully off-set the previous effects. Thus $\frac{dv(1,f)}{d\alpha_2} > 0 \; \forall f \in \{0,1\}$ for sufficiently high values of μ and s. By continuity, the result continues to hold if $1-p-q$ is sufficiently small, that is if $1-p-q < \eta$. This concludes the proof.

A.5 Proof of Proposition 4

The proof consists of two steps that deal with the impact on the fundamental thresholds and conditional distribution. First, we consider the fundamental thresholds in regions 2 after observing the outcome in region 1. From Lemma 3, we have that $f = 1$ coincides with $p' < p$, $q' > q$, and $\frac{p'}{1-q'} < \frac{p}{1-q}$, while the reverse inequalities hold if $f = 0$. Hence, observing $f = 1$ induces a lower weight on the first summand and a higher weight on the second summand of J in equation (22), while the effect of the third summand is ambiguous. Still, from $\frac{q'}{1-p'} > \frac{q}{1-p}$ it follows that the relative increase of the weight on the second summand must be higher when compared to the potential increase of the weight on the third summand.

Hence, using the comparative static result underlying Lemma 3, the $\Theta_2^*(0,m)$ that solves the version of equation (22) for region 2 and $n_2 = 0$ must be higher after observing a crisis in region 1 due to more aggressive attacks after unfavorable public information. A higher fundamental threshold is, ceteris paribus, associated with a higher conditional probability of a crisis in region 2.

Second, the distribution of the unobserved macro shock is updated after observing the outcome in region 1. Specially, the distribution of the fundamental of region 2 conditional on a crisis in region 1 ($f = 1$) is less favorable than the distribution of the fundamental of region 2 conditional on no crisis in region 1 ($f = 0$). The second effect strengthens the first effect. This concludes the proof.

A.6 Equilibrium in region 1 for the general case $n_1 \in [0,1]$

Figure 6 depicts the unique equilibrium fundamental thresholds for each realization of the macro shock over the range of $n_1 \in [0,1]$.[11] We modify the notation to account for the dependency on the proportion of informed investors. Let $\Theta_1^*(n_1,m)$ be the fundamental threshold in region 1 as a function of n_1, and $x_{1I}^*(n_1,m)$ and $x_{1U}^*(n_1,m)$ the signal thresholds of informed and uninformed investors, respectively. For $n_1 = 0$ the fundamental thresholds are equalized and follow Lemma

[11]The numerical simulations use equation (47) for $n_1 = 1$ and the system of equations for the case $n_1 \in [0,1)$. Notably values of n_1 close to, but smaller than, one require the use of interpolation methods since the term $1 - n_1$ shows up in denominators.

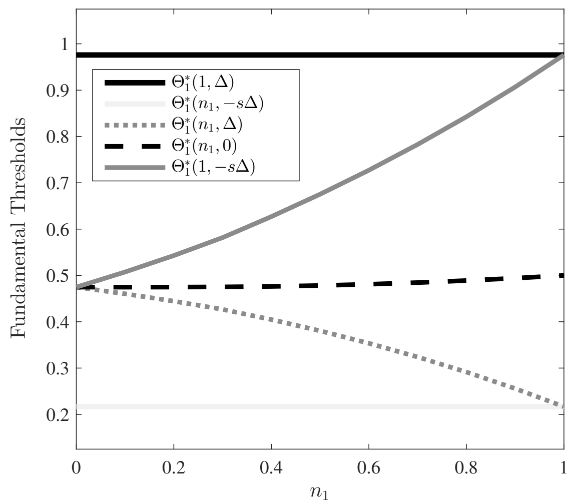

Figure 6: Fundamental thresholds are monotonic in the proportion of informed investors, n_1, converging to the thresholds when all investors are informed about the realization of m. Parameter values are $\alpha = \beta = 1$, $\mu = 1/2$, $p = 1/4$, $\Delta = 1/2$, $s = 3$, $\gamma_1 = \gamma_2 = 1/2$.

1. For $n_1 = 1$, the fundamental thresholds for each realization of the macro shock are equal and solve equations (14) and (15). We know from standard global games models that for equilibrium is characterized by:

$$\Phi\left(\frac{\alpha}{\sqrt{\alpha+\beta}}\left[\Theta_1^*(1,m)-(\mu+m)\right]-\sqrt{\frac{\beta}{\alpha+\beta}}\Phi^{-1}\left(\Theta_1^*(1,m)\right)\right)=\gamma_1 \tag{47}$$

and the signal threshold x_1^* is defined by $x_{1I}^*(1,m) = \Theta_1^*(1,m) + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Theta_1^*(1,m))$. Lemma 6 that the fundamental thresholds are bounded by $\Theta_1^*(1,\Delta)$ and $\Theta_1^*(1,-s\Delta)$ for intermediate values of n_1.

Lemma 6 ***Threshold ranking.*** *The equilibrium threshold found in Lemma 1 is a weighted average of the thresholds that prevail if investors were informed:*

$$\min_m\{\Theta_1^*(1,m)\} < \Theta_1^*(0,m) < \max_m\{\Theta_1^*(1,m)\}\ ,\forall m. \tag{48}$$

Proof By continuity, there exists a $\Theta_1^*(0,m)$ that solves $J(0,\Theta_1)=\gamma_1$. It is characterized by inequality (48). To see this, recall that $J(0,\Theta_1(0,m),x_{1U}(m))$ is a weighted average of the $\Psi(\Theta_1(0,m),m)$'s evaluated at the different levels of m. Given the strict difference in the $\Theta_1^*(1,m)$'s for different levels of m and due to $\frac{d\Psi(\Theta_1,m)}{d\Theta_1}, \frac{dJ(0,\Theta_1)}{d\Theta_1} < 0$, $\Theta_1^*(0,m)$ is a weighted average of the $\Theta_1^*(1,m)$'s. Inequality (48) follows.

We conclude by proving the existence of a unique monotone equilibrium in Lemma 7.

Lemma 7 ***Existence of a unique monotone equilibrium in region 1.*** *If private information is sufficiently precise, then there exists a unique monotone Bayesian equilibrium in region* 1 *for any proportion of informed investors,* $n_1 \in [0,1]$. *This equilibrium is characterized by signal thresholds for informed and uninformed investors,* $x^*_{1I}(n_1,m)$ *and* $x^*_{1U}(n_1)$*, and a fundamental threshold,* $\Theta^*_1(n_1,m)$*, for each realized macro shock,* $m \in \{-s\Delta, 0, \Delta\}$. *Investors attack whenever their private signal is sufficiently low,* $x_{i1} < x^*_{1U}(n_1)$ *if uninformed and* $x_{i1} < x^*_{1I}(n_1,m)$ *if informed. A crisis occurs whenever the fundamental is sufficiently low,* $\Theta_1 < \Theta^*_1(n_1,m)$.

Proof Following the same steps as in section A.2.1, we can first show that the equilibrium conditions can be expressed as a system of three equations in three unknowns, that is the fundamental thresholds $\left(\Theta^*_1(-s\Delta),\, \Theta^*_1(0),\, \Theta^*_1(\Delta)\right)$. Thereafter, we can prove existence of a unique monotone equilibrium following the same steps as in the Proof of Corollary 2.