

# Real Interest Rates, Bank Borrowing, and Fragility\*

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## Abstract

How do real interest rates affect bank fragility? We study this issue in a model in which bank borrowing is subject to rollover risk. Optimal borrowing trades off the benefit of additional profitable investments with the cost of greater risk of a bank run. Changes in the interest rate affect the price and amount of borrowing, both of which influence bank fragility in opposite directions. We show that the marginal impact of changes to the interest rate on bank fragility also depends on the *level* of the interest rate. Furthermore, we derive testable implications that may guide future empirical work.

**Keywords:** bank borrowing, rollover risk, fragility, real interest rates, global games, funding liquidity risk channel

**JEL classifications:** G01, G21, G28.

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# 1 Introduction

The relationship between interest rates and financial stability has been at the fore of policy and academic debates since the Global Financial Crisis of 2007-08. Recent research has improved our understanding of a risk-taking channel of monetary policy, which posits that lower interest rates increases the riskiness of banks' loan portfolios (e.g., [Dell'Ariccia et al., 2014](#); [Martinez-Miera and Repullo, 2017](#)). This literature is, however, silent on the implications of interest rates on fragility, i.e., banks' proclivity for sudden funding dry-ups ([Diamond and Dybvig, 1983](#); [Goldstein and Pauzner, 2005](#)). Fragility is an innate and distinct feature of banks that stems from their reliance on funding subject to rollover risk (e.g., short-term unsecured wholesale debt and interbank loans) to finance investments ([Adrian and Shin, 2011](#)).<sup>1</sup> Aside from loan risk, fragility is another key determinant of bank risk and consequently of financial stability. It is, thus, critical to complement our understanding of the risk-taking channel with an assessment for how interest rates affect bank fragility (Figure 1).

This paper constitutes a first attempt of such an assessment. In order to focus attention on interest rates and bank fragility, we abstract away from standard risk-taking incentives. The starting point for our analysis is an otherwise standard model of a bank subject to rollover risk ([Rochet and Vives, 2004](#); [Vives, 2014](#)). We extend the model by endogenizing both the price of debt and the bank's borrowing volume (Section 2). These features allow us to decompose the impact of increases in the real interest rate on fragility into distinct and opposing price and scale effects. We show that bank fragility increases when the *level* of the interest rate is low, but is

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<sup>1</sup>For the United States, [Hanson et al. \(2015\)](#) document that deposits represent over three quarters of funding for commercial banks, of which approximately half of those in large banks are uninsured. Across Europe, Certificates of Deposits (CDs) are a common form of unsecured funding for banks. [Perignon et al. \(2018\)](#) document that the market for CDs is roughly as large as that for repurchase agreements and about ten times as large as the unsecured interbank market.

decreasing when the level is high. Our analysis, thus, contributes to the debate on the financial stability consequences of monetary policy ‘normalization’ (Powell, 2019).

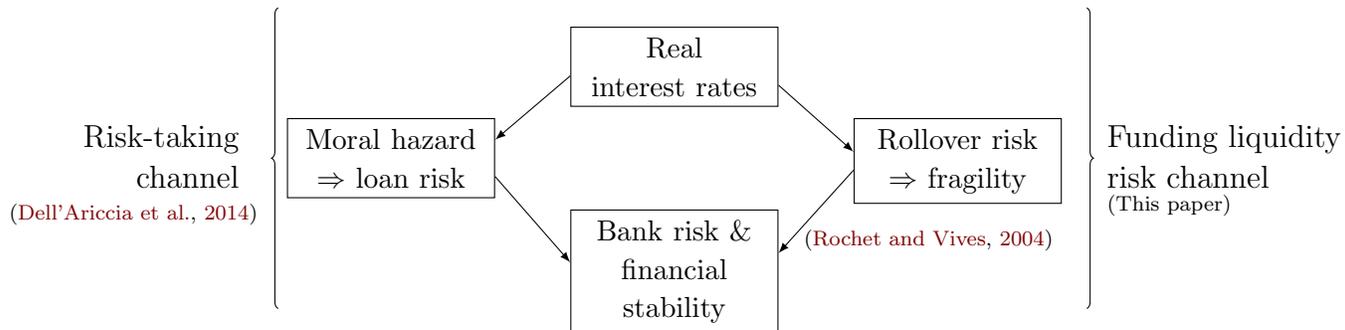


Figure 1: The risk-taking channel, e.g., Dell’Ariccia et al. (2014, 2017); Martinez-Miera and Repullo (2017) has focused on the relationship between interest rates, risk-taking incentives and loan risk. Rochet and Vives (2004), among others, study the relationship between rollover risk, fragility and bank risk. Building on their model, our paper highlights a funding liquidity risk channel, i.e., the relationship between interest rates, rollover risk and fragility.

In our model, a bank chooses how much uninsured demandable debt to issue in order to finance profitable investment that is costly to liquidate.<sup>2</sup> Following a shock to the bank’s balance sheet, concerns over the future viability of the bank can precipitate withdrawals, requiring the bank to liquidate investments. Bank failure is thus driven by both the exogenous balance sheet shock and losses from the costly liquidation of investments to serve endogenous withdrawals. Moreover, individual withdrawal incentives are further sparked by concerns about the withdrawals of other investors, so the run on the bank is also a consequence of a coordination failure.

Using global game techniques (Carlsson and van Damme, 1993; Morris and Shin, 2003; Vives, 2005), we derive the unique equilibrium in Section 3. The bank fails if

<sup>2</sup>Consistent with much evidence, unsecured bank debt is assumed to be demandable. Demandability is a feature of optimal debt contracts in models with uncertain liquidity needs (Diamond and Dybvig, 1983), agency conflicts (Calomiris and Kahn, 1991; Diamond and Rajan, 2001), or expropriation risk (Ahnert and Perotti, 2021). Accordingly, in what follows, we use “uninsured deposits” to refer to short-term or demandable debt including uninsured retail deposits or wholesale funding.

the shock exceeds a certain threshold. This fragility threshold decreases in the face value of debt, while the impact of greater borrowing is, in general, ambiguous. On the one hand, when the bank borrows more, it becomes exposed to a higher risk of a run. On the other hand, it also increases profitable investments and thereby raises the resources available to serve its debt. The former effect dominates if the required return of investors – and thus the equilibrium face value of debt – is sufficiently high.

In equilibrium, the bank chooses how much debt to borrow (and thus its investment scale) to maximize its expected equity value, taking into account its impact on the failure threshold and the face value of debt. The face value, in turn, is derived from a break-even condition that equates the expected return from lending to the bank with the risk-free real interest rate. This gives rise to the following trade-off: on the one hand, by scaling up its investments by issuing debt, the bank earns the intermediation margin (the difference between the investment return and the face value of debt). On the other hand, the bank is more likely to fail because greater debt increases the banks' susceptibility to a run. The risk of such a run leads the bank to scale down its borrowing compared to a situation without coordination failure.

We consider the implications of changes to the real interest rate in Section 4. First, we argue that a lower interest rate leads to greater borrowing and investment. These results are in line with empirical evidence of the standard bank lending channel (e.g., [Bernanke and Blinder, 1992](#); [Stein and Kashyap, 2000](#)). However, the mechanism in our model differs from other theories (e.g., [Peek and Rosengreen, 2013](#)). In particular, changes in the bank's borrowing and investment scale constitute an optimal response to a policy-induced change in the risk of experiencing a run.

Second, and at the crux of this paper, we consider how changes in the interest rate impact bank fragility. In general, a marginal change to the interest rate exerts

three effects on the fragility threshold. First, there is a direct *price effect*. A lower interest rate reduces the bank’s funding costs and thereby helps to reduce fragility. Second, there is an indirect *scale effect* whereby the lower interest rate induces the bank to increase its borrowing, thereby increasing fragility. Finally, there is an *amplification effect*, which captures the feedback between the risk-adjustment in the price of debt and the bank’s default risk. This amplification effect alters the magnitude, but not the sign, of the total effect of the interest rate change on fragility.

To understand how the interest rate influences fragility, we compare the price and scale effects. In general, the scale effect tends to dominate the price effect when the level of the interest rate is high. Thus, in a *high interest rate environment*, a lower interest rate increases fragility. By contrast, in a *low interest rate environment*, a lower interest rate reduces fragility. This latter result contrasts with the implications of the standard risk-taking channel often emphasized in the literature (e.g., [Cunliffe, 2019](#)). In particular, our analysis suggests that due to the dominating price effect, an ‘exit’ from a low interest environment is associated with adversely affecting fragility.

We consider two extensions of our baseline model in Section 5 to probe its robustness. First, we consider the bank to be endowed with equity. Since equity absorbs losses, it exerts a catalytic effect on borrowing and investment: equity mitigates concerns about the bank’s future solvency, thus reducing rollover risk and shifting bank borrowing and lending closer to the benchmark of perfect coordination.<sup>3</sup> Second, we study the consequences of having a multiplicative or scale-dependent shock, which introduces a new risk-taking incentive for the bank. Our baseline model, in contrast, assumes an additive or scale-invariant shock to the balance sheet, as in [Ahnert et al.](#)

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<sup>3</sup>This catalytic effect is similar to that explored by [Morris and Shin \(2006\)](#) in an international finance context. In their model, official sector assistance to a country in the midst of a financial crisis can spur debtor to lend more, thereby alleviating the crisis.

(2019), which allows us to better isolate the novel funding liquidity risk channel. We show that our insights are robust to the introduction of risk-taking on the asset side.

Finally, our model generates empirical predictions reviewed in Section 6. First, greater rollover risk and higher interest rates reduces bank borrowing. Second, banks with either more equity or lower exposure to rollover risk are less sensitive to interest rate changes. And finally, in a low interest rate environment, higher policy rates or lower market liquidity increase bank fragility. We link these predictions to existing empirical evidence and briefly discuss how they may inform future empirical work.

**Literature.** Our paper relates to several strands of the literature. A traditional literature on the bank lending channel posits that a monetary policy tightening leads to a shortfall of banks' deposits and reduces lending (e.g., [Bernanke and Gertler \(1995\)](#); [Stein and Kashyap \(1995\)](#); [Stein \(1998\)](#); [Boivin et al. \(2010\)](#); [Peek and Rosengreen \(2013\)](#)). [Drechsler et al. \(2017\)](#) suggest that banks respond to monetary policy shifts by exercising their market power in deposit markets. We complement this literature by arguing that banks' responses are also shaped by rollover risk.

We also contribute to the literature on bank runs and global games. In the unique equilibrium, the run is a consequence of a coordination failure following a large shock to the bank's balance sheet. In particular, we build on [Rochet and Vives \(2004\)](#) where investors delegate their rollover decisions to professional fund managers, so the decisions to roll over are global strategic complements.<sup>4</sup> We contribute by

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<sup>4</sup>[Goldstein and Pauzner \(2005\)](#) study one-sided strategic complementarity due to the sequential service constraint of banks ([Diamond and Dybvig, 1983](#)). [Liu \(2016\)](#) studies how runs on banks interact with freezes in the interbank market, giving rise to amplification. [Eisenbach \(2017\)](#) shows how rollover risk from demandable debt effectively disciplines banks for idiosyncratic shocks, while a two-sided inefficiency arises for aggregate shocks. [Ahnert et al. \(2019\)](#) consider how the introduction of senior secured debt influences rollover risk and banks' borrowing and investment choices. [Carletti et al. \(2020\)](#) study the role of liquidity regulation and its interaction with capital requirements. [Li and Ma \(2021\)](#) study the interaction between bank runs and falling asset prices in secondary markets and show the desirability of a committed liquidity support by a regulator.

showing how, via the endogenous borrowing choice and face value of debt, the effects of changes in the real interest rate (e.g., due to monetary policy) on fragility give rise to countervailing indirect effects absent in other studies with exogenous borrowing and investment (e.g., [Bebchuk and Goldstein, 2011](#); [Vives, 2014](#); [König, 2015](#)).

In a recent contribution, [Leonello et al. \(2022\)](#) consider how, when depositors do not internalize how their savings choices influence the amount of deposits held by a bank, this can give rise to excessively high bank fragility. Consequently, they argue that, when banks are exposed to rollover risk, banks that issue more deposits are less fragile. In contrast, we consider a setting where investors internalise how their funding choices influence bank fragility. Consequently, the equilibrium level of borrowing by the bank is below the perfect-coordination benchmark.

Our paper also connects to the literature on the risk-taking channel of monetary policy and its implications for financial stability. [Dell’Ariccia et al. \(2014, 2017\)](#) study the risk-shifting incentives of banks and argue that expansionary monetary policy increases bank leverage and risk-taking. [Martinez-Miera and Repullo \(2017\)](#) consider a ‘search-for-yield’ mechanism: when monetary policy reduces yields on safer assets compared to the interest on their long-term liabilities, financial institutions rebalance their asset portfolios towards riskier short-term assets. Our model complements this literature by focusing on a funding liquidity risk channel: higher failure risk stemming from expanding the bank’s balance sheet by increased short-term borrowing.

## 2 Model

The model builds on [Rochet and Vives \(2004\)](#), [Vives \(2014\)](#), and [Ahnert et al. \(2019\)](#). There are three dates  $t = 0, 1, 2$ , a single good for consumption an investment, a

continuum  $\omega > 0$  of investors and a representative bank owner / manager (henceforth banker for short). Each investor is risk-neutral, endowed with one unit of funds at  $t = 0$  and is indifferent between consuming at  $t = 1$  and  $t = 2$ . Investors can store their funds at  $t = 0$  to earn a (gross) risk-free return  $r > 0$  at  $t = 2$ . The banker is penniless but has access to illiquid investments that return  $R > r$  at  $t = 2$  per unit invested at  $t = 0$ . However, liquidating investments at  $t = 1$  yields only a fraction  $\psi \in (0, 1)$  of the investment's return. To finance investments, the banker issues  $D \geq 0$  of demandable debt to investors at  $t = 0$  with face value  $F > 0$ . The bank's initial balance sheet is  $I \equiv D$ , where  $I$  is the amount invested.

At  $t = 1$ , investors either redeem their claims against the bank or roll them over until  $t = 2$ . Each investor delegates the rollover decision to a professional fund manager who is rewarded for making the right decision: if the bank does not fail, a manager's payoff difference between withdrawing and rolling over is a cost  $c > 0$ ; if the bank fails, the differential payoff is a benefit  $b > 0$ .<sup>5</sup> The conservatism ratio,  $\gamma \equiv \frac{b}{b+c} \in (0, 1)$ , summarizes these payoffs, with more conservative managers (higher  $\gamma$ ) being less inclined to roll over.<sup>6</sup> For simplicity, we assume that the face value of debt,  $F$ , is independent of the withdrawal date.

The banker is protected by limited liability and is subject to a shock  $A$  at  $t = 2$ . This shock may improve the bank's balance sheet,  $A < 0$ , but operational, market, or legal risks may require writedowns,  $A > 0$ . The shock is drawn at  $t = 1$  from a continuous probability distribution with a decreasing reverse hazard rate,  $\frac{d}{dA} \frac{g(A)}{G(A)} < 0$ , where  $G(A)$  denotes the cumulative distribution and  $g(A)$  the probability density.<sup>7</sup>

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<sup>5</sup>As an example, assume the cost of withdrawal is  $c$ ; the benefit from getting the money back or withdrawing when the bank fails is  $b + c$ ; the payoff for rolling over when the bank fails is zero.

<sup>6</sup>Reviewing debt markets during the financial crisis, [Krishnamurthy \(2010\)](#) argues that investor conservatism was an important determinant of short-term lending behavior. See also [Vives \(2014\)](#).

<sup>7</sup>This property ensures a unique borrowing choice of the bank. It holds, for instance, for the exponential, (log-)normal, and uniform distributions.

Assets	Liabilities
$RI - A$	$FD$
	$E_2(A)$

Table 1: Balance sheet at  $t = 2$  after a small shock and all debt rolled over.

Table 1 shows the balance sheet at  $t = 2$  for a small shock and when all debt is rolled over. In this case, the value of bank equity is given by  $E_2(A) \equiv \max\{0, RI - A - FD\}$  and the bank fails whenever  $A > \bar{A} \equiv RI - FD$ . If, however, a fraction  $\ell \in [0, 1]$  of debt is withdrawn at  $t = 1$ , the banker liquidates a share  $\ell \frac{FD}{\psi RI}$  of the investment to repay debt holders. As a result, the banker has insufficient resources to repay the outstanding debt at  $t = 2$  and fails whenever

$$RI - A - \ell \frac{FD}{\psi} < (1 - \ell)FD, \quad (1)$$

or  $A > RI - FD [1 + \ell z]$ , where  $z \equiv \frac{1}{\psi} - 1 > 0$  is the per-unit cost of liquidation.<sup>8</sup> We assume zero recovery upon bank failure at the final date.<sup>9</sup>

At  $t = 1$ , fund managers base their withdrawal decisions on a private signal (Morris and Shin, 2003):

$$x_i = A + \epsilon_i, \quad (2)$$

where  $\epsilon_i$  is a mean-zero noise term that is independent of the shock  $A$  and identically and independently distributed across fund managers according to a continuous distribution  $H$  with support  $[-\epsilon, \epsilon]$  for  $\epsilon > 0$ . Table 2 summarizes the timeline.

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<sup>8</sup>The bank closes early if it runs out of funds at  $t = 1$ ,  $RI - A < \ell \frac{FD}{\psi}$ . The relevant failure condition, however, is equation (1), as in Rochet and Vives (2004) or Vives (2014).

<sup>9</sup>Our results are qualitatively unchanged for positive recovery values.

$t = 0$	$t = 1$	$t = 2$
1. Debt issuance	1. Balance sheet shock realizes	1. Investment matures
2. Investment	2. Private signals about shock	2. Shock materializes
	3. Withdrawals	3. Debt repayment
	4. Consumption	4. Consumption

Table 2: Timeline of events.

### 3 Equilibrium

We solve for the symmetric pure-strategy perfect Bayesian equilibrium in which fund managers at  $t = 1$  use threshold strategies, i.e. rolling over if and only if their signal indicates a healthy balance sheet,  $x_i \leq x^*$ , and the bank fails if and only if  $A > A^*$ .<sup>10</sup>

**Definition 1.** *The symmetric pure-strategy perfect Bayesian equilibrium comprises thresholds,  $x^*$  and  $A^*$ , borrowing volume  $D^*$ , and face value of debt  $F^*$  such that*

- a. *at  $t = 0$ , given the thresholds  $(x^*, A^*)$ , the banker maximizes his expected equity value by choosing borrowing  $D$  and the face value  $F$ , subject to a participation constraint of investors that ensures them an expected return of at least  $r$ ;*
- b. *at  $t = 1$ , given the banker's choices  $(D^*, F^*)$ , the threshold strategy,  $x^*$ , maximizes fund managers' expected payoffs, given that the banker fails for  $A > A^*$ .*

We solve the model backwards in two steps. First, we pin down  $x^*$  and  $A^*$  at the rollover stage at  $t = 1$ . Second, we characterize the banker's borrowing choice at  $t = 0$ , where the banker takes into account how his choices affect rollover risk.

<sup>10</sup>Since we assume that private information is sufficiently precise, the equilibrium is unique (Morris and Shin, 2003). It is also an extremal equilibrium that is in monotone strategies (Vives, 2005). Since the rollover decision is binary, our focus on threshold strategies is without loss of generality.

### 3.1 Rollover risk of debt

For the solution of the rollover subgame at date  $t = 1$ , we focus on the global game solution of vanishing private noise about the balance sheet shock,  $\epsilon \rightarrow 0$  (Eisenbach, 2017; Ahnert et al., 2019; Carletti et al., 2020). As a result, the rollover threshold converges to the failure threshold,  $x^* \rightarrow A^*$ . In this case, fund managers face only strategic uncertainty about the behavior of other managers at  $t = 1$  because the fundamental uncertainty about the magnitude of the shock vanishes.

**Proposition 1. Failure threshold.** *There exists a unique failure threshold  $A^*$ . All fund managers refuse to roll over and the banker defaults if and only if*

$$A > A^* \equiv RI - FD[1 + \gamma z]. \quad (3)$$

**Proof.** See Appendix A1. ■

Proposition 1 pins down the unique incidence of a run on bank debt. The failure threshold can be expressed as  $A^* = \bar{A} - \gamma z FD$ , so rollover risk causes a deviation of the failure threshold  $A^*$  from the perfect-coordination benchmark,  $\bar{A}$ . In particular, rollover risk is a consequence of the combination of fund manager conservatism,  $\gamma > 0$ , and asset illiquidity,  $\psi < 1$  (or  $z > 0$ ). Without rollover risk,  $\gamma z \rightarrow 0$ , the failure threshold thus converges to its perfect-coordination benchmark,  $A^* \rightarrow \bar{A}$ . We henceforth use  $A^*$  as our measure of bank fragility. An increase of  $A^*$  reduces fragility as it decreases the range of shocks  $(A^*, \infty)$  for which the bank fails.

**Lemma 1. Borrowing volume and fragility.** *An increase in bank borrowing increases fragility,  $\frac{\partial A^*}{\partial D} < 0$ , if and only if debt is sufficiently expensive,  $F > \frac{R}{1 + \gamma z}$ .*

**Proof.** See Appendix A1. ■

Greater borrowing induces two opposing effects on the failure threshold. First, greater borrowing allows the bank to increase the scale of its profitable investments,  $\frac{\partial I}{\partial D} = 1$ , which reduces fragility because it provides more resources that can be used to repay debt. Second, because the additional borrowing may be withdrawn, it increases the bank's exposure to a debt run, thus heightening fragility. Note that the effective per-unit cost of debt that impacts the bank's fragility at  $t = 1$  exceeds the face value  $F$  due to the possibility that assets are liquidated at a cost when the debt is withdrawn. This additional per-unit cost of debt is given by  $\gamma z F$ . As shown in Lemma 1, the second effect dominates the first if the total effective per-unit cost of debt at  $t = 1$ ,  $F(1 + \gamma z)$ , exceeds the per-unit return from the bank's investment. In this case, more borrowing unambiguously increases fragility. Moreover, a higher liquidation value mitigates the influence of fund manager conservatism and reduces fragility,  $\frac{\partial A^*}{\partial \psi} > 0$  (Rochet and Vives, 2004). Finally, with a higher face value of debt, the bank must liquidate a larger share of assets to meet early withdrawals,  $\frac{\partial A^*}{\partial F} < 0$ . A higher interest rate does not directly affect fragility,  $\frac{\partial A^*}{\partial r} = 0$ . But, as we subsequently show, it indirectly influences fragility via the volume of borrowing and price of debt.

### 3.2 Optimal borrowing and investment

The banker chooses the level of borrowing  $D$  and offers a face value  $F$  to investors in order to maximize his expected equity value at date  $t = 2$ , subject to the investor participation constraint in funding markets, the balance sheet identity, and the failure threshold  $A^*(D, F)$ . The banker's problem can be expressed as

$$\max_{D, F} \pi \equiv \int_{-\infty}^{\infty} E_2(A) dG(A) = \int_{-\infty}^{A^*(D, F)} [RI - A - FD] dG(A) \quad (4)$$

$$\text{s.t. } FG(A^*(F, D)) \geq r, \quad (5)$$

where the participation constraint in (5) states that investors' expected return from funding the bank must be at least the risk-free return earned on the outside option. Since the banker's profit function is strictly decreasing in  $F$  for all values of  $D$ , profit maximization implies that (5) must hold with equality. Proposition 2 characterizes the interior solution to the constrained optimization problem.

**Proposition 2. Bank borrowing and debt pricing.** *For any  $r \in (\underline{r}, \tilde{r})$  there exists a unique interior equilibrium with borrowing volume  $D^* \in (0, \omega)$  and face value  $F^* \in (\frac{R}{1+\gamma z}, R)$ .*

**Proof.** See Appendix A2. ■

To understand the intuition behind Proposition 2, note that, for a given face value  $F$ , a marginal increase in borrowing exerts two effects on the bank's expected equity value,  $\pi$ . First, an increase in  $D$  alters the failure threshold,  $A^*$ , and the set of shock realizations where the bank fails. As shown in Lemma 1, borrowing lowers the failure threshold if and only if the face value is sufficiently large,  $F > R/(1 + \gamma z)$ . Second, for a given realization of the shock, an increase in  $D$  changes the equity value conditional on the bank not failing,  $(R - F)D - A$ . In particular, greater borrowing increases the bank's equity value if and only if the intermediation margin is positive, i.e.,  $R - F > 0$ . Otherwise, debt is prohibitively expensive and the bank incurs a loss from borrowing any amount.

For an intermediate face value,  $R/(1 + \gamma z) < F < R$ , the banker's equity value strictly increases in  $D$ , while the failure threshold strictly decreases. Hence, the banker's optimal borrowing choice trades off the marginal benefit from increasing the equity value against the marginal cost which stems from the higher expected loss due to the increased likelihood of failure. For given  $F$ , the optimal choice of  $D$  satisfies

$$R - F = -(1 + \gamma z)FD \frac{g(A^*)}{G(A^*)} \frac{dA^*}{dD}. \quad (6)$$

The marginal benefit from additional borrowing on the left-hand side of equation (6) corresponds to the intermediation margin. The right-hand side is the expected loss due to a marginal change in the threshold  $A^*$ . The term  $\frac{g(A^*)}{G(A^*)} \frac{dA^*}{dD}$  measures the likelihood of default following the total change in the failure threshold conditional on not failing which multiplies the total value of losses in the event of failure. These include the reduction in the banker's equity,  $E_2(A^*) = \gamma zFD$ , and the losses to creditors,  $FD$ , that the banker internalizes through the adjustment in the price of debt via the participation constraint.<sup>11</sup>

The binding participation constraint in Equation (5) and condition in Equation (6) together determine the banker's choice of borrowing and the face value of debt.<sup>12</sup> Figure 2 shows the equilibrium. The solid line is the borrowing schedule  $D^*(F)$  derived from (6) and the dashed line  $F^*(D)$  is the participation constraint in (5). Both curves intersect at an intermediate  $r$  such that  $F^* \in (\frac{R}{1+\gamma z}, R)$  and  $D^* \in (0, \omega)$ .

Note that we could relax the assumption of a bank's asset return,  $R$ , being invariant to the scale of investment. In particular, suppose  $R \equiv R(I)$ , which satisfies the Inada conditions. Accordingly, the net marginal benefit for an additional unit of borrowing and investing—the left-hand side of equation (6)—is given by the sum of the intermediation margin,  $R(D) - F$ , and the marginal increase to the returns,  $R'(D)$ . At the same time, a marginal increase in bank borrowing reduces fragility

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<sup>11</sup>If  $F$  falls outside of the bounds in Proposition 2, then either debt is sufficiently cheap,  $F < R/(1+\gamma z) < R$ , and greater the banker borrows as much as possible,  $D^* = \omega$  (because borrowing reduces the likelihood that the bank fails and the equity value). Or, conversely, debt is very expensive,  $F > R$ , and the bank completely abstains from any borrowing,  $D^* = 0$  (because more borrowing raises the likelihood of failure *and* reduces the equity value).

<sup>12</sup>The interval in which an interior optimum exists is derived by using the creditors' participation constraint to obtain thresholds such that, at  $r = \underline{r}$ , there is no borrowing,  $D^* = 0$ , and at  $r = \tilde{r}$ , the banker borrows  $D^* = \omega$ .

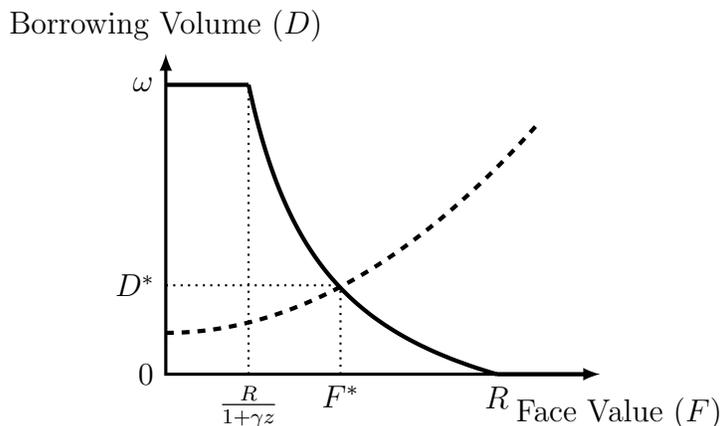


Figure 2: The unique equilibrium borrowing volume and debt face value are determined at the intersection of the creditors' participation constraint (dashed line) and the bank's demand for funds (solid line).

by an additional  $R'(D)D$ . The combined effect is to boost banking borrowing and investment relative to our current specification.

## 4 Effects of the real interest rate

In this section, we derive comparative statics for equilibrium borrowing and fragility. Our primary focus is on the risk-free real interest rate,  $r$ . Following [Dell'Ariccia et al. \(2014\)](#), we assume that the real interest  $r$  is determined at  $t = 0$  before the bank chooses borrowing and investment.

### 4.1 Benchmark: no rollover risk

It is instructive to first consider a benchmark without rollover risk. In particular, when  $\gamma = 0$ , fund managers are no longer subject to strategic complementarities in their rollover decisions and inefficient runs do not occur. Consequently, the bank fails

at  $t = 2$  whenever  $A > \bar{A}$ . Moreover, the bank borrows the maximal feasible amount,  $D^* = \omega$ , as long as  $F^* < R$ , i.e., whenever the net marginal benefit is positive.

**Proposition 3. No rollover risk.** *Without rollover risk,  $\gamma = 0$ , a marginal change to the real interest rate has no effect on the bank's borrowing and investment scale.*

In the absence of rollover risk, changes in the interest rate have no effect on the intensive margin of the banker's borrowing. However, by influencing the face value of debt, a change in the real interest rate can impact the extensive margin of bank borrowing. Whenever  $F^*$  increases and exceeds  $R$ , the bank stops borrowing.

## 4.2 Rollover risk and the funding liquidity risk channel

When  $\gamma > 0$ , the rollover decisions are subject to strategic complementarity. As a result, the bank fails whenever  $A > A^*$ . Moreover, for any  $F \in (R/(1 + \gamma z), R)$ , the banker optimally responds to an increase in the risk of failure by borrowing less. In other words, the presence of rollover risk gives rise to a 'scale effect' such that  $D^* < \omega$ . This, in turn, induces the optimal borrowing to respond to changes in interest rate. As such, we refer to this channel as the *funding liquidity risk channel*.

**Proposition 4. The funding liquidity risk channel.** *An increase in the real interest rate raises the face value of debt,  $\frac{dF^*}{dr} > 0$ , and reduces borrowing,  $\frac{dD^*}{dr} < 0$ .*

**Proof.** See Appendix A3. ■

An increase in the interest rate implies that investors require a higher face value in order to supply funding to the bank. A higher face value, in turn, raises the marginal cost of borrowing by increasing the likelihood of the bank failing due to run

risk. Moreover, the bank must forego greater equity value in the event of failure. The banker, thus, responds by reducing the amount of borrowing and investment, thereby counteracting the effect of the higher face value on the risk of a run.

Figure 3 shows the scale effect and funding liquidity risk channel. It plots the banker’s optimal borrowing against the real interest rate,  $r$ . The dashed line depicts the perfect-coordination benchmark in the absence of coordination risk ( $\gamma = 0$ ), while the solid line shows the case with rollover risk ( $\gamma > 0$ ). For values of  $r$  that are either sufficiently small ( $r < \underline{r}$ ) or sufficiently large ( $r > \tilde{r}$ ), rollover risk has no effect on the bank’s borrowing and investment scale. For intermediate values  $r \in (\underline{r}, \tilde{r})$ , however, the scale effect is present and is given by the distance between the dashed line and solid curve. Due to the scale effect, an increase in the interest rate is associated with lower borrowing as the bank trades off the net marginal benefit against the increased likelihood of failure due to a higher borrowing scale.

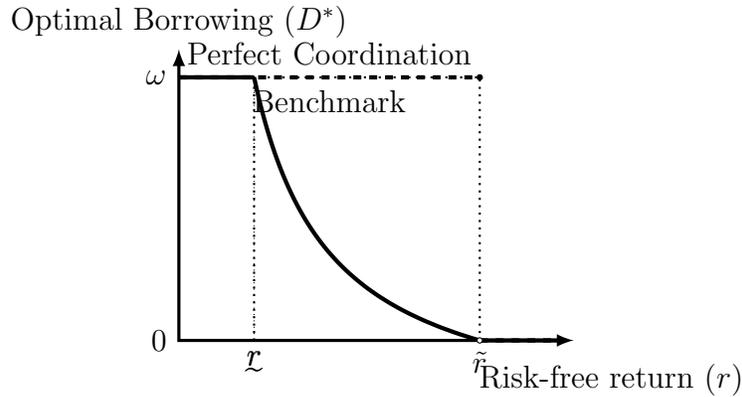


Figure 3: Rollover risk reduces borrowing relative to a perfect-coordination benchmark. Borrowing volume with (solid) and without rollover risk (dashed).

### 4.3 Funding liquidity risk channel and bank fragility

We next turn to the implications for bank fragility, which we measure by the bank's failure probability. Since the ex-ante failure probability is monotonically decreasing in the failure threshold  $A^*$ , to study the effects of the interest rate on bank fragility, it suffices to consider its effects on  $A^*$ .

The result in Proposition 4 implies that the total effect of the interest rate on bank fragility is a priori not clear. In particular, it follows from Lemma 1 that for  $F \in (R/(1 + \gamma z), R)$ , the failure threshold decreases in both  $F$  and  $D$ . But since changes in the risk-free return can have opposing effects on the face value of debt and the bank's borrowing volume, the total effects on the failure threshold are ambiguous. To see this, we decompose the total effect of a marginal increase in the risk-free return on the failure threshold into three separate terms:

$$\frac{dA^*}{dr} = \underbrace{\frac{1}{1 + \frac{g(A^*)}{G(A^*)} \frac{\partial A^*}{\partial F} F^*}}_{\text{Amplification effect (+)}} \times \left( \underbrace{\frac{\frac{\partial A^*}{\partial D} dD^*}{dr}}_{\text{Scale effect (+)}} + \underbrace{\frac{1}{G(A^*)} \frac{\partial A^*}{\partial F}}_{\text{Price effect (-)}} \right). \quad (7)$$

The first term in brackets is a scale effect, which affects fragility via the total adjustment in borrowing scale. Since  $\frac{dD^*}{dr} < 0$  and  $\frac{\partial A^*}{\partial D} < 0$ , it follows that the scale effect reduces fragility. The second effect in brackets, which we refer to as the price effect, captures the contribution of the higher face value of debt to the total effect on  $A^*$ . Since  $\frac{\partial A^*}{\partial F} < 0$ , the price effect raises the bank's fragility. Finally, the third effect (that multiplies into the previous two effects) is an amplification that accounts for the fact that an increase in fragility leads to an increase in the face value via the investors participation constraint, which further increases fragility.

As Proposition 5 shows, the effect of changes in the real interest rate on bank fragility (e.g., due to changes in monetary policy) depends on the relative magnitudes of the opposing scale and price effects. These are, in turn, shaped by the *level* of the interest rate.

**Proposition 5. The interest rate and bank fragility.** *There exists a unique  $\underline{r} \in (\underline{r}, \tilde{r})$ , where a marginal increase in the risk-free return reduces fragility,  $\frac{dA^*}{dr} > 0$ , if and only if  $r > \underline{r}$ .*

**Proof.** See Appendix A3. ■

The intuition behind the result of Proposition 5 is as follow. First note that the failure threshold,  $A^*$ , is proportional to the bank's total funding cost,  $FD$ . Thus, at any interior equilibrium, the impact of the scale effect in reducing fragility increases with the face value of debt since  $\partial A^*/\partial D = R - (1 + \gamma z)F < 0$ . At the same time, the negative impact of the price effect increases with the amount of bank borrowing since  $\partial A^*/\partial F = -(1 + \gamma z)D$ . Thus, whenever the risk-free return is sufficiently small, the face value of debt is low, while bank borrowing is relatively high. In such a low interest rate environment, the price effect dominates and financial fragility worsens following a reduction in the interest rate.

Conversely, in a high interest rate environment, the face value of debt, and thus the magnitude of the scale effect, are large compared to the borrowing volume and price effect. Consequently, an increase in the interest rate reduces financial fragility. At the threshold,  $\underline{r}$ , the face value is such that price and scale effects wash out.

The threshold itself crucially depends on both the extent of rollover risk,  $\gamma$ , and market liquidity,  $\psi$ . If, for example, market liquidity improves ( $\psi$  increases), the bank is better able to serve withdrawals and forestall failure. This, in turn, leads to a lower

face value,  $F$ . As a consequence, however, the impact of the scale effect in reducing fragility is less pronounced, while the price effect is amplified. In sum, the threshold,  $\underline{r}$ , at which the price and scale effect perfectly offset each other increases and so the range a lower interest rate reduces fragility becomes larger.<sup>13</sup>

**Corollary 1.** *The threshold  $\underline{r}$  is increasing as market liquidity improves,  $\frac{\partial \underline{r}}{\partial \psi} > 0$ .*

Figure 4 illustrates the implications for bank fragility emanating from the funding liquidity risk channel. Expansionary monetary policies (a decrease in  $r$ ) raise bank fragility via the scaling channel whenever  $r > \bar{r}$ . Thus, for ‘high interest environments’ the bank fragility consequences of our funding liquidity risk channel are in line with those of the standard risk-taking channel. However, in a ‘low interest rate environment’, i.e. for  $r < \underline{r}$ , reductions in the interest rate lower fragility and exert a stabilizing influence, thus contrasting the risk-taking channel.

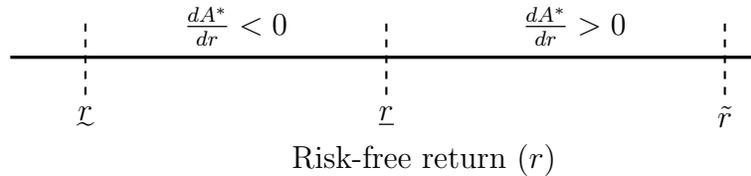


Figure 4: Effect of interest rate on bank fragility for different levels of interest rates.

To further illustrate this point, consider an increase in the interest rate from a low interest environment, e.g., a monetary policy ‘exit’ from low interest rates. While the standard risk-taking channel predicts that financial stability improves, our funding liquidity risk channel implies the opposite, i.e., a worsening of bank fragility due to the dominance of the price effect.

<sup>13</sup>Since  $\psi$  and  $\gamma$  only appear together as  $\gamma z$ , a similar logic applies when rollover risk  $\gamma$  is reduced.

## 5 Extensions

In this section we explore two important extensions to our baseline model. First, we consider how the presence of equity influences the bank's decision to issue additional debt. And second, we show that our analysis is robust when we consider a scale-dependent shock and additional asset-side risk.

### 5.1 Equity

In this section, we assume that the banker is endowed with initial equity  $E > 0$  at  $t = 0$ . This endowment has no effect on the analysis of the sub-game equilibrium at  $t = 1$  except for the fact that the balance sheet identity at  $t = 0$  now reads  $I \equiv D + E$  and, after substituting into equation (3), the failure threshold changes to

$$A^* \equiv R(D + E) - FD[1 + \gamma z]. \quad (8)$$

Since the banker invests all funds into profitable investments, the additional resources  $RE > 0$  at  $t = 2$  can also be used to serve debt. Hence, equity allows the banker to withstand larger shocks and lowers fragility,  $\frac{\partial A^*}{\partial E} = R > 0$ . Moreover, the presence of equity requires to distinguish between the level of borrowing,  $D^*$ , and the level of investment,  $I^* = D^* + E$ . Proposition 6 states the resulting optimal borrowing and investment of the banker, generalizing our previous result.

**Proposition 6. Bank equity.** *There exist values  $\underline{r}_E > \underline{r}$  and  $\tilde{r}_E > \tilde{r}$  such that the bank's optimal borrowing is given by Equation (6), with  $\underline{r}$  being replaced by  $\underline{r}_E$  and  $\tilde{r}$  replaced by  $\tilde{r}_E$ . Greater equity increases borrowing,  $dD^*/dE > 0$ , makes debt cheaper,  $dF^*/dE < 0$ , and decreases fragility,  $dA^*/dE > 0$ .*

**Proof.** See Appendix A4. ■

Bank equity exerts a catalytic effect on bank scale as Figure 5 illustrates. That is, a marginal increase in equity induces the banker to increase borrowing and investment. Along the extensive margin, greater equity shifts the bounds  $\underline{r}_E$  and  $\tilde{r}_E$  outwards. The increase in the lower bound implies that additional equity enables the banker to borrow the maximum  $D^* = \omega$  for a larger range of risk-free returns. The increase in the upper bound implies that the range of risk-free returns where borrowing becomes prohibitively costly shrinks. And, on the intensive margin, for any given  $r \in (\underline{r}_E, \tilde{r}_E)$ , an increase in equity increases bank borrowing further,  $dD^*/dE > 0$ .

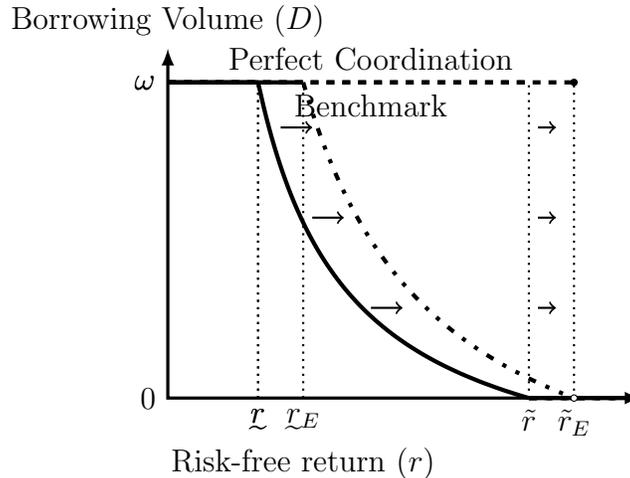


Figure 5: Equity increases the range where over which maximum borrowing takes place and also shifts the borrowed amount outwards (dash-dotted curve). Borrowing volume with (solid curve) and without rollover risk (dashed line).

How does the presence of bank equity influence the pass-through of changes in the interest rate via the funding liquidity risk channel? Figure 5 conveys the main insights. First, while both bounds,  $\underline{r}_E$  and  $\tilde{r}_E$  increase following a marginal increase in bank capital, the upper bound increases by more. Thus the gap,  $\tilde{r}_E - \underline{r}_E$ , also increases. Second, since  $dD^*/dE > 0$  for all  $r \in (\underline{r}_E, \tilde{r}_E)$ , this suggests that, typically,

the borrowing schedule  $D^*(r)$  becomes flatter. This implies that marginal changes in the interest rate have a smaller impact on bank borrowing when banks hold more equity. This result follows since more bank equity reduces rollover risk. Hence, the interest rate has a smaller impact on the bank's fragility and therefore dampens the bank's corresponding adjustment of borrowing and investment. In Section 6, we discuss how this result may be empirically tested.

## 5.2 Scale-dependent shock and asset-side risk

Our preceding analysis assumed a scale-invariant (or additive) shock to the banker's balance sheet. This allowed us to derive the key implications of the funding liquidity risk channel by fully abstracting from the effect of the bank's decisions on asset-side risk. In this section, we touch upon the robustness of our analysis by relaxing this assumption and introduce a scale-dependent (or multiplicative) shock whose impact depends on the scale of the banker's borrowing. Specifically, we suppose that the investment return,  $R$ , is a random variable with distribution  $G(R)$  (density  $g(R)$ ) with an increasing hazard rate. Thus, the size of the shock experienced by the banker depends on the scale of his investments and hence on the banker's choice of borrowing.

Allowing for equity  $E > 0$  on the banker's balance sheet, the failure condition is given by  $RI - \ell \frac{FD}{\psi} < (1 - \ell)FD$ . The global games analysis continues to apply, giving rise to a threshold such that the bank fails if and only if  $R < R^*$ . As in [Rochet and Vives \(2004\)](#), this threshold becomes

$$R^* = \frac{DF}{D + E}(1 + \gamma z), \quad (9)$$

where we substituted the balance sheet identity  $I \equiv D + E$  into the denominator on the right-hand side. More borrowing increases leverage and unambiguously increases bank fragility,  $\frac{\partial R^*}{\partial D} > 0$ , but at a diminishing rate,  $\frac{\partial^2 R^*}{\partial D^2} > 0$ .<sup>14</sup>

The banker's optimization problem at  $t = 0$  is

$$\max_{F,D} \int_{R^*}^{\infty} [R(D + E) - FD] dG(R) \quad \text{s.t.} \quad F(1 - G(R^*)) = r. \quad (10)$$

The analogue of condition (6) is now

$$\mathbf{E} \left[ \tilde{R} \mid R \geq R^* \right] - F = (1 + \gamma z) FDH(R^*) \frac{dR^*}{dD} \quad (11)$$

where  $H(x) \equiv \frac{g(x)}{1-G(x)}$  is the hazard rate of the investment return and  $dR^*/dD = \partial R^*/\partial D + \partial R^*/\partial F \times dF/dD$ . The key trade-off underlying the funding liquidity risk channel, i.e. balancing the benefit of scaling up investments with the cost of more expensive debt and losing equity due to higher rollover risk, remains largely unchanged. What differs, however, is that the benefit (left-hand side) now also depends on the scale of the bank's investment,  $I$  via the dependency of the left-hand side on  $R^*$ . In particular, since in this case the investment itself is the source of risk, the benefit from borrowing more is the expected return conditional on the bank not failing. As such, the left-hand side of equation (11) also depends on the failure threshold,  $R^*$ .

What is the influence of this additional effect on the benefits? The expected return on investment increases when bank fragility increases, i.e.  $\frac{d\mathbf{E}[\tilde{R} \mid R \geq R^*]}{dR^*} > 0$ , which strengthens the bank's incentives to increase its borrowing. This suggests that our earlier analysis with a scale-invariant shock provided a lower bound for the

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<sup>14</sup>The concavity of the threshold is a consequence of the scale-dependency of the shock. In contrast, the scale-invariant threshold  $A^*$  decreased linearly in  $D$  only if the face value of debt was sufficiently large (i.e.  $r > \underline{r}$ ).

effects on bank borrowing. Moreover, it allowed us to study the funding liquidity risk channel in isolation without having to account for asset-side risk adjustments arising from limited liability.

## 6 Testable implications

In this section, we present several empirical predictions from our model. Wherever possible we provide references to existing empirical work and discuss how our predictions square with these findings. However, our model also generates new predictions which, to our knowledge, have not yet been tested empirically (notably, Predictions 2(ii) and 3). Hence, these predictions may inform future empirical work.

**Prediction 1.** *Bank borrowing and lending decreases if (i) rollover risk increases, or (ii) the interest rate increases.*

Prediction 1 is consistent with a number of recent empirical findings. With respect to Prediction 1(i), [Jasova et al. \(2018\)](#) document that a reduction in rollover risk due to the ECB's very long-term refinancing operations increases bank lending. [Iyer et al. \(2013\)](#) find that bank lending to firms is reduced if a larger share of its funding comes from the unsecured interbank market in which it is exposed to rollover risk. [de Haas and van Lelyfeld \(2014\)](#) find that lending growth of banks slows more after a shock for banks with higher reliance on wholesale funding. [Ivashina and Scharfstein \(2010\)](#) and [Cornett et al. \(2011\)](#) document a larger reduction in domestic credit following the financial crisis of 2007-08 for U.S. banks with a higher reliance on wholesale funding.

Regarding Prediction 1(ii), [Choi and Choi \(2021\)](#) document how, following a monetary policy tightening, U.S. banks substitute low-interest retail deposits with high-interest wholesale deposits. But, as the substitution is not perfect, there is an overall reduction of bank borrowing, which translates into a reduction of bank lending. [Drechsler et al. \(2017\)](#) document a related result whereby a monetary policy tightening lead to increases in the deposit rate and outflows, thereby leading to a decrease in bank lending. [Girotti \(2021\)](#) also finds a similar result.

**Prediction 2.** *The borrowing and lending by banks responds less to an increase in the interest rate if (i) the bank has more equity, or (ii) the bank is exposed to higher rollover risk.*

Greater bank equity increases the interval of interest rates over which bank borrowing and lending is strictly positive. Moreover, the size of the interval grows as bank equity increases. As a consequence, the bank's borrowing becomes less responsive to changes in the interest rate. The lower interest-sensitivity of the bank's borrowing and lending stems from the following two observations. First, an increase in rollover risk leads to an increase in the range of the interest rate where bank borrowing is positive and below the perfect-coordination benchmark. Graphically, an increase in rollover risk leads to a decrease in the lower bound,  $\underline{r}$ , while the upper bound,  $\tilde{r}$ , remains unchanged in [Figure 3](#). And second, since the total supply of funding is fixed, the schedule for the bank's borrowing becomes, generically, flatter. Consequently, marginal changes in the interest rate elicit weaker responses in bank borrowing.

Prediction 2(i) is also in line with several empirical studies. For example, [van den Heuvel \(2012\)](#) finds that output growth of banking sectors in U.S. states that have low bank capital-to-asset ratios are more sensitive to changes in the Federal funds rate. [Jiménez et al. \(2012\)](#) show that monetary policy tightening episodes

have larger negative effects on bank lending for banks with lower capital. Using a different identification strategy, Jiménez et al. (2014) show that monetary policy cuts have a more pronounced impact on lowly capitalised banks to lend more. Acharya et al. (2020) demonstrate that ‘full allotment’ liquidity provision by the ECB resulted in a lowering of deposit spreads and a further extension of credit by highly capitalised banks to their borrowers. Both these results are consistent with the predictions of our model wherein an increase in bank capital leads to a reduction in the face value of debt claims and an increase in lending.

**Prediction 3.** *In a low interest rate environment, bank fragility increases if (i) the interest rate increases, or (ii) the bank holds more illiquid assets.*

Similar to the effect of interest rates on fragility studied in Proposition 5, the marginal impact of market liquidity on bank fragility can be decomposed into a scale, price and amplification effects (see Appendix A5 for details). In contrast to before, however, bank fragility is increasing in the scale effect and decreasing in the price effect. Thus, in a low interest rate environment – where the scale effect is less pronounced – bank fragility is reduced as market liquidity improves.

## 7 Conclusion

We considered a parsimonious model for bank borrowing and investment in the presence of rollover risk to study the relationship between real interest rates and fragility. Changing the interest rate creates an ambiguous effect on bank fragility which depends on the prevailing level of the interest rate. In particular, increasing the interest rate from a low-interest rate environment can impair bank fragility.

To isolate our main channel, we made several simplifying assumptions with respect to the dead-weight losses borne by creditors and the term-structure of debt. Nevertheless, one may relax these assumptions without impacting the core trade-off. For example, in the event of bank failure, one can assume that creditors obtain a pro-rata share of the bank's liquidation value. The qualitative effects for the transmission of the interest rate, however, would remain unchanged.

## References

- Acharya, V., B. Imbierowicz, S. Steffen, and D. Teichmann (2020). Does the lack of financial stability impair the transmission of monetary policy. *Journal of Financial Economics* 138, 342–365.
- Adrian, T. and H. S. Shin (2011). Financial intermediaries and monetary economics. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics* 3, pp. 601–650. Elsevier.
- Ahnert, T., K. Anand, J. Chapman, and P. Gai (2019). Asset Encumbrance, Bank Funding and Fragility. *Review of Financial Studies* 32(6), 2422–2455.
- Ahnert, T. and E. Perotti (2021). Cheap but flighty: A theory of safety-seeking capital flows. *Journal of Banking and Finance* 131, 106211.
- Bebchuk, L. A. and I. Goldstein (2011). Self-fulfilling credit market freezes. *Review of Financial Studies* 24(11), 3519–55.
- Bernanke, B. and A. Blinder (1992). The federal funds rate and the channels of monetary transmission. *American Economic Review* 82(4), 901–921.
- Bernanke, B. and M. Gertler (1995, Fall). Inside the black box: The credit channel of monetary policy transmission. *Journal of Economic Perspectives* 9, 27–48.
- Boivin, J., M. Kiley, and F. Mishkin (2010). How has the monetary transmission mechanism evolved over time? In B. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*, Volume 3. North-Holland.
- Calomiris, C. and C. Kahn (1991). The role of demandable debt in structuring optimal banking arrangements. *American Economic Review* 81(3), 497–513.
- Carletti, E., I. Goldstein, and A. Leonello (2020, March). The interdependence between bank capital and liquidity. mimeo.

- Carlsson, H. and E. van Damme (1993). Global games and equilibrium selection. *Econometrica* 61(5), 989–1018.
- Choi, D. B. and H.-S. Choi (2021). The effect of monetary policy on bank wholesale funding. *Management Science* 67, 388–416.
- Cornett, M., J. McNutt, P. Strahan, and H. Tehranian (2011). Liquidity risk management and credit supply in the financial crisis. *Journal of Financial Economics* 101(2), 297–312.
- Cunliffe, J. (2019). Financial stability and low for long. Speech delivered to Society of Professional Economists Annual Conference.
- de Haas, R. and I. van Lelyfeld (2014). Multinational banks and the global financial crisis: Weathering the perfect storm? *Journal of Money, Credit and Banking* 46, 333–64.
- Dell’Ariccia, G., L. Laeven, and R. Marquez (2014, January). Real interest rates, leverage, and bank risk-taking. *Journal of Economic Theory* 149, 65 – 99.
- Dell’Ariccia, G., L. Laeven, and G. Suarez (2017, April). Bank leverage and monetary policy’s risk-taking channel: Evidence from the united states. *Journal of Finance* 72(2), 613 – 654.
- Diamond, D. and P. Dybvig (1983). Bank runs, deposit insurance and liquidity. *Journal of Political Economy* 91, 401–419.
- Diamond, D. and R. Rajan (2001). Liquidity risk, liquidity creation, and financial fragility: A theory of banking. *Journal of Political Economy* 109(2), 287–327.
- Drechsler, I., A. Savov, and P. Schnabl (2017). The deposits channel of monetary policy. *The Quarterly Journal of Economics* 132, 1819–1876.
- Eisenbach, T. (2017). Rollover risk as market discipline: A two-sided inefficiency. *Journal of Financial Economics* 126(2), 252–269.

- Girotti, M. (2021). How monetary policy changes bank liability structure and funding cost. *Oxford Economic Papers* 73, 49–75.
- Goldstein, I. and A. Pauzner (2005). Demand deposit contracts and the probability of bank runs. *Journal of Finance* 60(3), 1293–1327.
- Hanson, S., A. Shleifer, J. Stein, and R. Vishny (2015). Banks as patient fixed-income investors. *Journal of Financial Economics* 117, 449–469.
- Ivashina, V. and D. Scharfstein (2010). Bank lending during the financial crisis of 2008. *Journal of Financial Economics* 97(3), 319 – 338. The 2007-8 financial crisis: Lessons from corporate finance.
- Iyer, R., J.-L. Peydró, S. da Rocha-Lopes, and A. Schoar (2013). Interbank Liquidity Crunch and the Firm Credit Crunch: Evidence from the 2007–2009 Crisis. *The Review of Financial Studies* 27(1), 347–72.
- Jasova, M., C. Mendicino, and D. Supera (2018, January). Rollover risk and bank lending behavior: Evidence from unconventional central bank liquidity. mimeo.
- Jiménez, G., S. Ongena, J.-L. Peydró, and J. Saurina (2012). Credit supply and monetary policy: Identifying the bank balance-sheet channel with loan applications. *American Economic Review* 102(5), 2301–2326.
- Jiménez, G., S. Ongena, J.-L. Peydró, and J. Saurina (2014, March). Hazardous times for monetary policy: What do twenty-three million bank loans say about the effects of monetary policy on credit risk-taking? *Econometrica* 82(2), 463–505.
- König, P. (2015). Liquidity requirements: A double-edged sword. *International Journal of Central Banking* 11(4), 129–68.
- Krishnamurthy, A. (2010). How debt markets have malfunctioned in the crisis. *Journal of Economic Perspectives* 24, 3–28.

- Leonello, A., C. Mendicino, E. Panetti, and D. Porcellacchia (2022). Savings, efficiency and the nature of bank runs. Technical Report 2636, European Central Bank.
- Li, Z. and K. Ma (2021). Interbank market freezes and creditor runs. *Accepted at Management Science*.
- Liu, X. (2016). Interbank market freezes and creditor runs. *The Review of Financial Studies* 29, 1860–1910.
- Martinez-Miera, D. and R. Repullo (2017). Search for yield. *Econometrica* 85, 351–378.
- Morris, S. and H. Shin (2003). Global games: Theory and applications. In M. Dewatripont, L. Hansen, and S. Turnovsky (Eds.), *Advances in Economics and Econometrics (Proceedings of the 8th World Congress of the Econometric Society)*. Cambridge University Press.
- Morris, S. and H. S. Shin (2006). Catalytic finance: When does it work? *Journal of International Economics* 70, 161 – 177.
- Peek, J. and E. Rosengreen (2013). The role of banks in the transmission of monetary policy. Technical Report 13-5, Federal Reserve Bank of Boston.
- Perignon, C., D. Thesmar, and G. Vuillemey (2018). Wholesale funding dry-ups. *The Journal of Finance* 73(2), 575–617.
- Powell, J. H. (2019). Monetary policy: Normalization and the road ahead. Speech at the 2019 SIEPR Economic Summit, Stanford Institute of Economic Policy Research, Stanford, California.
- Rochet, J.-C. and X. Vives (2004). Coordination failures and the Lender of Last Resort: was Bagehot right after all? *Journal of the European Economic Association* 2(6), 1116–47.
- Stein, J. (1998). An adverse selection model of bank asset and liability management with implications for the transmission of monetary policy. *RAND Journal of Economics* 29(3), 466–486.

- Stein, J. and A. Kashyap (1995). The impact of monetary policy on bank balance sheets. *Carnegie Rochester Conference Series on Public Policy* 42, 151–195.
- Stein, J. C. and A. K. Kashyap (2000, June). What Do a Million Observations on Banks Say about the Transmission of Monetary Policy? *American Economic Review* 90(3), 407–428.
- van den Heuvel, S. (2012). Banking conditions and the effects of monetary policy: Evidence from u.s. states. *The B.E. Journal of Macroeconomics* 12(5), Article 5.
- Vives, X. (2005). Complementarities and games: New developments. *Journal of Economic Literature* 43, 437–479.
- Vives, X. (2014). Strategic complementarity, fragility, and regulation. *Review of Financial Studies* 27(12), 3547–92.

# A Proofs

## A1 Proof of Proposition 1 and Lemma 1

The proof is in two steps. First, we show that the failure threshold based on condition (1) implies the existence of well-defined dominance regions. In these regions, fund managers have a dominant action to roll over or withdraw irrespective of the actions of other managers. Suppose that all debt is rolled over,  $\ell = 0$ . The bank still fails if  $A > \bar{A} = RI - FD$ , so withdrawing is a dominant strategy for fund managers for  $x > \bar{A} + \epsilon$ . Similarly, suppose that no debt is rolled over,  $\ell = 1$ . The bank still does not fail if  $A < \underline{A} \equiv RI - \frac{FD}{\psi}$ . Thus, rolling over is a dominant strategy for  $x < \underline{A} - \epsilon$ . For  $A \in [\underline{A}, \bar{A}]$ , whether the bank fails depends on the withdrawal proportion of fund managers. If the realization of  $A$  was common knowledge, the model would exhibit multiple self-fulfilling equilibria (Figure 6).

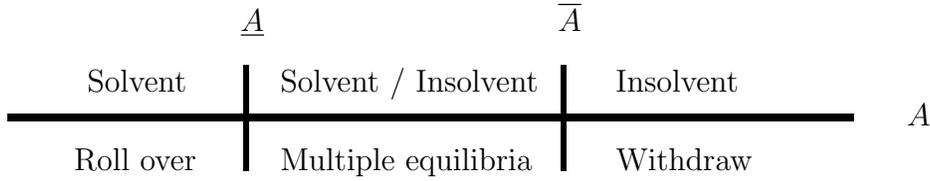


Figure 6: Tripartite classification of the balance sheet shock

Second, we characterize the equilibrium under incomplete information. Suppose fund managers use a symmetric threshold strategy around  $x^*$ . For a given realization  $A \in [\underline{A}, \bar{A}]$ , the proportion of fund managers who do not roll over debt is  $\ell(A, x^*) = \text{Prob}(x_i > x^* | A) = \text{Prob}(\epsilon_i > x^* - A) = 1 - H(x^* - A)$ . Using (1), the failure threshold  $A^*$  solves

$$A^* = RI - (1 + z\ell(A^*, x^*))FD. \quad (\text{A1})$$

For given  $x^*$ , the left-hand side of (A1) is strictly increasing in  $A^*$  and rises from  $-\infty$  to  $\infty$ . The right-hand side is strictly decreasing in  $A^*$  and is bounded above and below by  $(1 + z)FD$  and  $FD$ , respectively. Hence, there exists a unique failure threshold  $A^*$ .

The posterior distribution of the shock conditional on the private signal can be derived using Bayes' rule. The threshold  $x^*$  is pinned down by indifference between rolling over and withdrawing of a fund manager who observes  $x_i = x^*$ :  $1 - \gamma = \Pr(A > A^* | x_i = x^*)$ . For small  $\epsilon$ , the latter can be written as  $\gamma = \Pr(A < A^* | x_i = x^*) = 1 - H(x^* - A^*)$ . The indifference condition therefore implies  $x^* - A^* = H^{-1}(1 - \gamma)$ . Inserting it into  $\ell(A^*, x^*)$ , the withdrawal proportion at the threshold  $A^*$  becomes  $\ell(A^*, x^*) = 1 - H(x^* - A^*) = 1 - H(H^{-1}(1 - \gamma)) = \gamma$ . The failure threshold  $A^*$  stated in Proposition 1 follows immediately.

The fact that there are no other equilibria in non-threshold strategies follows from standard arguments (Morris and Shin, 2003; Vives, 2005). It exploits the fact that the regions above  $\bar{A}$  and below  $\underline{A}$  admit equilibria in strictly dominated strategies. Finally, the partial derivatives of  $A^*$  are immediate:  $\frac{\partial A^*}{\partial F} = -(1 + \gamma z)D < 0$  and  $\frac{\partial A^*}{\partial D} = R - (1 + \gamma z)F$ .

## A2 Proof of Proposition 2

The bank's problem is given by (4). Denote by  $\mu$  the Lagrange multiplier of the investor participation constraint. The first-order conditions for an interior optimum are:

$$D : \quad \frac{\partial \mathcal{L}}{\partial D} = (R - F)G(A^*) + (\mu F + FD\gamma z)g(A^*)\frac{\partial A^*}{\partial D} = 0 \quad (\text{A2})$$

$$F : \quad \frac{\partial \mathcal{L}}{\partial F} = (\mu - D)G(A^*) + (\mu F + FD\gamma z)g(A^*)\frac{\partial A^*}{\partial F} = 0 \quad (\text{A3})$$

$$\mu : \quad \frac{\partial \mathcal{L}}{\partial \mu} = -r + FG(A^*) = 0 \quad (\text{A4})$$

Combining equations (A2) and (A3) yields  $\mu^* = D + \frac{(R-F)\frac{\partial A^*}{\partial F}}{\frac{\partial A^*}{\partial D}}$ . Substituting  $\mu^*$  into (A2) yields

$$\frac{G(A^*)}{g(A^*)} = \frac{\gamma z F D (-\frac{\partial A^*}{\partial D} + R)}{R - F} = \frac{\gamma z (1 + \gamma z) F^2 D}{R - F}. \quad (\text{A5})$$

Equations (A4) and (A5) pin down the interior critical point  $(F^*, D^*)$ . An interior optimum requires  $R/(1 + \gamma z) < F < R$ . Note that (A3) becomes strictly negative if  $F \geq R$ . Similarly,  $\frac{\partial A^*}{\partial D} > 0$  if  $F < R/(1 + \gamma z)$ , implying that (A2) becomes strictly positive.

The critical point  $(F^*, D^*)$  constitutes a maximum as the bordered Hessian, evaluated at  $(F^*, D^*)$ , is strictly positive. To see this, note that the second derivatives are given by

$$\frac{\partial^2 \mathcal{L}}{\partial D \partial F} = (\mu - D) \frac{\partial G(A^*)/g(A^*)}{\partial A^*} \frac{\partial A^*}{\partial D} - \frac{G(A^*)}{g(A^*)} + (\mu + \gamma z D) F \frac{\partial^2 A^*}{\partial F \partial D} + \gamma z F \frac{\partial A^*}{\partial F} \quad (\text{A6})$$

$$\frac{\partial^2 \mathcal{L}}{\partial D^2} = (R - F) \frac{\partial G(A^*)/g(A^*)}{\partial A^*} \frac{\partial A^*}{\partial D} + \gamma z F \frac{\partial A^*}{\partial D} \quad (\text{A7})$$

$$\frac{\partial^2 \mathcal{L}}{\partial F^2} = (\mu - D) \frac{\partial G(A^*)/g(A^*)}{\partial A^*} \frac{\partial A^*}{\partial F} + (\mu + \gamma z D) \frac{\partial A^*}{\partial F} \quad (\text{A8})$$

Since  $\mu^* > D$  and  $\frac{R}{1+\gamma z} < F < R$ , equations (A6) to (A8) are all negative. Using that the participation constraint is  $V(F, D) = FG(A^*) - r$ , we further obtain:

$$\frac{\partial V}{\partial F} = G(A^*) + Fg(A^*) \frac{\partial A^*}{\partial F}, \quad \frac{\partial V}{\partial D} = Fg(A^*) \frac{\partial A^*}{\partial D}. \quad (\text{A9})$$

Thus,  $\frac{\partial V}{\partial D}$  is negative for  $F > R/(1 + \gamma z)$ , while  $\frac{\partial V}{\partial F}$  is positive when evaluated at  $(F^*, D^*)$ . To see this, we can express  $\frac{\partial V}{\partial F} = g(A^*) \left[ \frac{G(A^*)}{g(A^*)} + F \frac{\partial A^*}{\partial F} \right]$  and substitute (A5) for  $G/g$  to get:

$$\frac{\partial V}{\partial F} = g(A^*) \left[ \frac{FD(1 + \gamma z)}{R - F} (F(1 + \gamma z) - R) \right] > 0$$

since  $F > R/(1 + \gamma z)$ . Hence, the bordered Hessian matrix is

$$H = \begin{vmatrix} 0 & \frac{\partial V}{\partial F} & \frac{\partial V}{\partial D} \\ \frac{\partial V}{\partial F} & \frac{\partial^2 \mathcal{L}}{\partial D^2} & \frac{\partial^2 \mathcal{L}}{\partial D \partial F} \\ \frac{\partial V}{\partial D} & \frac{\partial^2 \mathcal{L}}{\partial D \partial F} & \frac{\partial^2 \mathcal{L}}{\partial F^2} \end{vmatrix} > 0.$$

Finally, note that for  $F^* \rightarrow R$ , the left-hand side of (A5) converges to  $+\infty$  while the right-hand side remains bounded, implying that  $D^* \rightarrow 0$ . For  $F < R/(1 + \gamma z)$ , the first-order necessary conditions imply that  $D^* = \omega$ . Using the participation constraint, we can derive bounds on  $r$ , given by  $\underline{r} \equiv RG(0)/(1 + \gamma z)$  and  $\tilde{r} \equiv RG(0)$ , such that for  $r \in (\underline{r}, \tilde{r})$ , it must be that  $F \in (R/(1 + \gamma z), R)$ . Clearly, if  $r > \tilde{r}$ , then  $F > R$  and if  $r < \underline{r}$ , then  $F < R/(1 + \gamma z)$ . This completes the characterization of the bank's optimal choice.

### A3 Proof of Propositions 4 – 5

Let the vector of parameters be  $\mathbf{v} = (r, \psi)$ . The first-order condition for borrowing is

$$\phi(D, F; \mathbf{v}) \equiv \frac{G(A^*)}{g(A^*)}(R - F) - \gamma z F D \left( -\frac{\partial A^*}{\partial D} + R \right) = 0, \quad (\text{A10})$$

with

$$\frac{\partial \phi}{\partial D} = \frac{\partial A^*}{\partial D} \frac{d(G/g)}{dA^*} (R - F) + \left( \frac{\partial A^*}{\partial D} - R \right) \gamma z F < 0$$

and

$$\frac{\partial \phi}{\partial F} = \frac{\partial A^*}{\partial F} \left( \frac{d(G/g)}{dA^*} (R - F) + 2\gamma z F \right) - \frac{G}{g} < 0.$$

We apply the implicit function theorem (IFT) to  $\phi(D^*, F^*; \mathbf{v}) = 0$  and the participation constraint  $V(D^*, F^*; \mathbf{v}) = 0$ . The Jacobian of this system of equations is

$$\mathbf{J} = \begin{pmatrix} \frac{\partial V(D^*, F^*; \mathbf{v})}{\partial F} & \frac{\partial V(D^*, F^*; \mathbf{v})}{\partial D} \\ \frac{\partial \phi(D^*, F^*; \mathbf{v})}{\partial F} & \frac{\partial \phi(D^*, F^*; \mathbf{v})}{\partial D} \end{pmatrix}. \quad (\text{A11})$$

At the equilibrium point  $(D^*, F^*)$ , the determinant of the Jacobian is negative,  $|\mathbf{J}| < 0$ , since all entries are negative except for  $\frac{\partial V(D^*, F^*; \mathbf{v})}{\partial F} > 0$ . Henceforth, we evaluate all expressions at the equilibrium but suppress the arguments of the derivatives for brevity.

**Real interest rate.** With  $\partial V / \partial r = -1$  and  $\frac{\partial \phi}{\partial r} = 0$ , the IFT yields

$$\frac{dD^*}{dr} = -\frac{\begin{vmatrix} \frac{\partial V}{\partial F} & -1 \\ \frac{\partial \phi}{\partial F} & 0 \end{vmatrix}}{|\mathbf{J}|} < 0, \quad \frac{dF^*}{dr} = -\frac{\begin{vmatrix} -1 & \frac{\partial V}{\partial D} \\ 0 & \frac{\partial \phi}{\partial D} \end{vmatrix}}{|\mathbf{J}|} > 0.$$

**Liquidation value.** The parameter  $\psi$  appears via  $z$ , i.e.,  $z \equiv \frac{1-\psi}{\psi}$  with  $dz/d\psi < 0$ . Since  $\frac{\partial A^*}{\partial z} = -\gamma F D < 0$  and  $\frac{\partial^2 A^*}{\partial D \partial z} = -\gamma F < 0$ , we have  $\frac{\partial V}{\partial z} = F g \frac{\partial A^*}{\partial z} < 0$  and  $\frac{\partial \phi}{\partial z} = (R - F) \frac{d(G/g)}{dA^*} \frac{\partial A^*}{\partial z} + \gamma F D \left( \frac{\partial A^*}{\partial D} - R \right) + \gamma z F \frac{\partial A^*}{\partial z} < 0$ . The IFT yields  $\frac{dD^*}{dz} < 0$  and because  $z$

depends negatively on  $\psi$ , we have  $\frac{dD^*}{d\psi} > 0$ . The sign of  $\frac{dF^*}{dz}$  is ambiguous:

$$\frac{dF^*}{dz} = -\frac{\left(\frac{\partial V}{\partial z} \frac{\partial \phi}{\partial D} - \frac{\partial V}{\partial D} \frac{\partial \phi}{\partial z}\right)}{|\mathbf{J}|} = \frac{g(A^*)(F^*)^2(1+\gamma z)^2 \frac{\partial A^*}{\partial z} \left(\frac{R(1+2\gamma z)}{(1+\gamma z)^2} - F^*\right)}{|\mathbf{J}|} \gtrless 0 \Leftrightarrow F^* \lesseqgtr \bar{F} \quad (\text{A12})$$

where

$$\bar{F} \equiv \frac{R(1+2\gamma z)}{(1+\gamma z)^2}. \quad (\text{A13})$$

Observe that  $\bar{F} \in \left(\frac{R}{1+\gamma z}, R\right)$ . Evaluating (A5) at  $\bar{F}$  yields the corresponding debt level  $\bar{D}$ . Evaluating the participation constraint at  $(\bar{F}, \bar{D})$  yields  $\bar{r} \in (\underline{r}, \bar{r})$  which solves  $V(\bar{D}, \bar{F}; \bar{r}) = 0$ .  $\bar{F}$  and  $\bar{D}$  are independent of  $r$ . Since  $F^*$  strictly increases in  $r$ , it follows that  $\frac{dF^*}{d\gamma} < 0$  if and only if  $r > \bar{r}$ , where  $\bar{r} > \underline{r}$ . Since  $z$  depends negatively on  $\psi$ , we have  $\frac{dF^*}{d\psi} \gtrless 0 \Leftrightarrow r \lesseqgtr \bar{r}$ .

**Total effects on fragility.** The total effects of  $\mathbf{v}$  on fragility can be computed directly from the participation constraint  $FG(A^*) - r = 0$ . We have

$$\frac{dA^*}{d\psi} = -\frac{G(A^*)}{g(A^*)} \frac{dF^*}{F^*} \quad \text{and} \quad \frac{dA^*}{dr} = -\frac{G(A^*)}{g(A^*)} \frac{\frac{dF^*}{dr} - \frac{1}{F^*}}{F^*}.$$

It follows immediately from (A12) and the previous result that  $\frac{dA^*}{d\psi} > 0 \Leftrightarrow r < \bar{r}$ . Next, consider  $\frac{dA^*}{dr}$ . Substituting for  $dF^*/dr$  (using the previous results) yields

$$\begin{aligned} \frac{dA^*}{dr} &\propto \frac{\partial \phi}{\partial D} - \frac{|\mathbf{J}|}{G(A^*)} = -\frac{\partial A^*}{\partial F} \frac{\partial \phi}{\partial D} + \frac{\partial A^*}{\partial D} \frac{\partial \phi}{\partial F} \\ &= \frac{\partial A^*}{\partial F} \left(-\frac{\partial A^*}{\partial D} + R\right) \gamma z F^* + 2\gamma z F^* \frac{\partial A^*}{\partial F} \frac{\partial A^*}{\partial D} - \gamma z F^* D^* \frac{\partial A^*}{\partial D} \frac{-\frac{\partial A^*}{\partial D} + R}{R - F^*} \\ &= \frac{\gamma z F^*}{R - F^*} \left(-\frac{\partial A^*}{\partial D} + R\right) \left(\frac{\partial A^*}{\partial F} (R - F^*) - D^* \frac{\partial A^*}{\partial D}\right) + 2\gamma z F^* \frac{\partial A^*}{\partial D} \frac{\partial A^*}{\partial F} \\ &= \frac{\gamma z F^* (1 + \gamma z) F^*}{R - F^*} \left(\frac{\partial A^*}{\partial F} (R - F^*) - D^* \frac{\partial A^*}{\partial D} - \frac{2D^* (R - F^*)}{F^*} \frac{\partial A^*}{\partial D}\right) \\ &\propto RD^* (F^* + (1 + \gamma z) F^* - 2R) = (2 + \gamma z) RD^* (F^* - \underline{F}), \end{aligned}$$

where

$$\underline{F} \equiv \frac{R}{1 + \frac{\gamma z}{2}}. \quad (\text{A14})$$

Note that the first line follows by substituting the Jacobian, the second line follows from using optimality condition (A5) to substitute for  $G(A^*)/g(A^*)$  and substituting the expressions for  $\partial\phi/\partial F$  and  $\partial\phi/\partial D$  and the fifth line follows after substituting the expressions for  $\partial A^*/\partial D$  and  $\partial A^*/\partial F$  and re-arranging.

Observe that  $\underline{F} \in (R/(1+\gamma z), R)$ . Evaluating (A5) at  $\underline{F}$  yields the corresponding debt level  $\underline{D}$ . Evaluating the participation constraint at  $(\underline{F}, \underline{D})$ , we obtain a critical value  $\underline{r} \in (\underline{r}, \tilde{r})$  that solves  $V(\underline{D}, \underline{F}; \underline{r}) = 0$ . Observe further that  $\underline{F}$  and  $\underline{D}$  are independent of  $r$ . Since  $F^*$  strictly increases in  $r$ , it follows that  $F^* > \underline{F}$  and therefore  $\frac{dA^*}{dr} > 0$  if and only if  $r > \underline{r}$ .

## A4 Proof of Proposition 6

The failure threshold follows from substituting  $I = D + E$  in equation 3. Moreover, the proof of Proposition 2 continues to apply except for the generalized bounds  $\underline{r}_E$  and  $\tilde{r}_E$  because  $E$  enters  $A^*$  and  $E_2$  linearly, so the first-order conditions in Appendix A2 hold. To calculate the bounds  $\underline{r}_E$  and  $\tilde{r}_E$ , observe that for  $F = \frac{R}{1+\gamma z}$ , the first-order conditions still imply  $D^* = \omega$  and thus  $A^* = RE$ . The investor participation constraint implies  $\underline{r}_E = \frac{RG(RE)}{1+\gamma z}$ . By the same logic, for  $F \geq R$ , the FOCs imply  $D^* = 0$  and thus  $\tilde{r}_E = RG(RE)$ . Hence  $\underline{r} < \underline{r}_E$  and  $\tilde{r} < \tilde{r}_E$  because  $G(\cdot)$  monotonically increases. The comparative statics of these bounds with respect to equity follow immediately.

To compute the comparative statics of  $(D^*, F^*, A^*)$  w.r.t. equity, note that  $E$  enters only via  $A^*$ , so equations (A10) and (A11) still apply. The partial derivatives with respect

to  $E$  are given by  $\frac{\partial \phi}{\partial E} = \frac{d(G/g)}{dA^*} \frac{\partial A^*}{\partial E} (R - F)$  and  $\frac{\partial V}{\partial E} = gF \frac{\partial A^*}{\partial E}$ . Hence:

$$\frac{dD^*}{dE} = -\frac{\begin{vmatrix} \frac{\partial V}{\partial F} & \frac{\partial V}{\partial E} \\ \frac{\partial \phi}{\partial F} & \frac{\partial \phi}{\partial E} \end{vmatrix}}{|\mathbf{J}|} > 0, \quad \frac{dF^*}{dE} = -\frac{\begin{vmatrix} \frac{\partial V}{\partial E} & \frac{\partial V}{\partial D} \\ \frac{\partial \phi}{\partial E} & \frac{\partial \phi}{\partial D} \end{vmatrix}}{|\mathbf{J}|} < 0,$$

where the sign of the latter expressions follows from  $\frac{\partial V}{\partial E} \frac{\partial \phi}{\partial D} - \frac{\partial V}{\partial D} \frac{\partial \phi}{\partial E} = -g(F^*)^2 \gamma z (1 + \gamma z) R < 0$ . Finally, as  $E$  enters the participation constraint only via  $A^*$ , we have  $\frac{dA^*}{dE} = -\frac{G}{gF} \frac{dF^*}{dE} > 0$ .

## A5 Bank fragility and market liquidity

We can decompose the impact of a marginal increase in  $\psi$  on fragility as follows:

$$\frac{dA^*}{d\psi} = \underbrace{\frac{1}{1 + \frac{\partial A^*}{\partial F} \frac{g(A^*)}{G(A^*)} F^*}}_{\text{Amplification effect (+)}} \times \left( \underbrace{\frac{\partial A^*}{\partial D} \frac{dD^*}{d\psi}}_{\text{Scale effect (-)}} + \underbrace{\frac{\partial A^*}{\partial \psi}}_{\text{Price effect (+)}} \right). \quad (\text{A15})$$

In contrast to  $dA^*/dr$ , the scale effect increases fragility as  $\frac{dD^*}{d\psi} > 0$ , while the price effect reduces fragility as  $\frac{\partial A^*}{\partial \psi} > 0$ . The price effect is proportional to total funding costs since  $\partial A^*/\partial \psi = -\gamma F D/\psi^2$ . Thus, its magnitude becomes larger. This implies that a larger face value is required to equalize scale and price effects. Thus the critical threshold becomes larger,  $\bar{r} > r$ .